Operators from H^p to ℓ^q for $0 < p < 1 \le q < \infty$

Oscar Blasco

Abstract. We give some estimates for the norm of operators $T : H^p \to \ell^q$ for $0 < p < 1 \le q < \infty$ in terms of the norm of the rows and columns of the matrix $T(u_n) = (t_{kn})_{k\in\mathbb{N}}$, $u_n(z) = z^n$, in certain vector-valued sequence spaces.

1. Introduction

Throughout the paper X stands for a quasi-Banach space and we denote, for $1 \le s < \infty$ and $1/s + 1/s' = 1$, by $\ell^s(X)$, $\ell^s_{weak}(X)$ and $\ell(s,\infty,X)$, the spaces of sequences $(A_k) \subset X$ such that

$$\|(A_k)\|_{\ell^s(X)} = (\sum_k \|A_k\|^s)^{1/s} < \infty,$$

$$\|(A_k)\|_{\ell^s_{weak}(X)} = \sup_{\|(\lambda_k)\|_{s'}=1} \|\sum_k \lambda_k A_k\| < \infty \text{ and}$$

$$\|(A_k)\|_{\ell(s,\infty,X)} = \sup_{j\in\mathbb{N}} (\sum_{n=2^{j-1}-1}^{2^j} \|A_n\|^s)^{1/s} < \infty.$$

We write $\ell^s, \ell(s,\infty)$ in the case $X = \mathbb{C}$. Of course $\ell^s(X) \subset \ell(s,\infty,X) \cap \ell^s_{weak}(X)$.

For each $0 < p \le \infty$, H^p denotes the Hardy space on the unit disk, i.e. space of holomorphic functions on $\mathbb{D}$ such that $\sup_{0<r<1} \|f_r\|_{L^p(\mathbb{T})} < \infty$ where $f_r(z) = f(rz)$. For a given bounded operator $T : H^p \to \ell^q$, $0 < p, q \le \infty$, one can associate the matrix $(t_{kn})_{k,n}$ such that $T(u_n) = \sum_{k\in\mathbb{N}} t_{kn} e_k$, where $u_n(z) = z^n$ for $n \ge 0$. Let $T_k = (t_{kn})_{n\ge 0}$ and $x_n = (t_{kn})_{k\in\mathbb{N}}$ denote its rows and columns respectively.

Several theorems concerning upper and lower estimates of the norm $\|T\|$ in terms of

$$\|(T_k)\|_{\ell^r(\ell^s)} = (\sum_{k=1}^{\infty}(\sum_{n=0}^{\infty} |t_{kn}|^s)^{r/s})^{1/r}$$

were proved by B. Osikiewicz. Let me collect the results in [**14**] using the notation $a^+ = max\{a, 0\}$.

1991 *Mathematics Subject Classification.* Primary 47B38 ; Secondary 30D55, 47A30, 47B37.
Key words and phrases. Hardy spaces, vector-valued sequence spaces.
The author was supported in part by Proyecto MTM2005-08350-C03-03.

In the case $1 \leq p \leq 2$, $1 \leq q \leq \infty$ and $1/r = (1/q - 1/2)^+$ it was shown (see [**14**], Theorem 2.1 and Theorem 2,2) that

$$\|(T_k)\|_{\ell^r(\ell^2)} \leq \|T\| \leq \|(T_k)\|_{\ell^q(\ell^p)}. \tag{1.1}$$

Also for $2 \leq p < \infty$, $1 \leq q \leq \infty$ and $1/s = (1/q - 1/p')^+$ it was shown (see [**14**], Theorem 2.3 and Theorem 2,4) that

$$\|(T_k)\|_{\ell^s(\ell^p)} \leq \|T\| \leq \|(T_k)\|_{\ell^q(\ell^2)}. \tag{1.2}$$

The reader is referred to [**5**] for some improvements of these results. The objective of this note is to study the case $0 < p < 1$.

The main result is the following:

THEOREM 1.1. *Let $0 < p < 1 \leq q < \infty$ and $T : H^p \to \ell^q$ be a bounded operator. Define the matrix $(a_{kn}) = ((n+1)^{1/p-1} t_{nk})$ and set A_k and B_n the rows and columns of the matrix. There exists $C > 0$ such that*

$$\|T\| \leq C \min\{\|(A_k)\|_{\ell^q(\ell(1,\infty))}, \|(B_n)\|_{\ell(1,\infty,\ell^q)}\} \tag{1.3}$$

and

$$\max\{\|(A_k)\|_{\ell^q_{weak}(\ell(2,\infty))}, \|(B_n)\|_{\ell(q_0,\infty,\ell^q)}\} \leq C\|T\|, \tag{1.4}$$

where $q_0 = \max\{q, q'\}$

Let us write down the just mentioned result in the particular cases $q = 1$ and $q = 2$.

COROLLARY 1.2. Let $0 < p < 1$ and $T : H^p \to \ell^1$ be a bounded operator. Let A_k and B_n the rows and columns of the matrix $(a_{kn}) = ((n+1)^{1/p-1} t_{nk})$. There exists $C > 0$ such that

$$C^{-1} \max\{\|(B_n)\|_{\ell^\infty(\ell^1)}, \sup_{\|(\lambda_k)\|_\infty = 1} \|\sum_k \lambda_k A_k\|_{\ell(2,\infty)}\} \leq \|T\| \leq C\|(B_n)\|_{\ell(1,\infty,\ell^1)}.$$

COROLLARY 1.3. Let $0 < p < 1$ and $T : H^p \to \ell^2$ be a bounded operator. Let A_k and B_n the rows and columns of the matrix $(a_{kn}) = ((n+1)^{1/p-1} t_{nk})$. There there exists $C > 0$ such that

$$C^{-1}\|(A_k)\|_{\ell^2_{weak}(\ell(2,\infty))} \leq \|T\| \leq C\|(A_k)\|_{\ell^2(\ell(1,\infty))}, \tag{1.5}$$

$$C^{-1}\|(B_n)\|_{\ell(2,\infty,\ell^2)} \leq \|T\| \leq C\|(B_n)\|_{\ell(1,\infty,\ell^2)}. \tag{1.6}$$

We shall now recall some facts to be used in the sequel.

Let us first mention the following duality result (see [**8**]): Let $0 < p < 1$ and $1/m + 1 \leq p < 1/m$, $m \in \mathbb{N}$. $\Phi \in (H^p)^*$ if and only if there exist a function g and a constant $C > 0$ such that

$$|g^{(m+1)}(z)| \leq \frac{C}{(1 - |z|)^{m+2-1/p}}, \tag{1.7}$$

for which

$$\Phi(f) = \lim_{r \to 1} \int_0^{2\pi} f(re^{it})\overline{g(re^{it})} \frac{dt}{2\pi}$$

for all $f \in H^p$.

Moreover

$$\|\Phi\|_{(H^p)^*} \approx \max\{|g(0)|, |g'(0)|, ..., |g^m(0)|, \sup_{|z|<1} (1 - |z|)^{m+2-1/p} |g^{(m+1)}(z)|\}.$$

Throughout the paper we identify g and Φ.

The estimates in Theorem 1.1 and Corollaries 1.2 and 1.3 are, in some special cases, sharp and allow to give some consequences on Taylor coefficient of functions in H^p-spaces for $0 < p < 1$ in some other cases.

REMARK 1.4. Given $g \in (H^p)^*$ and $(\lambda_k) \in \ell^q$ define $T : H^p \to \ell^q$ by

$$T(f) = \langle g, f\rangle(\lambda_k)_k.$$

Obviously has

$$\|T\| = \|g\|_{(H^p)^*}\|(\lambda_k)\|_{\ell^q}.$$

This example corresponds to the case $(t_{nk}) = (\alpha_n\lambda_k)$ where $g(z) = \sum_{n=0}^{\infty}\alpha_n z^n$, $B_n = (n+1)^{1/p-1}\alpha_n(\lambda_k)_k$ and $A_k = \lambda_k((n+1)^{1/p-1}\alpha_n)_n$. The reader can compare the norm with the estimates from Theorem 1.1 in this case.

REMARK 1.5. Let $0 < p < 1$, (λ_n) be a sequence and $T : H^p \to \ell^1$ given by

$$T(f) = (\lambda_n a_n), \quad f(z) = \sum_{n=0}^{\infty} a_n z^n.$$

Then $\|T\| \approx \|(B_n)\|_{\ell(1,\infty,\ell^1)} = \|((n+1)^{1/p-1}\lambda_n)\|_{\ell(1,\infty)}$.

Indeed, note that in this case $B_n = (n+1)^{1/p-1}\lambda_n e_n$ and (t_{kn}) is a diagonal matrix. Hence T is bounded if and only if $\{(\lambda_n)_n\}$ there exists $C > 0$

$$\sum_{n=0}^{\infty}|\lambda_n a_n| \leq C\|\sum_{n=0}^{\infty} a_n z^n\|_{H^p},$$

that is to say (λ_n) belongs to the space of multipliers (H^p, ℓ^1). Now invoke the result by P. Duren and A. Shields (see [**9**]) establishing that for $0 < p < 1$

$$(H^p, \ell^1) = \{(\lambda_n) : ((n+1)^{1/p-1}\lambda_n) \in \ell(1,\infty)\}, \tag{1.8}$$

with equivalent norms, to get the desired result.

Let us give the following new application of Corollary 1.3.

COROLLARY 1.6. Let $0 < r < \frac{2}{3}$ and let $g(z) = \sum_{n=0}^{\infty}\alpha_n z^n \in (H^r)^*$. Then there exists $C > 0$ such that

$$C^{-1}\|((n+1)^{1/r-3/2}(\sum_{j\geq n}|\alpha_j|^2)^{1/2})_n\|_{\ell(2,\infty)} \leq \|g\|_{(H^r)^*},$$

$$\|g\|_{(H^r)^*} \leq C\|((n+1)^{1/r-3/2}(\sum_{j\geq n}|\alpha_j|^2)^{1/2})_n\|_{\ell(1,\infty)}.$$

PROOF. Condition $0 < r < \frac{2}{3}$ allows to get $0 < p < 1$ such that $1/p+1/2 = 1/r$. Using factorization of Hardy spaces (see [**7**]) one has that $H^r = H^pH^2$. Consider now the operator $T : H^p \to \ell^2$ defined by the matrix $(t_{nk}) = (\bar{\alpha}_{n+k})$, in other words

$$T(f) = (\sum_{n=0}^{\infty} a_n\bar{\alpha}_{n+k})_k.$$

Clearly, if $(\beta_k)_k \in \ell^2$ is a finite sequence and $h(z) = \sum_{k=0}^{N}\bar{\beta}_k z^k$ apolynomial then

$$\langle T(f), (\beta_k)\rangle = \sum_{n,k}\bar{\alpha}_{n+k}a_n\beta_k = \int_{\mathbb{T}}\bar{g}(\xi)f(\xi)h(\xi)d\xi.$$

Using the factorization $H^r = H^p H^2$ we easily conclude that $\|T\| = \|g\|_{(H^r)^*}$. Using now that $\|B_n\|_{\ell^2} = (n+1)^{1/r-3/2}(\sum_{j\geq n}|\alpha_j|^2)^{1/2}$, the result follows from (1.6). □

The paper is organized as follows. Section 2 contains some preliminary and introductory results and Section 3 is devoted to the proof of Theorem 1.1.

Throughout the paper we use the notation $M_q(F,r) = (\int_0^{2\pi}\|F(re^{i\theta})\|^q \frac{d\theta}{2\pi})^{1/q}$ for analytic functions $F : \mathbb{D} \to X$, q' stands for the conjugate exponent of q and, as usual, the constant C may vary from line to line.

2. Preliminary results

Definition 2.1. Let $0 < p < 1 \leq q \leq \infty$ and let $T : H^p \to \ell^q$ be a bounded operator. Denote $u_n(z) = z^n$ for $n \geq 0$, $(e_k)_{k\in\mathbb{N}}$ the standard basis of ℓ^q and $\xi_k((\lambda_j)) = \langle(\lambda_j), e_k\rangle = \lambda_k$.

Consider the functional $\xi_k T(f) = \langle T(f), e_k\rangle \in (H^p)^*$ and denote by g_k the analytic function representing $\xi_k T$. Assume

$$g_k(z) = \sum_{n=0}^{\infty} t_{kn} z^n. \tag{2.1}$$

We can now define

$$F_T(z) = (g_k(z))_{k\in\mathbb{N}} \tag{2.2}$$

Hence to each operator T we can associate a matrix $(a_{kn}(T)) = (t_{kn})$ given by

$$T(u_n) = \sum_{k=1} t_{kn} e_k \tag{2.3}$$

where the rows $T_k = (t_{kn})_{n\geq 0}$ are the Taylor coefficients of the sequence of functions $g_k = g_k(T) \in (H^p)^*$ and the columns $x_n = (t_{kn})_{k\in\mathbb{N}}$ are the Taylor coefficients of the vector-valued analytic function $F_T : \mathbb{D} \to \ell_q$ given by and

$$F_T(z) = \sum_{n=0}^{\infty} x_n z^n, \; x_n = \sum_{k=1}^{\infty} t_{kn} e_k. \tag{2.4}$$

It is well known that the boundedness of operators $T : H^p \to X$, where X is a Banach space and $0 < p < 1$, is equivalent to the boundedness of its extension $T : B^p \to X$ where B^p is the Banach envelope of H^p (see [**8**]) and coincides with the space of analytic functions such that

$$\int_0^1 (1-r^2)^{1/p-2} M_1(f,r)dr < \infty.$$

Taking into account that B^p is a weighted Bergman space $B_1(\rho)$ for $\rho(t) = t^{1/p-1}$ and, due to the results in [**1**] (see also [**4**] for alternative approaches and more references), the boundedness of operators T from $B_1(\rho)$ into X can be described by the behavior of certain fractional derivative of the vector valued function whose Taylor coefficients $T(u_n) = x_n$ where $u_n(z) = z^n$. Therefore the theorem could be achieved using this general approach, but we present here a direct proof using only classical and elementary facts from Theory of Hardy spaces.

Let us now mention some facts which will be needed later on.

LEMMA 2.2. *(see [**7**]) Let $\beta > 0$ and let (α_n) be a sequence of non-negative numbers. Then*

$$\|((n+1)^{-\beta}\alpha_n)\|_{\ell(1,\infty)} \approx \|((n+1)^{-\beta}\sum_{j=0}^{n}\alpha_j)\|_{\ell^\infty} \approx \sup_{0<r<1}(1-r)^\beta(\sum_n \alpha_n r^n). \tag{2.5}$$

Let us mention that in some cases Haussdorff-Young's inequality holds for vector-valued Lebesgue spaces (see [**16**]). In particular, next lemma is well known.

LEMMA 2.3. *(see [**3**]) Let $1 < p \le 2$, $p \le q \le p'$ and let $F(z) = \sum_{n=0}^\infty x_n z^n$ with $x_n \in \ell^q$ for $n \ge 0$. Then*

$$(\sum_{n=0}^\infty \|x_n\|_{\ell^q}^{p'} r^{np'})^{1/p'} \le M_p(F,r).$$

3. Proof of Theorem 1.1

We shall need the following result.

LEMMA 3.1. *Let $0 < p < 1 \le q < \infty$, $1/m+1 \le p < 1/m$ for some $m \in \mathbb{N}$ and let $T : H^p \to \ell^q$ be a linear operator. Set $x_n = T(u_n)$ and $F_T(z) = \sum_{n=0}^\infty x_n z^n$. Then*

$$\|T\| \approx \max\{\|x_0\|_{\ell^q}, \|x_1\|_{\ell^q}, ..., \|x_m\|_{\ell^q}, \sup_{|z|<1}(1-|z|)^{m+2-1/p}\|F_T^{(m+1)}(z)\|_{\ell^q}\}.$$

PROOF. For each $(\lambda_k) \in \ell^{q'}$ we denote $T_\lambda(f) = \sum_{k=1}^\infty \lambda_k \xi_k T(f)$. We have that

$$\|T\| = \sup\{\|T_\lambda\|_{(H^p)^*} : \|(\lambda_k)\|_{\ell^{q'}} = 1\}.$$

Using (1.7) one has that $T_\lambda \in (H^p)^*$ if and only if it is represented by $g_\lambda = \sum_{k=1}^\infty \lambda_k g_k$ and there exists $C > 0$ such that $|g_\lambda^{(m+1)}(z)| \le \frac{C\|T_\lambda\|_{(H^p)^*}}{(1-|z|)^{m+2-1/p}}$, and

$$\|T_\lambda\|_{(H^p)^*} \approx \max\{|g_\lambda(0)|, |g'_\lambda(0)|, ..., |g_\lambda^{(m)}(0)|, \sup_{|z|<1}(1-|z|)^{m+2-1/p}|g_\lambda^{(m+1)}(z)|\}$$

Observe that $g_\lambda^{(j)}(z) = \sum_{k=1}^\infty \lambda_k g_k^{(j)}(z)$ and $F_T^{(j)}(z) = (g_k^{(j)}(z))$. Taking supremun over $\|(\lambda_k)\|_{\ell^{q'}} = 1$ one gets that the result. □

Proof of (1.3). Of course one can write

$$\|T\| \le (\sum_{k=1}^\infty \|\xi_k T\|^q)^{1/q} = (\sum_{k=0}^\infty \|g_k\|_{(H^p)^*}^q)^{1/q}.$$

Using the continuous inclusion $(H^p, \ell^1) \subset (H^p)^*$ and (1.8) we have the estimate $\|g_k\|_{(H^p)^*} \le \|g_k\|_{(H^p,\ell^1)} \le C\|A_k\|_{\ell(1,\infty)}$ where $A_k = ((n+1)^{1/p-1}t_{kn})_n$.
Therefore

$$\|T\| \le C\|(A_k)\|_{\ell^q(\ell(1,\infty))}. \tag{3.1}$$

On the other hand, note that $F_T(z) = \sum_{n=0}^\infty x_n z^n$ where $x_n = (t_{kn})_{k\in\mathbb{N}} \in \ell^q$ for all $n \ge 0$. Hence

$$\sup_{0\le k\le m}\|x_k\|_{\ell^q} \le \|((n+1)^{1/p-1}x_n\|_{\ell(1,\infty,\ell^q)}.$$

On the other hand

$$F_T^{(m+1)}(z) = \sum_{n=m+1}^\infty n(n-1)....(n-m)x_n z^{n-(m+1)}.$$

Hence

$$\begin{aligned}
\|F_T^{(m+1)}(z)\|_{\ell^q} &\leq \sum_{n=m+1}^{\infty} n^{m+1}\|x_n\|_{\ell^q}|z|^{n-(m+1)} \\
&\leq C \sum_{j=[log_2(m+2)]}^{\infty} 2^{j(m+1)}2^{-j(1/p-1)}\Big(\sum_{n=2^{j-1}-1}^{2^j}(n+1)^{1/p-1}\|x_n\|_{\ell^q}\Big)|z|^{2^j-(m+2)} \\
&\leq C\|((n+1)^{1/p-1}x_n)\|_{\ell(1,\infty,\ell^q)} \sum_{j=[log_2(m+2)]}^{\infty} 2^{j(m-1/p)}|z|^{2^j-(m+2)} \\
&\leq \frac{C\|((n+1)^{1/p-1}x_n)\|_{\ell(1,\infty,\ell^q)}}{(1-|z|)^{m+2-1/p}}
\end{aligned}$$

From Lemma 3.1 one obtains

$$\|T\| \leq C\|((n+1)^{1/p-1}x_n)\|_{\ell(1,\infty,\ell^q)}. \tag{3.2}$$

Now (3.1) and (3.2) give (1.3).

Proof of (1.4). Let us take $(\lambda_k) \in \ell^{q'}$ (or $(\lambda_k) \in c_0$ for $q = 1$). Using (1.7) again there exists $C > 0$ such that

$$|g_\lambda^{(m+1)}(z)| \leq \frac{C\|T_\lambda\|}{(1-|z|)^{m+2-1/p}}.$$

In particular, for $g_\lambda(z) = \sum_k \lambda_k g_k(z) = \sum_{n=0}^\infty (\sum_k \lambda_k t_{nk})z^n$,

$$M_2(g_\lambda^{(m+1)}, r) \leq \frac{C\|T_\lambda\|}{(1-r)^{m+2-1/p}}.$$

Therefore

$$(\sum_{n=m}^{\infty}(n+1)^{2(m+1)}|\sum_k \lambda_k t_{kn}|^2 r^{2n})^{1/2} \leq \frac{C\|T_\lambda\|}{(1-r)^{m+2-1/p}}.$$

Applying now Lemma 2.2 for $\beta = 2(m+2-1/p)$ one concludes that $((n+1)^{2(1/p-1)}|\sum_k \lambda_k t_{kn}|^2)_n \in \ell(1,\infty)$ and

$$\|((n+1)^{(1/p-1)}\sum_k \lambda_k t_{kn})_n\|^2_{\ell(2,\infty)} \leq C\|T_\lambda\|^2.$$

This shows

$$\sup_{\|(\lambda_k)\|_{\ell^{q'}}=1} \|\sum_{k=1}^{\infty}\lambda_k A_k\|_{\ell(2,\infty)} \leq C\|T\|.$$

Hence

$$\|(A_k)\|_{\ell^q_{weak}(\ell(2,\infty))} \leq C\|T\|.$$

Let us now show that $\|(B_n)\|_{\ell(\max\{q,q'\},\infty,\ell^q)} \leq C\|T\|$.

Assume first $q = 1$. Recall that from (3.1) one has $\|F_T^{(m+1)}(z)\|_{\ell^q} \leq \frac{C\|T\|}{(1-|z|)^{m+2-1/p}}$.

Now use, for $n \geq m$,

$$\|x_n\|_{\ell^1} n^{m+1}|z|^{n-m} \leq C\|F_T^{(m+1)}(z)\|_{\ell^1} \leq \frac{C\|T\|}{(1-|z|)^{m+2-1/p}}.$$

Selecting $|z| = 1 - 1/(n+1)$ to obtain

$$\|B_n\|_{\ell^1} = \|x_n\|_{\ell^1}(n+1)^{1/p-1} \leq C\|T\|$$

which is the desired estimate.

Assume now $q > 1$. Denote $t = \min\{q, q'\}$ and $q_0 = \max\{q, q'\}$ and apply (3.1) to obtain

$$M_t(F_T^{(m+1)}, r) \leq M_\infty(F_T^{(m+1)}, r) \leq \frac{C\|T\|}{(1-r)^{m+2-1/p}}.$$

Using Lemma 2.3 one can write

$$(\sum_{n=0}^{\infty}(n+1)^{(m+1)q_0}\|x_n\|_{\ell^q}^{q_0} r^{nq_0})^{1/q_0} \leq \frac{C\|T\|}{(1-r)^{m+2-1/p}}.$$

Now apply Lemma 2.2 for $\beta = q_0(m+2-1/p)$ to get $(\|(n+1)^{1/p-1}x_n\|_{\ell^q}^{q_0}) \in \ell(1,\infty)$ and the corresponding estimate for the norm. This finishes the proof. □

References

[1] O. Blasco, *Operators on weighted Bergman spaces and applications*, Duke Math. J. **66**, (1992), 443-467.

[2] O. Blasco, *Vector-valued analytic functions of bounded mean oscillation and geometry of Banach spaces*, Illinois J. Math. **41**, (1997), 532-558.

[3] O. Blasco, *Remarks on vector-valued BMOA and vector-valued multipliers*, Positivity **4**, (2000), 339-356.

[4] O. Blasco, *Operators on weighted Bergman spaces*, Contemp. Math. **404**, (2006), 53-66.

[5] O. Blasco, *Norm estimates for operators from H^p to ℓ^q*, Submitted.

[6] D.M. Campbell, R.J. Leach, *A survey of H^p multipliers as related to classical function theory*, Complex Variables **3** (1984), 85-111.

[7] P. Duren, *Theory of H^p-spaces*, Academic Press (1970).

[8] P.L. Duren, B.W. Romberg, A.L. Shields, *Linear functionals on H^p spaces with $0 < p < 1$*, J. Reine Angew. Math. **238** (1969), 32-60.

[9] P.L. Duren, A.L. Shields, *Coefficient multipliers of H^p and B^p spaces*, Pacific J. Math. **32** (1970), 69-78.

[10] C.N. Kellogg, *An extension of the Hausdorff-Young theorem*, Michigan Math. J. **18** (1971), 121-127.

[11] Y. Kim, *Coefficient multipliers of H^p and G^p spaces*, Math. Japonica **30** (1985),no 5, 671-679.

[12] M. Mateljevic, M. Pavlovic, *L^p-behaviour of the integral means of analytic functions*, Studia Math. **84**, (1976), 45-90.

[13] B. Osikiewicz, A. Tonge, *An interpolation approach to Hardy-Littlewood inequalities for norms of operators on sequence spaces*, Linear Alg. and Appl. **331** (2001), 1-9.

[14] B. Osikiewicz, *Norm estimates for operators $T; H_p \to l_q$ involving the $L_r(l_s)$ norms*, Quaestiones Math. **28** (2005), 263-270.

[15] B. Osikiewicz, *Multipliers of Hardy spaces*, Quaestiones Math. **27** (2004), 57-73.

[16] J. Peetre, *Sur la transformation de Fourier des fonctions a valeurs vectorielles*, Rend. Sem. Mat. Univ. Padova **42** (1969), 15-46.

Departmento de Análisis Matemático, Universidad de Valencia, Burjassot (Valencia), Spain 46100

E-mail address: oblasco@uv.es