On coefficients of vector valued Bloch functions

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Abstract

Let X be a complex Banach space and let $Bloch(X)$ denote the space of X-valued analytic functions on the unit disc verifying that $sup_{|z|<1}(1-|z|^2)||f'(z)|| < \infty$. A sequence $(T_n)_n$ of bounded operators between two Banach spaces X and Y defines a multiplier between $Bloch(X)$ and $\ell_1(Y)$ if for any function $f(z) = \sum_{n=0}^{\infty} x_n z^n$ in $Bloch(X)$ we have that $(T_n(x_n))_n \in \ell_1(Y)$. It is shown that if X is a Hilbert space then $(T_n)_n$ is a multiplier from $Bloch(X)$ into $\ell_1(Y)$ if and only $sup_k \sum_{n=2^k}^{2^{k+1}} ||T_n||^2 < \infty$. Several results about Taylor coefficient of vector-valued Bloch functions relying unpon properties on X, such as Rademacher and Fourier type p, are presented.

AMS Subj. Class: 46E40, 46B20
Key words: Bloch functions, type and cotype

1 Introduction.

Throughout the paper X stands for a complex Banach space and we write $Bloch(X)$ for the space of X-valued analytic functions on the unit disc verifying that $||f||_{Bloch(X)} = ||f(0)|| + sup_{|z|<1}(1-|z|^2)||f'(z)|| < \infty$.

Clearly one has that $f \in Bloch(X)$ if and only if $x^*f(z) = \langle f(z), x^* \rangle \in Bloch$ for all $x^* \in X^*$. Moreover $||f||_{Bloch} \approx \sup_{||x^*||=1} ||x^*f||_{Bloch}$.

For $1 \leq p, q \leq \infty$ we shall be denoting by $\ell_p(X)$ and $\ell(p, q, X)$ the spaces of sequences $(x_n)_n$ in X such that $||(x_n)||_p = (\sum_{n=0}^{\infty} ||x_n||^p)^{1/p} < \infty$ and $\left(\|(x_n)_{n \in I_k}\|_{\ell_p}\right)_k \in \ell_q$, where $I_k = \{n \in \mathbb{N};\ 2^{k-1} \leq n < 2^k\}$ for $k \in \mathbb{N}$ and $I_0 = \{0\}$.

*Partially supported by Proyecto BMF2002-04013

For $p,q<\infty$ we write $\|(x_n)\|_{p,q} = \Big(\sum_{k=0}^\infty (\sum_{n\in I_k}\|x_n\|^p)^{q/p}\Big)^{1/q}$, $\|(x_n)\|_{\infty,q} = \Big(\sum_{k=0}^\infty \sup_{n\in I_k}\|x_n\|^q\Big)^{1/q}$ and $\|(x_n)\|_{p,\infty} = \sup_k(\sum_{n\in I_k}\|x_n\|^p)^{1/p}$. As usual, when $X=\mathbb{C}$ we simply write $\ell(p,q)$. These classes were first considered in [25].

Let us recall the following well known facts on Taylor coefficients of Bloch functions.

If $f(z)=\sum_{n=0}^\infty x_n z^n$ with $x_n\in X$ then, for each n and $r\in(0,1)$, we have that

$$x_n r^n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(re^{i\theta})e^{-in\theta}d\theta.$$

This shows that if $f\in Bloch(X)$ then $\|x_n\|nr^n \le C(1-r)^{-1}$ for all $n\in\mathbb{N}$ and $0<r<1$. Choosing $r=1-1/n$ we obtain that

$$f\in Bloch(X) \text{ then } (x_n)\in\ell_\infty(X). \tag{1}$$

On the other hand, for $f(z)=\sum_{n=0}^\infty x_n z^n$

$$||f'(z)|| \le \sum_k\sum_{n\in I_k} n||x_n|||z|^n \le 2\sup_k(\sup_{n\in I_k}||x_n||)\sum_k 2^k|z|^{2^k} \le \frac{||(x_n)_n||_{1,\infty}}{1-|z|}.$$

Hence

$$\text{If } (x_n)_n\in\ell(1,\infty,X) \text{ then } \phi(z)=\sum_{n=0}^\infty x_n z^n\in Bloch(X). \tag{2}$$

The reader is referred to [2, 3, 7] for the basic theory we shall be using on Bloch functions. Let us now recall the following basic result on multipliers (see [3]):

$$(Bloch,\ell_1)=\ell(2,1) \tag{3}$$

where $(Bloch,\ell_1)$ stands for the space of sequences (λ_n) such that the operator $f(z)=\sum_n \alpha_n z^n \to (\lambda_n\alpha_n)_n$ is bounded from $Bloch$ into ℓ_1.

As a consequence we have the following fact

$$\text{If } \phi(z)=\sum_{n=0}^\infty \alpha_n z^n\in Bloch \text{ then } (\alpha_n)_n\in\ell(2,\infty). \tag{4}$$

Note that $f(z) = \sum_{n=1}^\infty e_n z^n = (z^n)_n$ is a c_0-valued bounded analytic function, in particular $f \in Bloch(c_0)$, and $(e_n) \notin \ell(p, \infty, c_0)$ for any $p < \infty$.

Vector valued Bloch functions has been used in different papers and for different reasons (see [4, 5, 8, 9, 10, 11, 12, 13]). We refer the reader to [6, 14] for new results on the subject.

The aim of this paper is to understand whether (4) and (3) have natural extensions to vector-valued functions and how the vector-valued analogues of them depend on the geometrical properties of the Banach spaces.

The first problem to deal with consists of finding out Banach spaces X for which

$$f(z) = \sum_{n=0}^{\infty} x_n z^n \in Bloch(X) \text{ implies } (x_n)_n \in \ell(2, \infty, X). \tag{5}$$

To this aim let us give the following definition.

Definition 1.1 *Let X be a complex Banach space. We define $\Lambda_{Bloch,\ell_1}(X)$ as the space scalar-valued sequences $(\lambda_n)_n$ such that the operator $\sum_{n=0}^\infty x_n z^n \to (\lambda_n x_n)_n$ is bounded from $\in Bloch(X)$ into $\ell_1(X)$*

The second problem to deal with is to describe $\Lambda_{Bloch,\ell_1}(X)$. Clearly, from (3), $\Lambda_{Bloch,\ell_1}(X)$ is a solid sequence space such that

$$\ell_1 \subseteq \Lambda_{Bloch,\ell_1}(X) \subseteq \ell(2, 1).$$

Hence (5) is equivalent to the following question: When is

$$\Lambda_{Bloch,\ell_1}(X) = \ell(2, 1)? \tag{6}$$

We start by mentioning a couple of examples of vector valued functions Bloch functions to keep in mind for checking the validity of results in the vector valued setting.

Example 1.1 *(see [14], Example 3.1) Let $1 \le p \le \infty$ and define $f_p : \mathbb{D} \to \ell_p$ by $f_p(z) = \sum_{n=1}^\infty n^{-1/p} e_n z^n$ where e_n stands for the canonical basis. Then $f_p \in Bloch(\ell_p)$.*

Example 1.2 *(see [14] Example 3.2) Let $1 \le p < \infty$ and define $F_p : \mathbb{D} \to L^p(\mathbb{T})$ by $F_p(z)(\xi) = \frac{1}{(1-\bar{\xi}z)^{1/p}}$. Then $F_p \in Bloch(L^p(\mathbb{T}))$.*

Now observe that $f_p(z) = \sum_{n=1}^{\infty} x_n z^n$ with $||x_n|| = n^{-1/p}$. Hence

$$\Lambda_{Bloch,\ell_1}(\ell_p) \subsetneq \ell(2,1) \text{ for } p < 2.$$

On the other hand $F_p(z) = \sum_{n=1}^{\infty} x'_n z^n$ with $||x'_n|| \approx n^{-1/p'}$ Hence

$$\Lambda_{Bloch,\ell_1}(L^p(\mathbb{T})) \subsetneq \ell(2,1) \text{ for } p > 2.$$

There are many properties on the Banach spaces that are describe for the validity of certain multiplier results in the setting of vector-valued functions. This is the case of the UMD property (for the Hilbert transform), the notion of Fouier type (for the Hausdorff-Young inequality), ... For vector valued Hardy and Bergman spaces some properties were introduced in the collaborations with A. Pelczynsky (see [16]) and J.L. Arregui (see [5]) respectively.

Another possible generalization is to consider sequences of bounded operators $(T_n)_n$ in $\mathcal{L}(X, Y)$ between two Banach spaces X and Y and to define operator-valued multipliers. This approach for different spaces of analytic functions and multipliers can be found in [4, 5, 10, 11, 13, 14].

Definition 1.2 *A sequence $(T_n)_n$ in $\mathcal{L}(X, Y)$ is said to be a multiplier between $Bloch(X)$ and $\ell_1(Y)$, to be denoted $(T_n) \in \big(Bloch(X), \ell_1(Y)\big)$, if for any function $f(z) = \sum_{n=0}^{\infty} x_n z^n$ in $Bloch(X)$ we have that the sequence $(T_n(x_n))$ belongs to $\ell_1(Y)$.*

This is equivalent to the existence of a constant $C > 0$ such that

$$\sum_{n=0}^{N} ||T_n(x_n)|| \leq C sup_{|z|<1}(1 - |z|^2)|| \sum_{n=1}^{N} n x_n z^{n-1}|| \quad (7)$$

for any $N \in \mathbb{N}$ and $x_0, x_1, ..., x_N$ elements in X.

The infimum of the constants C verifying (7) is the multiplier norm, which coincides with the operator norm between $Bloch(X)$ and $\ell_1(Y)$.

The third problem to deal with is to find conditions on the Banach spaces X and Y to have

$$\big(Bloch(X), \ell_1(Y)\big) = \ell(2, 1, \mathcal{L}(X, Y)). \quad (8)$$

Let us now collect several definitions of properties of Banach spaces to be used in the sequel.

Definition 1.3 *Let $1 \leq p \leq 2 \leq q < \infty$ and let X be a complex Banach space.*

X is said to have Fourier type p (see [28]) if there exists a constant C such that

$$(\sum_{n=-\infty}^{\infty} ||\hat{f}(n)||^{p'})^{1/p'} \leq C||f||_{L^p(\mathbb{T},X)} \tag{9}$$

for all function $f \in L^p(\mathbb{T}, X)$.

X is said to have Rademacher type p (see [27])if there exists a constant C such that

$$\int_0^1 ||\sum_{j=1}^n x_j r_j(t)||dt \leq C\big(\sum_{j=1}^n \|x_j\|^p\big)^{1/p}$$

for any finite family $x_1, x_2, \dots x_n$ of vectors in X where r_j stand for the Rademacher functions on $[0, 1]$.

X is said to have Rademacher cotype q (see [27]) if there exists a constant C such that

$$\big(\sum_{j=1}^n \|x_j\|^q\big)^{1/q} \leq C\int_0^1 ||\sum_{j=1}^n x_j r_j(t)||dt$$

for any finite family $x_1, x_2, \dots x_n$ of vectors in X where r_j stand for the Rademacher functions on $[0, 1]$.

The notion of Fourier type was first introduced by J. Peetre ([28]) and we refer the reader to the survey [20] for a complete study and references about this property.

Just mention that X has Fourier type p if and only if X^* does have it.

The notions of type and cotype were introduced by B. Maurey and G. Pisier and were shown to be rather important in Banach space theory. Let simply recall that Fourier type p implies Rademacher type p and that if X^* has type p then X has cotype p'.

The main examples of spaces of Fourier type p are $L^r(\mu)$ for any $p \leq r \leq p'$ or interpolation spaces $[X_0, X_1]_\theta$ between any Banach space X_0 and any Hilbert space X_1 where $1/p = 1 - \theta/2$.

Recall also that $L^r(\mu)$ has Rademacher type $\min p, 2$ and Rademacher cotype $\max p, 2$.

Throughout the paper we shall use the notation B_1 for the Bergman space of analytic functions on the unit disc $\mathbb{D}$ which are integrable against the normalized Lebesgue measure in the disc $\mathbb{D}$ denoted by $dA(z)$, we write $M_p(f,r)$, as usual, for $(\int_0^{2\pi} ||f(re^{it})||^p \frac{dt}{2\pi})^{1/p}$, $1 \leq p < \infty$, and C represents a constant that may vary from one place to another.

2 Taylor coefficients.

We first start by showing that (5) holds for Hilbert spaces. The proof that we shall present here is based upon Grothendieck's inequality.

Theorem 2.1 *Let H be a Hilbert space. Then there exists a constant $C > 0$ such that*

$$||(x_n)_n||_{2,\infty} \leq C||f||_{Bloch(H)}$$

for all $f(z) = \sum_{n=0}^{\infty} x_n z^n \in Bloch(H)$, i.e. $\Lambda_{Bloch,\ell_1}(H) = \ell(2,1)$.

PROOF. Given $f \in Bloch(H)$ we can define $T_f : B_1 \to H$ by the formula $T_f(u_n) = x_n$, where $u_n(z) = (n+1)z^n$.

Now we extend the definiton to polynomials, by linearity, and for $\phi(z) = \sum_{n=0}^{N} \alpha_n z^n$, we have

$$T_f(\phi) = \sum_n \frac{x_n \alpha_n}{n+1} = \int_{\mathbb{D}} \phi(\bar{z}) f(z) dA(z)$$

Recall first that the pairing

$$\langle \phi, \psi \rangle = \sum_{n=0}^{\infty} \frac{\alpha_n \bar{\beta}_n}{n+1} = \int_{\mathbb{D}} \phi(z)\overline{\psi(z)} dA(z), \tag{10}$$

for any $\phi(z) = \sum_{n=0}^{\infty} \alpha_n z^n$ and $\psi(z) = \sum_{n=0}^{\infty} \beta_n z^n$, gives the identification $(B_1)^* = Bloch$ (see [35]).

Since $\langle T_f(\phi), x^* \rangle = \langle x^* f, \phi \rangle$ and polynomials are dense in B_1 we can continuously extend T_f to B_1 as a bounded operator and $||T_f|| \leq C||f||_{Bloch(H)}$.

On the other hand it is known (see [33] or [35]) that B_1 is isomorphic to ℓ_1. Hence from Grothendieck theorem (see [17]) we have that T_f is absolutely summing.

Let $||(\lambda_n)||_{2,1} \leq 1$. It follows from (3) that

$$\sup_{||g||_{(B_1)^*} \leq 1} \sum_n |\langle \lambda_n u_n, g \rangle| \leq C.$$

This leads to

$$\sum_n |\lambda_n| ||T(u_n)|| \leq C$$

for all $||(\lambda_n)||_{2,1} \leq 1$. Or in other words $(x_n) \in \ell(2, \infty, X)$ and

$$||(x_n)_n||_{2,\infty} \leq C||T_f|| = C||f||_{Bloch(H)}.$$

$\square$

Due to the fact that $(\ell(p,q,X))^* = \ell(p',q',X^*)$ for $1/p + 1/p' = 1/q + 1/q' = 1$ for $1 \leq p, q < \infty$ under the natural pairing

$$\langle (x_n), (x_n^*) \rangle = \sum_n \langle x_n, x_n^* \rangle$$

(where we also use $\langle .,. \rangle$ for the dual pairing in X), it is convenient to get a predual of $Bloch(X^*)$ under such a pairing (see [6] for the duality $(B_1(X))^* = Bloch(X^*)$).

We shall be denoting $J_1(X)$ the space of X-valued analytic functions f on the disc $\mathbb{D}$ such that $\int_{\mathbb{D}} ||f'(z)|| dA(z) < \infty$. We endow it with the norm $||f||_{J_1(X)} = ||f(0)|| + \int_0^1 M_1(f', r)dr$.

Now we have $(J_1(X))^* = Bloch(X^*)$ under the pairing

$$\langle f, g \rangle = \sum_{n=0}^{\infty} \langle x_n^*, x_n \rangle \tag{11}$$

for any $g(z) = \sum_{n=0}^{\infty} x_n^* z^n \in Bloch(X^*)$ and $f(z) = \sum_{n=0}^{\infty} x_n z^n \in J_1(X)$.

The reader is referred to [2] for this duality result in the scalar case and to [8, 9] for its vector valued extension and generalizations.

Clearly one has the following elementary estimates: There exist $C_1, C_2 > 0$ such that

$$C_1 \Big\| (x_n) \Big\|_{\infty,1} \leq ||f||_{J_1(X)} \leq C_2 \Big\| (x_n) \Big\|_1 \tag{12}$$

for any $f \in J_1(X)$ with Taylor coefficients (x_n).

In order to improve estimates (12) under some assumtions on the Banach space X we shall need the following lemma.

Lemma 2.2 *(see [12] or [27]) Let* (α_n) *be sequence of nonnegative numbers and* $0 < q, \beta < \infty$. *Then*

$$\int_0^1 (1-r)^{\beta q-1}(\sum_{n=1}^{\infty} \alpha_n r^n)^q dr \approx \sum_{k=1}^{\infty} (\sum_{n\in I_k} \frac{\alpha_n}{n^\beta})^q. \tag{13}$$

Theorem 2.3 *Let* X *be a Banach space of Fourier type* p.

(i) There exists a constant $C > 0$ *such that*

$$||f||_{J_1(X)} \leq C\Big\|(x_n)\|_{p,1}$$

for all $(x_n) \in \ell(p,1,X)$ *and* $f(z) = \sum_{n=1}^\infty x_n z^n$.

(ii) There exists a constant $C > 0$ *such that*

$$\Big\|(x_n)\|_{p',\infty} \leq C||f||_{Bloch(X)}$$

for all $f(z) = \sum_{n=1}^\infty x_n z^n \in Bloch(X)$.

(iii) $\ell(p,1) \subseteq \Lambda_{Bloch,\ell_1}(X)$.

PROOF. Let us now show (i). Note that

$$||f||_{J_1(X)} \leq ||f(0)|| + \int_0^1 M_{p'}(f',r)dr \leq C(||f(0)|| + \int_0^1 (\sum_n n^p ||x_n||^p r^{np})^{1/p} dr).$$

Now applying Lemma 2.2 for $\beta = p$ and $q = 1/p$ we obtain (i).

(ii) follows by duality, because $Bloch(X)$ is isometrically included into $(J_1(X^*))^*$.

(iii) follows from (ii), since for all $(\lambda_n)_n \in \ell(p,1)$ and $f(z) = \sum_n x_n z^n$ we have

$$\sum |\lambda_n|||x_n|| \leq ||(x_n)||_{p',\infty}||(\lambda_n)||_{p,1} \leq C||(\lambda_n)||_{p,1}||f||_{Bloch(X)}.$$

□

Remark 2.1 *Observe that Theorem 2.3 gives an alternative proof of Theorem 2.1.*

The reader is referred to Proposition 3.3 in [14] for a closely related result.

Theorem 2.4 *Let $1 \le p < 2$ and let X be a Banach space.*

(i) If $\ell(p,1) \subseteq \Lambda_{Bloch,\ell_1}(X)$ then X has cotype p'.

(ii) If $\ell(2,1) = \Lambda_{Bloch,\ell_1}(X)$ then X has Orlicz property, i.e. if $\sum_n |\langle x_n, x^*\rangle| < \infty$ then $\sum_n ||x_n||^2 < \infty$.

PROOF. We shall see in both cases that if $\sum_n |\langle x_n, x^*\rangle| < \infty$ then $\sum_n ||x_n||^{p'} < \infty$. For the equivalence of this fact and X having coytpe p' in the case $p < 2$ we refer to the work of M. Talagrand (see [30, 31]).

Let $x_1, .., x_N \in X$ such that $\sum_{n=1}^N |\langle x_n, x^*\rangle| = 1$ we construct $f(z) = \sum_{n=2^k+1}^{2^k+N} x_{n-2^k} z^n$ for $2^k \le N < 2^{k+1}$. Hence f belongs to $Bloch(X)$ (becasuse $x^* f \in Bloch$ for all $x^* \in X^*$) and the assumption $\ell(p,1) \subseteq (Bloch(X), \ell_1(X))$ easily gives $\sum_{n=1}^N ||x_n||^{p'} \le C$.

□

To obtain certain converse estimates we shall need to use next lemma (see also [5] for similar result for Bergman spaces).

Lemma 2.5 *Let X be a Banach space and let $1 < q < \infty$. Then*

$$\int_0^1 \int_0^1 (1-r)^{-1/q} M_q(f', rs) dr ds \le C||f||_{J_1(X)}.$$

PROOF. It was shown in Theorem 2.1 of [15] that

$$\Big(\int_0^1 (1-r)^{-1/q} M_q(f, r) dr\Big)^{1/p} \le C||f||_{H_1(X)},$$

where $H_1(X)$ stands for the vector-valued Hardy space.

Now, since $M_1(f', r) = ||f_r||_{H_1(X)}$ where $f_r(z) = f(rz)$ then the result follows by integrating over $(0,1)$. □

Theorem 2.6 *Let X be a Banach space with Fourier type p.*

(i) There exists a constant $C > 0$ such that

$$\Big\|\Big(\frac{x_n}{n^{1/p'}}\Big)\Big\|_{p',1} \le C||f||_{J_1(X)}$$

for all $f(z) = \sum_{n=1}^\infty x_n z^n$.

(ii) There exists a constant $C > 0$ such that

$$||f||_{Bloch(X)} \le C\Big\|(n^{1/p'} x_n)\Big\|_{p,\infty}$$

for all $f(z) = \sum_{n=1}^\infty x_n z^n$.

PROOF. (i) Since X has Fourier type p, we have

$$\Big(\sum_{n=1}^{\infty}\|x_n\|^{p'}r^{p'n}\Big)^{1/p'} \leq CM_p(f,r)\,,$$

Using Lemma 2.5 and Lemma 2.2 we get

$$\begin{aligned}
||f||_{J_1(X)} &\geq C\int_0^1\int_0^1 (1-r)^{-1/p}M_p(f',rs)drds \\
&\geq C\int_0^1\int_0^1 (1-r)^{-1/p}\Big(\sum_{n=1}^{\infty}n^{p'}\|x_n\|^{p'}s^{p'n}r^{p'n}\Big)^{1/p'}drds \\
&\geq C\big(\int_0^1 (1-r)^{-1/p}\sum_{k=1}^{\infty}\big(\sum_{n\in I_k}\|r^n x_n\|^{p'}\big)^{1/p'}dr \\
&\geq C\big(\int_0^1 (1-r)^{-1/p}\sum_{k=1}^{\infty}\big(\sum_{n\in I_k}\|x_n\|^{p'}\big)^{1/p'}r^{2^k}dr\big)^{1/p} \\
&\geq C\sum_{k=1}^{\infty}\big(\sum_{n\in I_k}\|x_n\|^{p'}\big)^{1/p'}\big(\int_0^1 (1-r)^{-1/p}r^{2^k}dr\big) \\
&\geq C\sum_{k=1}^{\infty}\big(\sum_{n\in I_k}\|x_n\|^{p'}\big)^{1/p'}2^{-k(1/p')} \\
&\geq C\Big\|\big(\frac{x_n}{n^{1/p'}}\big)\Big\|_{p',1}.
\end{aligned}$$

(ii) follows by duality. □

3 Multipliers.

Now we analyze the interplay between geometry of Banach spaces and questions (6) and (8).

Repeating the argument in Theorem 2.4 with $T_n = \lambda_n T$ for a fixed operator T we obtain the following result.

Proposition 3.1 *Let $1 \leq p \leq 2$ and let X and let Y be Banach spaces. If*

$$\ell(p,1,\mathcal{L}(X,Y)) \subseteq \big(Bloch(X),\ell_1(Y)\big)$$

then $\Pi_{p,1}(X,Y) = \mathcal{L}(X,Y)$, where $\Pi_{p,1}(X,Y)$ stands for the space of $(p,1)$-summing operators (see [17] for definition and properties).

Theorem 3.2 *Let X and Y be Banach spaces and assume that X has Fourier type p. Then*

$$\ell(p,1,\mathcal{L}(X,Y)) \subseteq \big(Bloch(X), \ell_1(Y)\big).$$

PROOF. This follows easily from Theorem 2.3. □

Theorem 3.3 *Let X^* be a complex Banach space of Rademacher cotype p' and Y be any Banach space. Then*

$$\big(Bloch(X), \ell_1(Y)\big) \subset \ell(p',1,\mathcal{L}(X,Y)).$$

PROOF. Let (T_n) be a sequence of operators in $\big(Bloch(X), \ell_1(Y)\big)$. Using a simple duality argument we have that

$$||\sum_{n=1}^{\infty} \epsilon_n T_n^*(y_n^*) z^n||_{J_1(X^*)} \leq C$$

for all $\epsilon_n \in \{-1,1\}$ and $||y_n^*|| = 1$.

Now writting $\epsilon_n = r_n(t)$ for $t \in [0,1]$, and $f_t(z) = \sum_{n=1}^{\infty} r_n(t) T_n^*(y_n^*) z^n$ we have

$$\int_0^1 ||f_t||_{J_1(X^*)} dt = \int_0^1 \int_0^1 |M_1(f_t', r)| dr dt \geq C \int_0^1 (\sum_n n^{p'} ||T_n^*(y_n^*)||^{p'} r^{np'})^{1/p'} dr.$$

Applying Lemma 2.2 we obtain $(T_n^*(y_n^*)) \in \ell(p',1,X^*)$ uniformly for $||y_n^*|| = 1$. Hence $(T_n) \in \ell(p',1,\mathcal{L}(X,Y))$. □

Combining now Theorems 3.2 and 3.3 we get our final corollary

Corollary 3.4 *Let H be a Hilbert space and let Y be a Banach space. Then*

$$\big(Bloch(H), \ell_1(Y)\big) = \ell(2,1,\mathcal{L}(X,Y)).$$

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