The dynamics of personal norms and the emergence of an organizational culture

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Culture is the degree to which members of a group (organization, society) share similar behavior and values or beliefs. Cultural diversity or homogeneity is an important topic in social sciences (social psychology, sociology...) but also in management. Recently it has become a hot topic in economics. Management literature: Corporate culture as homogeneity in behavior and values of the members of an organization. (Schwartz and Davis 1981, Schein 1985, Van den Steen 2010).
Introduction

- Do all groups and societies tend to homogeneity?
- Why some organizations and societies are more homogeneous and other are more diverse?
- What are the determinants of the levels of homogeneity and/or diversity of a culture? The strength of a culture.
- What are the economic consequences of cultures with different levels of diversity? How does diversity affect ultimate group performance and its members' levels of satisfaction or dissatisfaction?
Empirical research on cultural organizational differences reveals four broadly accepted findings (Bednar, Bramson, Jones-Rooy and Page, 2010):

1- Cultures do exhibit homogeneity both in behavior and in beliefs or preferences.

2- Behavior and preferences exhibit coherence.

3- Despite the previous findings, cultures exhibit substantial within-group diversity or heterogeneity and behavior and preferences also exhibit varying levels of incoherence.

4- Cultures differ (inter-group diversity).

Therefore, empirical evidence shows that cultures exist, they differ from one another, they are coherent and yet diversity and incoherence persists within them.
A candidate for a "good" theory or model on the emergence and formation of organizational cultures should explain all these findings, show the determinants of the levels of diversity and incoherence of different cultures, and most importantly from the economist’s point of view, should explain the consequences of the different levels of diversity and incoherence on aggregate production and social welfare.
Our approach views the diversity of a culture as endogenous product of individuals’ interactions (Lewis 1969; Ullmann-Margalit 1977; Vandershraaf 1995; Bicchieri 2006).

We analyze the emergence and evolution of a culture in an organization or a group in which members are guided by economic incentives and also follow personal norms of behavior.

We assume the definition of personal norms of Schwartz (1977) as "internal standard for a particular conduct" or "individual internalized moral rules".
Agents participate each period in a **team production game** (similar to a public goods game) by choosing their level of costly effort and are motivated by the two previously mentioned forces.

We allow for a huge heterogeneity in the initial condition of the group concerning not only the individuals’ personal norms (or intrinsic motivation) but also concerning:

- their levels of **materialism**: the weight they assign in their utility function to their material interest;
- their individual **skills** or productivities
- and their shares in the total **income distribution** (remunerations).
We take materialism, skills and the income distribution rule as given, but personal norms evolve along the life-cycle of the individuals according to two psychological forces: cognitive dissonance or consistency and informational conformity.

**Consistency** is an individual force that drives personal norms towards actual behavior.

**Conformity**, by driving personal norms towards the average group behavior, captures how social interaction impact on diversity.

We characterize the long-run outcomes of the group (the steady states of the dynamics of personal norms) and study how the levels of diversity, both in personal norms and in behavior and the level of incoherence between both variables are determined by the primitives of the model: the income distribution in the group, the distribution of skills and the levels of materialism, conformity and consistency.

We also analyze how these parameters affect the group aggregate production and social welfare.
Model

- Consider a social group or organization composed by $N$ agents.
- Each agent $i$ has a productivity (skill) of $s_i \geq 0$.
- Agents choose (simultaneously) a (non-verifiable) level of effort of $e_i \geq 0$, that is costly, $c(e_i) = (1/2)e_i^2$ and participates in a team production game.
- Total revenue is $y = \sum_{i=1}^{n} s_i e_i$.
- Total revenue generated by the team (or group) is divided according to a sharing rule $\mathbf{w}$, vector of real numbers that assigns to each agent a share $w_i \in [0, 1]$ of the total revenue, with $\sum_{i=1}^{n} w_i = 1$.
- Agents are motivated by two elements: material incentives and personal norms.
Each agent has a personal norm $\hat{e}_i \geq 0$ concerning effort: any deviation of actual behavior from $\hat{e}_i$ will yield disutility. The loss function is: $-\frac{1}{2} (e_i - \hat{e}_i)^2$.

Each agent assigns a weight $\beta_i \in [0, 1]$ to the material payoffs and $(1 - \beta_i)$ to the intrinsic motivation (personal norms $\hat{e}_i$).

Therefore we will denote $\beta_i$ as the level of materialism of player $i$. Standard selfish preferences: special case for $\beta_i = 1$.

Note that agents might differ not only in personal norms $\hat{e}_i$, but also in $\beta_i$, in the income share $w_i$ and in the skill $s_i$.

The utility function of a player in the team production game:

$$u_i(e) = \beta_i [w_i \sum_{j=1}^{N} s_j e_j - \frac{1}{2} e_i^2] - (1 - \beta_i) \frac{1}{2} (e_i - \hat{e}_i)^2$$  \hspace{1cm} (1)
The Nash equilibrium (NE) $\bar{e}_t^i$ of the simultaneous game in each period $t$ is given by:

$$\bar{e}_t^i = \beta_i(w_is_i) + (1 - \beta_i)\hat{e}_t^i, \ i = 1, 2, ..., N.$$  

(2)

For instance, if the same weight is assigned to both material payoffs and personal norms (i.e., $\beta_i = 1/2$) and with equal sharing (i.e., $w_i = 1/N$), then NE is given by: $\bar{e}_i = \frac{(s_i/N) + \hat{e}_t^i}{2}$.

Notice that the efficient effort that maximizes social welfare is

$$e_i^* = s_i \sum_j \beta_j w_j + (1 - \beta_i)\hat{e}_i$$

We have that $e_i^* \geq \bar{e}_i$, so that the effort level chosen in the NE is always smaller or equal than the one chosen to maximize social welfare.
Model

- We consider a two-speed dynamics: gradual changes in preferences (personal norms) are accompanied by immediate behavioral adjustment in each period’s equilibrium play.

- We assume that individuals may change their personal norms through **two psychological mechanisms**:

  a) **cognitive dissonance (or consistency)**. People tend to seek consistency in their beliefs and behavior. When there is a discrepancy between beliefs and behaviors, something must change in order to eliminate or reduce the dissonance. Personal norms move towards the actual behavior of the agent. (Akerlof and Dickens 1982, Kuran and Sandholm 2008, Nordblom and Zamac 2011).

  b) **informational conformism (conformity)**. Conformity is a type of social influence involving a change in belief or behavior in order to fit in with a group. Personal norms move towards the average of the actual behavior of the organization. (Bernheim 1994, Kandel and Lazear 1992, Akerlof 1997, Fischer and Huddart 2008).
Dynamics of Personal Norms with only consistency

- Cognitive Dissonance is reduced by making preferences of each agent $i$ to evolve in the direction of his actual NE behavior:

$$\hat{e}_i^{t+1} = \gamma_i \hat{e}_i^t + (1 - \gamma_i) \hat{e}_i^t$$

(3)

- where $\gamma_i \in (0, 1)$ is the weight that agent $i$ assigns to the actual equilibrium behavior, and $1 - \gamma_i$ indicates how much the agent is anchored to own past personal norm.

- Using the continuos time limit of the dynamics we get the following set of independent linear ODE:

$$\frac{d\hat{e}_i}{dt} = (\gamma_i \beta_i) (w_i s_i - \hat{e}_i^t)$$
If the dynamic of personal norms is governed by consistency, then personal norms $\hat{e}_i^\infty$ and equilibrium behavior $\bar{e}_i^\infty$ converge to a level equal to the Nash equilibrium in material payoffs of the team production game ($\hat{e}_i^\infty = \bar{e}_i^\infty = w_i s_i$). Moreover, $\hat{e}_i^\infty$ is a globally stable steady state.

- Personal norms tend to the individual marginal revenue.
- All agents end up choosing an effort that only depend on the own skills and on the sharing rule.
- At the steady state, personal norms dictate inefficient efforts. For example, even if agents start with personal norms for efficiency ($\hat{e}_i^0 = s_i$), in the steady state personal norms get eroded and tend to the individual marginal revenue $w_i s_i$. 
Cognitive dissonance

- Consistency provides a dynamic foundation of (conventional) Nash equilibrium.
- Diversity of both behavior and values equals the dispersion of the individual marginal revenues in the society.
- Notice that in the long run there is complete coherence between behavior and values, i.e. there is not cognitive dissonance.
- Notice that if some agent has $\gamma_i = 0$, $\beta_i = 0$ or $\beta_i = 1$, this only has an effect on his own dynamics maintaining unchanged the dynamics of personal norms and behaviors of all the other agents.
By conformity, personal norms tend to move towards the average of the actual behavior of the organization.

The evolution of personal norms in this case takes the following form:

$$\hat{e}_{i}^{t+1} = \alpha_i \langle \bar{e}^t \rangle + (1 - \alpha_i) \hat{e}_i^t, \ i = 1, 2, \ldots N$$

where $\alpha_i \in (0, 1)$ measures the degree of conformism and $\langle \bar{e}^t \rangle$ is the average of the NE of the group in period $t$.

(We denote by $\langle b \rangle$ the average of a variable $b$.)
Lemma

*If the dynamics of personal norms is governed by conformism, then in the steady state the personal norm of all agents converges to the same value, \( \hat{e}_i^\infty = \frac{\langle \beta ws \rangle}{\langle \beta \rangle}, \forall i \).*

We assume that the random variables \( \beta_i \) and \( w_is_i \) are statistically independent, so that we can approximate \( \langle \beta ws \rangle \simeq \langle \beta \rangle \langle ws \rangle \). Then:

**Corollary**

*If personal norms are governed by a dynamics of conformism and the variables \( \beta_i \) and \( w_is_i \) are statistically independent, then the personal norm of each agent in the steady state is \( \hat{e}_i^\infty = \langle ws \rangle, \forall i \) and the equilibrium behavior is \( \bar{e}_i^\infty = \beta_i w_is_i + (1 - \beta_i) \langle ws \rangle \), for \( i = 1, 2, \ldots, N \).*

- This result states that having just conformism, the group tends to complete homogeneity in values.
- Notice that with this dynamics, even if all agents have the same personal norm, each agent performs a different action given the different incentives deriving from skills, sharing rules and preference.
In the previous case all agents had a conformist influence since $\alpha_i > 0 \ \forall i$.

Next we analyze cases in which not all agents have positive values for conformism, $\alpha_i$ or they have different values of $\beta_i$.

We will analyze the influence of a subset of agents in the group with a different set of parameters both on the evolution of personal norms and the equilibrium behavior in the long run.
Dynamics of Personal Norms with Conformity

Case 1: The effect of non-conformist agents in the group.

A number $N_0$ of non-conformists agents with $\alpha_i = 0$, but $\beta_i > 0$, $\forall i$.

Lemma

If personal norms are governed by a dynamics of informational conformism, but a number $N_0$ of agents are non-conformist ($\alpha_i = 0$), then in steady state,

Non-conformist agents: $\hat{e}_i^\infty = \hat{e}_i^0$ and Conformist agents:

$$\hat{e}_i^\infty = \frac{\langle \beta ws \rangle + \frac{N_0}{N} \langle (1 - \beta) \hat{e}_i^0 \rangle_{N_0}}{\langle \beta \rangle + \frac{N_0}{N} \langle 1 - \beta \rangle_{N_0}}$$

- Even in presence of some non-conformist agents, all conformist agents have the same personal norm homogeneity concerning values. This value, however, depends on the initial level of personal norms of non-conformists.
A number $N_0$ of non-materialist agents with $\beta_i = 0$.

**Lemma**

If personal norms are governed by a dynamics of conformism, but a number $N_0$ of agents are non-materialist ($\beta_i = 0$), then the steady state personal norm is $\hat{e}^\infty = \langle ws \rangle_{N/N_0}$.

In this case only the $(N - N_0)$ agents with $\beta_i > 0$ influence the steady state homogeneous culture.
Dynamics of Personal Norms with Conformity

Case 3: The effect of selfish agents in the group.

- A number $N_0$ of agents with $\beta_i = 1$ (are exclusively concerned by material payoffs) and $N - N_0$ agents with $\beta_i \in (0, 1)$. However every non-selfish agent has $\alpha_i > 0$.
- Obviously, selfish agents do not have any $\hat{e}_i$, or it is irrelevant.

Lemma

If personal norms are governed by a dynamics of informational conformism, and there is a number $N_0$, of selfish agents ($\beta_i = 1$), then in steady state:

Selfish agents: no personal norm and $\bar{e}_i^\infty = w_is_i$

Rest of agents: Personal norms $\hat{e}_i^\infty = \frac{\langle \beta ws \rangle}{\langle \beta \rangle}$ and

$\bar{e}_i^\infty = \beta_i w_is_i + (1 - \beta_i) \frac{\langle \beta ws \rangle}{\langle \beta \rangle}$
Notice that in all the previous cases if personal norms evolve exclusively because of Conformism, a strong culture arises in the long-run.

- An homogeneous culture in personal norms (values) but not in behavior.
Notice that in all the previous cases if personal norms evolve exclusively because of Conformism, a strong culture arises in the long-run.

- An homogeneous culture in personal norms (values) but not in behavior.
- The presence of non-conformist agents or non-materialistic agents or selfish agents affects the particular value of the homogeneous personal norm in the steady state.
Complete Dynamics: Cognitive Dissonance and Informational Conformity.

- With probability \((1 - \tau_i)\) the personal norm \(\hat{e}_i^{t+1}\) of individual \(i\) moves towards the average equilibrium behavior in the group, and with probability \(\tau_i\) the personal norm moves in the direction of the individual equilibrium behavior.

\[
\hat{e}_i^{t+1} = \tau_i (\gamma_i \bar{e}_i^t + (1 - \gamma_i) \hat{e}_i^t) + (1 - \tau_i) (\alpha_i \langle \bar{e}^t \rangle + (1 - \alpha_i) \hat{e}_i^t)
\]

- We denote by \(a_i \equiv \tau_i \gamma_i\) the weight associated to consistency and by \(b_i \equiv \alpha_i (1 - \tau_i)\) the weight associated to conformism for each player, where both \(a_i, b_i \in (0, 1)\).

- For simplicity the level of materialism \(\beta\) will be the same for all players \(\beta_i = \beta \forall i\).
**Lemma**

If personal norms are governed by a dynamics that is a mix of consistency and informational conformism, the steady state personal norm of each agent is

\[ \hat{e}_i^\infty = \left( \frac{a_i \beta}{a_i \beta + b_i} \right) (w_i s_i) + \left( \frac{b_i}{a_i \beta + b_i} \right) \langle ws \rangle, \text{for } i = 1, 2, \ldots N. \]

Moreover, the equilibrium behavior is

\[ \bar{e}_i^\infty = (\beta + (1 - \beta) \left( \frac{a_i \beta}{a_i \beta + b_i} \right)) (w_i s_i) + (1 - \beta) \left( \frac{b_i}{a_i \beta + b_i} \right) \langle ws \rangle, \text{for } i = 1, 2, \ldots N. \]

- Therefore in the steady state, personal norms are a convex combination between individual marginal revenue and average marginal revenue.
- Note that the first part is driven by consistency and the latter by conformity.
The diversity of culture.

Now we are ready to define a measure of the diversity of a culture in the long run.

The lower the variation of personal norms, the stronger (the more homogeneous) is the culture.

The natural candidate for this measure of dispersion is the variance.

Consider first the variance of personal norms in the steady-state. We denote this variance as \( \sigma^2(\hat{e}^\infty) \) which is given by:

\[
\sigma^2(\hat{e}^\infty) = \frac{1}{N} \sum_{i=1}^{N} (\hat{e}_i^\infty - \langle \hat{e}^\infty \rangle)^2
\] (6)
The diversity of culture.

- Assuming statistical independence between \( a_i, b_i \) and \( w_i s_i \), we can approximate

\[
\sigma^2(\hat{e}^\infty) = \left\langle \left( \frac{a\beta}{a\beta + b} \right)^2 \right\rangle \sigma^2(ws)
\]

- Note that \( \left\langle \left( \frac{a\beta}{a\beta + b} \right)^2 \right\rangle < 1 \), so \( \sigma^2(\hat{e}^\infty) \leq \sigma^2(ws) \).

- Variance of personal norms in the steady state is smaller than the variance of the individual marginal revenues of the agents.

- Rough intuition: the dynamics of personal norms creates some sort of homogeneity in the population personal norms.
The determinants of diversity.

- **An increase in** $\sigma^2(ws)$, will yield an increase in the variance of personal norms.

- We know that the variance of this product is (if $w$ and $s$ are statistically independent):
  \[
  \sigma^2(ws) = \frac{1}{N^2} \sigma^2(s) + \langle s \rangle^2 \sigma^2(w) + \sigma^2(w) \sigma^2(s)
  \]

- An increase both in the variance of the distribution of skills and in the variance of the sharing rule (a more unequal income distribution) will cause an increase in the variance of the product and in turn a rise in the variance of personal norms.

- This means that a society in which skills are unevenly distributed will show a weaker culture (more diversity).

- In the same way, if the variance of the remunerations distribution is high (sharing rule with high variance), we can expect a reduction in the strength of the culture.
The determinants of diversity.

- An **increase in the level of conformism** $b$ has the effect of decreasing the variance of personal norms.
- An increment in the level of conformism in the group will lead the agents to adopt similar personal norms reducing their variance. In this case the society will show a stronger culture, that is less diversity in values.
- An increase in **the level of consistency** $a$ will raise the variance of personal norms, reducing the strength of the social norm.
- This is due to the fact that, by consistency, each agent will have its own personal norm moving towards the level predicted by his own material incentives.
The determinants of diversity.

- We will analyze the effects of a change in the dispersion of the population distribution of conformism.
- In order to make meaningful comparisons we analyze the impact of Mean Preserving Spread changes in the dispersion of the distribution of the parameter $b_i > 0$.
- There is a well known result from Rothschild and Stiglitz (1970) that states that a MPS increases the expected value of a convex function. It is easy to check that the second derivative of the expression with respect to $b$ is positive, thus the function is convex and, consequently, an increase in the dispersion of the distribution of $b_i$, will increase the variance of the personal norms and this in turn implies an increase in the diversity.
- The intuition of this result relies on the fact that in a society with a less dispersed level of conformism, culture is stronger (less diversity).
Finally, an **increase in the level of materialism** ($\beta$) will lead to an increase in the diversity.

An increase in the weight assigned to the material payoffs is equivalent to a reduction in the influence of personal norms and hence, a reduction in the pressure of the dynamics, resulting in a greater influence of productivity and/or sharing rule.

Summarizing, for a higher $\beta$, the individuals are more selfish and less influenced by personal norms, weakening the culture.

Note that in the opposite case when $\beta$ tends to 0, then $\sigma^2(\hat{e}^\infty)$ tends to 0. In this case $\bar{e}_t^i = \hat{e}_t^i, \forall t$, therefore, only conformism matters.

Societies with very low levels of materialism have stronger (more homogeneous) cultures.
The diversity of behavior.

- We will analyze a measure of the "observed" **diversity of behavior in the group** given by the variance of the equilibrium actions in the steady state. We denote this variance as $\sigma^2(\bar{e}^\infty)$ which is given by:

$$\left[ \sigma^2(\bar{e}^\infty) = \frac{1}{N} \sum_{i=1}^{N} (\bar{e}_i^\infty - \langle \bar{e}^\infty \rangle)^2 \right]$$

- Hence, with the statistical independence between $a_i$ and $b_i$ and $w_is_i$ we obtain

$$\sigma^2(\bar{e}^\infty) \sim \left\langle \left( \frac{\beta(a + b)}{a\beta + b} \right)^2 \right\rangle \sigma^2(ws)$$

- Now it is easy to check that the variance of the equilibrium behavior is increasing with the productivity and sharing rule ($ws$) variance, with the degree of materialism ($\beta$) and with the level of consistency ($a$) and is decreasing with the level of conformism ($b$).
The diversity of a culture.

Relation between the diversity of Personal Norms and Behavior

\[ \sigma^2(\tilde{e}^\infty) = \frac{\left\langle \left( \frac{\beta(a+b)}{a\beta+b} \right)^2 \right\rangle}{\left\langle \left( \frac{a\beta}{a\beta+b} \right)^2 \right\rangle} \sigma^2(\hat{e}^\infty) \]

Notice that \( \sigma^2(\tilde{e}^\infty) \geq \sigma^2(\hat{e}^\infty) \), that is the diversity of behavior is always greater than the diversity of values in any group or organization.
The level of incoherence in a culture.

- What is the degree of **incoherence between behavior and values** in a culture?
- We define the level of incoherence of a culture as the variance of the distances between $\bar{e}_i^\infty$ and $\hat{e}_i^\infty$.

$$\rho^2 = \frac{1}{N} \sum_{i=1}^{N} (\bar{e}_i^\infty - \hat{e}_i^\infty)^2$$

- After some calculations,

$$\rho^2 \approx \left\langle \left( \frac{\beta b}{a\beta + b} \right)^2 \right\rangle \sigma^2(\text{ws})$$

- The level of incoherence of a culture represents a psychological cost and it increases with $\sigma^2(\text{ws})$, with the level of materialism $\beta$, with the level of conformism $b$ and decreases with the level of consistency $a$. 
The determinants of diversity and incoherence.

Our model shows **how the diversity and coherence** of an organizational culture **depends on primitive economic and behavioral variables** and establishes some clear predictions.

- An increase in the variance of the distribution of skills $\sigma^2(s)$ yields a culture with more diversity in behavior and values and more incoherence.
- An increase in the variance of the income distribution $\sigma^2(w)$, that is, a more unequal distribution, yields a culture with more diversity in behavior and values and more incoherence.
- An increase in the level of materialism of the society $\beta$ yields a culture with more diversity in behavior and values and more incoherence.
- An increase in the level of consistency of the society $a$ yields a culture with more diversity in behavior and values and less incoherence.
- An increase in the level of conformism of the society $b$ yields a culture with less diversity in behavior and values and more incoherence.
How does the diversity of a culture influence the **total production and social welfare of the organization**?

Social Welfare in the group is a weighted average of gross output $y(\bar{e}^\infty)$, aggregate material costs $C(\bar{e}^\infty)$ and the psychological costs associated to incoherence $L(\rho^2)$.

$$SW = \beta y(\bar{e}) - \beta C(\bar{e}^\infty) - (1 - \beta) L(\rho^2)$$
Aggregate production and Social Welfare.

\[ y = N \left[ \frac{\langle s \rangle^2}{N} + (\beta + (1 - \beta)A)(\frac{\sigma^2(s)}{N} + \text{Cov}(w, s^2)) + (1 - \beta)(1 - A)\langle s \rangle \text{Cov}(w, s) \right] \]

- **Aggregate output** is an increasing function of \( \langle s \rangle, \sigma^2(s), \text{Cov}(w, s) \) and \( \text{Cov}(w, s^2) \). Notice that \( A = \frac{\beta a}{\beta a + b} \).

\[ C = \frac{\langle s \rangle^2}{2N} + \frac{N}{2} (A + \beta (1 - A)^2 \cdot \sigma^2(ws) + \frac{N}{2} (\text{Cov}(w, s))^2 + \langle s \rangle \text{Cov}(w, s) = \]

\[ = \frac{\langle s \rangle^2}{2N} + \frac{N}{2} \sigma^2(\bar{e}^\infty) + \frac{N}{2} (\text{Cov}(w, s))^2 + \langle s \rangle \text{Cov}(w, s). \]

- **The material cost function** is increasing in \( \langle s \rangle, A, \beta, \sigma^2(ws) \) and \( \text{Cov}(w, s) \).

\[ L(\rho^2) = \frac{N}{2} \rho^2 = \frac{N}{2} \beta^2 (1 - A)^2 \cdot \sigma^2(ws) \]

- **The loss function** is increasing in \( \beta \) and \( \sigma^2(ws) \) and decreasing in \( A \).
At first sight notice that an increase in diversity, $\sigma^2(\bar{e}^\infty)$ increases material costs $C(\bar{e}^\infty)$ and therefore lowers Social Welfare and an increase in Incoherence $\rho^2$, increases the psychological losses and also diminishes Social Welfare.

The final effects on SW depend nevertheless on which is the primitive parameter that changes, the determinants of diversity and incoherence.

For instance, in general $\sigma^2(ws)$ depends on $\langle s \rangle$, $\sigma^2(s)$, and $\sigma^2(w)$, but also on $Cov(w,s)$ if the remuneration is positively correlated to the skills.
Aggregate production and Social Welfare.

- Assume for instance that $w$ and $s$ are statistically independent ($\text{Cov}(w, s) = 0$). An example is a group where $s_i = s$ for $\forall i$. Then,

$$y(\bar{e}) = \langle s \rangle^2 + (\beta + (1 - \beta)A) \cdot \sigma^2(s)$$

$$C(\bar{e}^\infty) = \frac{\langle s \rangle^2}{2N} + \frac{\sigma^2(s)}{2N}(A + \beta(1 - A)^2) + \frac{N}{2}(A + \beta(1 - A)^2)(\langle s \rangle^2 \sigma^2(w) + \sigma^2(w)\sigma^2(s))$$

$$L(\rho^2) = \frac{N}{2} \beta^2 (1 - A)^2 \cdot \left( \frac{\sigma^2(s)}{N^2} + \langle s \rangle^2 \sigma^2(w) + \sigma^2(w)\sigma^2(s) \right)$$

- The egalitarian distribution $w_i = 1/N$ for $\forall i$, is the optimal one that maximizes social welfare in this case.
Assume now that \( w \) and \( s \) are not statistically independent \( (\text{Cov}(w, s) \neq 0) \). For simplicity let us assume a continuous of individuals in the unit interval. Skills are uniformly distributed in the interval \([0, 1]\) and \( A \) is constant.

Let us analyze the following family of sharing rules: \( w_i = s_i^\lambda, \lambda \geq 0 \) and s.t. \( w_i > w_j \) iff \( s_i > s_j \).

If \( \lambda = 0 \), all agents are paid the same. The higher is \( \lambda \) the higher is the payoff for individuals with higher skills and the lower is the payoff for those with lower skills.
Aggregate production and Social Welfare: an example

On the x-axis there is $\lambda$ and on the y-axis there is $D = (\beta + (1 - \beta)A)$. We show the optimal level of $\lambda$ that maximizes total net production.

Figure 1. Optimum level of $\gamma$
The optimal level of $\lambda$ is always less than one. If $D$ is small, then it is always optimal to set $\lambda = 0$.

In general it is optimal to remunerate marginal skills in a decreasing manner. The remuneration increases at a smaller rate than the rate of increase of skills.

The psychological cost function $L$ is decreasing with $\lambda$, so that it is minimized by setting $\lambda = 0$.

The value of $\lambda$ that maximizes Social Welfare will be smaller than the values that maximizes total net production.
Further Research

- **Leadership** and influential individuals (see Hernandez, Olcina and Toral 2014).
- An organization as a **Network**.
- Exogenous shocks of productivity. **Changes in the environment** of the group and the value of flexibility.