Education and family income: can poor children signal their talent?

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Abstract

The aim of this paper is to explain how financial constraints and family background characteristics affect the signalling educational investments of individuals born in low-income families.

We show that talented students who are poor are unable to signal their talent, as the maximum level of education they can attain may also be achieved by less talented students who are rich. Under this approach, a decrease in inequalities across generations cannot be expected. The paper also shows that an increase in educational standards would help poor individuals with high-ability if it is combined with other non-monetary measures.

Keywords: signalling education, segregation, educational standard, equal opportunities

JEL classification: I20, C70

1 Introduction

Economists have long expressed concern about the inequality of opportunities between rich and poor. Children raised in high-income families earn more than children raised in low-income families; there is a strong correlation of about 0.4 between a father and a son's permanent earnings (Solon, 1992; Altonji and Dunn, 1991; Zimmerman, 1992).[1] Despite this evidence, it remains unclear how parents' income determines children's outcome, and it is important to unravel how poverty is transmitted in order to design suitable policies for reducing inequality.

One of the most common policies to provide equal opportunities to all individuals has been to subsidize formal education. The justification is that capital market imperfections prevent agents from borrowing against future human capital incomes.[2] Moreover, simultaneously to this extension of subsidies, loans or public education, a large number of prestigious MA courses and private Colleges, Universities or Schools, that not all people can afford, have appeared. Having a college degree from these institutions or a Masters from some schools, is a sure-fire ticket to a high-paid job; people who attend the top 50 schools are virtually guaranteed a very comfortable job.

In this paper we try to explain the persistence of inequality across generations and the proliferation of exclusive and prestigious Universities. Inequality is mainly transmitted trough educational attainments. Children from poorer backgrounds achieve lower educational levels. Two main reasons can be found. First, as Becker and Tomes (1986) pointed out, poorer families are financially constrained or find it more difficult access to credit markets

[1] See Solon (1999) for a survey of intergenerational correlation in earnings.

[2] If capital markets are imperfect and poor parents are financially constrained, they invest less in children's education (Becker and Tomes, 1986.)

which increases the monetary cost of financing education for their children. This effect of a family's income on a child's achievement is direct. Second, poorer parents may have some characteristics that make them less successful on the labour market and which affect children. Family background characteristics affect motivation, access to career information, constancy, discipline and other learning skills. Thus, although money matters, as proved by extensive literature, other unobservable characteristics correlated with family income are also important.[3]

Thus, we consider that a family's background is a determinant factor in educational achievement together with innate ability. The correlation between innate ability and family wealth is controversial to assume.[4] We

[3] Empirical studies have a great deal of difficulty in detecting any systematic influence of parents' income in children's human capital. While, for example, Gaviria (2002) and Maurin (2002) found evidence that borrowing constraints prevent parents from investing optimally in their children's human capital, Cameron and Taber (2000) or Mulligan (1997) do not find evidence of borrowing constraints. Shea (2000) found that changes in parents' income due to luck have a negligible impact on children's human capital for most families.

The reason is that although money could be important, other unobservable characteristics correlated with family income may also be important. Thus, Chevalier and Lanot (2002) found that family characteristic effects dominate the financial constraint effects in schooling investment. PISA (2003) documents the importance of family background (and specifies what is included under this concept) in children's educational achievement in OECD countries.

A survey of the research on the relationship between parents' social status and their children's achievement can be found in Haveman and Wolfe (1995) or in the Introduction of Maurin (2002). See Shea (2000) and Chevalier and Lanot (2002) for additional references and also for a discussion on this matter.

[4] One of the most cited studies related with this matter (The Bell Curve, Hernstein and Murray, 1994) argue that ability is correlated with income. Specifically, the authors suggest that a child's economic success is mostly explained by cognitive ability (IQ), that

will assume that talented children are born in rich and poor families with equal probability. However, talent only leads to high wages if it can be signalled in the labour market.[5] If poor talented children cannot signal their talent, they will never achieve the same labour market outcomes as rich talented children.

The aim of this paper is to explain how financial constraints and family background characteristics affect the signalling possibilities of poor children. This will help to explain the transmission of inequality across generations and also why inequality has persisted over time despite the implementation in most Western nations of policies to facilitate access to formal education. Specifically, the paper presents a signalling model in the labour market in which firms interpret education as a signal of ability and therefore offer a higher wage to an individual with more education. Unlike Spence (1973), individuals differ not only in their innate ability but also in their families' socioeconomic background.

In our model we will show, in the first place, that talented rich people will always attain the highest education level, which allows them to signal their talent, under some reasonable educational cost conditions. It means that any

is, by one factor transmitted by genes. Therefore, roughly speaking we could say that rich families pass "better" genes to their children (compared to poor families). Hence, poverty is also transmitted. However, the statement in The Bell Curve has been widely reviewed and reexamined. Other authors find that the socioeconomic status of people's parents and their broader social environment is at least as important, and may be more important than IQ in determining social and economic success in adulthood (see Fisher, 1996; Koreman, 1995; Currie and Thomas, 1995; Heckman, 1995). So, in this paper we will not assume that children's ability is correlated with parents' income.

[5] Despite the difficulty of testing the signalling hypothesis (Spence, 1973, 1974), it is commonly assumed that high-ability leads to higher wages in the labour market. See Riley (2001) for an excellent modern survey on signalling.

other type does not find it profitable to meet the education cost (monetary or non-monetary) of such a level, given the potential gains. Imitation is prevented. This guaranties them the highest wage in the labour market. Second, we will show that poor people, even those with great talent, will not be able to signal their talent and achieve the same wage as talented rich people. The reason is that the maximum education level they can attain may also be achieved by less talented students who are rich. The latter prefers to mimic the talented poor people than to reveal himself. This results in children from poorer backgrounds not achieving the same labour market outcomes as the richer children.

If education works as a relevant signalling device, a decrease in inequalities across generations cannot be expected (unlike Becker and Tome' (1979) model of human capital), as talented rich individuals always have an incentive to send a signal -education- that no other types would ever mimic.[6] As much as public education spreads to higher educational levels, rich talented people will "fly" to prestigious but expensive educational institutions in order to preserve their position in the educative ranking. Education becomes defensive expenditure, as Thurow (1975) argue. Unless (i) large amounts of money are transferred to poor families to facilitate their access to any type of college or university, (ii) educational policies are oriented towards increasing the role of ability in educational achievement (by raising educational standards, for example), (iii) the segregation of students among

[6] In Becker and Tomes' (1979) model, convergence to the mean of earnings is expected, as high-income parents stop investing in their children's education when the marginal rate of return of human capital equals that of physical capital. Moreover, the effectiveness of their higher human capital investments to increase children's earnings is lower (decreasing marginal returns of human capital are assumed).

schools on based on families' socioeconomic status decreases, or (iv) some other scholar support is implemented in order to reduce disparities due to unobservable family characteristics, the richest-high ability individuals will find room to signal their talent -and they will be the only ones able to do so. Consequently, they will preserve the position their parents had in the labour market.

The paper is organized as follows: section 2 outlines the model. Section 3 contains a general analysis and the basic results. In Section 4, we present the equilibrium of the model and in Section 5 some policies to guarantee equal opportunities are discussed. Conclusions are offered in Section 6.

2 The model

Consider that there are two types of families in the society depending on their level of wealth. Rich families, r, represent a proportion $(1-\delta)$ and poor families, p, a proportion δ of the total number of families which is normalized to one, $\delta \in (0,1)$. Each family has a child with an innate ability, which can be either high h or low l with equal probability. Given this ability, that is private information, families invest in their children's education, e. Firms observe this education levels and offer wages to the individuals. The variable e measures the number of diplomas, the number and kind of extra-academic courses taken and the caliber of grades and distinctions earned during an academic year, taken into account the quality of the school. A given educational level e could reflect, for example, the best graduate or a given school but also a degree in the best schools. Apart from that, in this model $e=0$ gets the interpretation of having just compulsory education.

Thus, there are four types of individuals depending on their innate ability and their family's wealth. Let ij denote an individual with j-ability from a i-type family. Thereby the types are denoted by $\{pl, ph, rl, rh\}$.

It is assumed that firms compete for workers in the labour market. Hence, the wage offered by a firm to a worker with education e, $w(e)$, equals his expected productivity. Given the firms' belief about the worker's ability after observing e, the wage offer will be $w(e) = \mu(ih/e)H + \mu(il/e)L$, where $\mu(ih/e)$ is the firms' assessment of the probability that the worker's ability is h (or l for $\mu(il/e)$), and H and L are the productivity of a worker with high and low ability, respectively. So the firms' payoff is given by the productivity of a worker minus the paid wage.

Families are altruistic and finance the education cost of their children and also value the wage their children will obtain in the labour market. Rich families have a wealth R and poor families have P. The utility function of a family with an ij-child type is:

$$U_{ij}(e, w) = W^i + E[w/e] - c(e, i, j),$$

where $W^i \in \{R, P\}$ denotes the i family's wealth, $E[w/e]$ is the expected wage if the child reaches the labour market with education e and $c(e, i, j)$ is the cost of a level of education e for an i family with a child of $j-$ability.

Individuals require a certain innate ability and some monetary support from their family as inputs to obtain education. The education production function we propose includes these two inputs, but also captures some stylized facts concerning the influence of family wealth on the educational achievements of their children. On the one hand, education depends, as we said, on monetary investment and poorer families are financially constrained or find it more difficult the access to the credit market, thus increasing the monetary cost of financing education. On the other hand, family background characteristics may affect children's performance at school. Most of these characteristics are correlated with family income and wealth. For instance, the occupational status of parents, their higher educational levels and the families' cultural possessions are all aspects that can influence students' performance (PISA, 2003). Rich families are better able to take advantage of the educational system or schools find it easier to educate them.

We summarize the influence of family socioeconomic background on the production of education as an input and denote it by index i. Thus, the input i, $i = \{r, p\}$ captures the learning skills which depend of family background. We assume that children from rich families do better at school, *ceteris paribus*, than those from poor families, that is $p < r$.

We assume the existence of an educational function:

$$\begin{aligned} e &= F(s, i, j), \qquad (1) \\ s.t.\, F_s &> 0,\ F_j > 0,\ F_i > 0, F_{ss} \leq 0, F_{jj} \leq 0,\ F_{ii} \leq 0,\ F_{ij} > 0,\ F_{si} > 0,\ F_{sj} > 0. \end{aligned}$$

where e is the educational achievement of an individual (for convenience, educational achievement of the ij child may be also denoted by e_{ij}) and s is the parental monetary support, that is, the actual educational investment. Therefore, some substitutability exits among inputs. The higher the innate ability, *ceteris paribus*, the higher the educational level and, the more family support, *ceteris paribus*, the higher the educational achievement.

Monetary support is decided by the family. Poorer families face borrowing constraints. Hence, we assume that the gross interest rate is higher for poor families, π^p, than for rich families, π^r. The relationship between a family's educational investment, s, and educational cost is given by the equation:

$$C = \pi^i s.$$

To move from the education production function (1) to a cost function involves solving equation (1) for s and then plugging into the above equation. This gives the education cost function for the ij type:

$$c = \pi^i F^{-1}(e, j, i). \tag{2}$$

From now on, the cost of a level of education e for an i-family with a child of j-ability will be denoted by $c(e, i, j; \pi^i)$. And the marginal cost of education for the ij-type will be given by $c_e(e, i, j)$

$$c_e(e, i, j) = \pi^i \frac{\partial \left[F^{-1}(e, j, i)\right]}{\partial e} \text{ or } c_e^{ij} = \frac{\pi^i}{F_s(s, j, i)}.$$

So, the marginal cost of education for the four types of individuals are given by

$$\begin{aligned}
c_e(e, p, l) &= \frac{\pi^p}{F_s(s, l, p)}, && \text{(a)}\\
c_e(e, p, h) &= \frac{\pi^p}{F_s(s, h, p)}, && \text{(b)}\\
c_e(e, r, l) &= \frac{\pi^r}{F_s(s, l, r)}, && \text{(c)}\\
c_e(e, r, h) &= \frac{\pi^r}{F_s(s, h, r)}. && \text{(d)}
\end{aligned}$$

Therefore, for a given ability, the marginal cost of education for any educational level $\bar{e}$ is higher for poor than for rich families. Notice that:

$$c_e(\bar{e}, p, j) = \frac{\pi^p}{F_s(s^{pj}(\bar{e}), j, p)} > \frac{\pi^r}{F_s(s^{rj}(\bar{e}), j, r)} = c_e(\bar{e}, r, j)$$

where $s^{ij}(\bar{e})$ is the monetary support required by the $ij-$type to finance the education level $\bar{e}$ that solves $\bar{e} = F(s, j, i)$. As $F_i > 0$ and $p < r$, then $s^{pj}(\bar{e}) > s^{rj}(\bar{e})$ for a given ability. Moreover, given that $F_{si} > 0$ and $F_{ss} \leq 0$, we obtain $F_s(s^{pj}(\bar{e}), j, p) < F_s(s^{rj}(\bar{e}), j, r)$. Since $\pi^p > \pi^r$, the above inequality is fulfilled.

Moreover, for any educational level $\bar{e}$ and for any family wealth, the marginal cost of education is higher for low than for high ability individuals:

$$c_e(\bar{e}, i, l) = \frac{\pi^i}{F_s(s^{il}(\bar{e}), l, i)} > \frac{\pi^i}{F_s(s^{ih}(\bar{e}), h, i)} = c_e(\bar{e}, i, h)$$

since $s^{il}(\bar{e}) > s^{ih}(\bar{e})$, $F_{sj} > 0$ and $F_{ss} \leq 0$.

Finally, let us compare the marginal cost of education for "intermediate" types. As we will show in the next section, this relation turns out to be crucial for the results of our model. The marginal education cost for a poor individual of high ability, $c_e(\bar{e}, p, h)$, is higher than for a rich individual of low-ability, $c_e(\bar{e}, r, l)$ if:

$$c_e(\bar{e}, p, h) = \frac{\pi^p}{F_s(s(\bar{e}), h, p)} > \frac{\pi^r}{F_s(s(\bar{e}), l, r)} = c_e(\bar{e}, r, l), \tag{A.1}$$

which requires that the marginal increase in educational achievement of the last m.u. invested by the poor family to be lower than the marginal increase obtained by the rich family.

It is worthy to comment that although π^p is close to π^r, (i.e., borrowing constraints are not important), the marginal cost between these two types of families can be quite different if families' backgrounds are different enough. Formally, for a given $\bar{e}$, if $\frac{\pi^p}{\pi^r} = 1$, the crucial assumption can be verified if $\frac{F_s(s(\bar{e}),h,p)}{F_s(s(\bar{e}),l,r)} < 1$ (or if $F_s(s(\bar{e}), h, p) < F_s(s(\bar{e}), l, r)$), which requires a "high" difference $[s^{rl}(\bar{e}) - s^{ph}(\bar{e})]$, given that $F_{ss} < 0$. Thus, if the families' backgrounds are different enough, the monetary support needed for children from poorer families' background will be much higher than for richer children and, consequently, the marginal productivity of the last m.u. invested by the poor family will be lower. In this context, the crucial assumption may be fulfilled.

In most of the paper we will assume that (A.1) holds. Thereby, low-income families face higher marginal education costs than high-income families, $c_{ei} < 0$, regardless of their children's ability.

Finally, let us recall the timing of the game. Firstly, nature determines an individual's ability. Secondly, the family chooses a level of education $e \geq 0$. Thirdly, firms observe the worker's education and then simultaneously make wage offers to the worker. Finally, the worker accepts the highest wage offer.

3 Analysis of the model

3.1 Solution concept

An equilibrium exits in this model when (i) the educational level chosen by each family type is optimal, given the wages they anticipate the firms will offer; (ii) a firm's wage offer is the expected productivity of the worker, given their beliefs about which type of individual they are dealing with; (iii) firms form their beliefs in a reasonable way; and (iv) the aggregate level of education is the minimum compatible with (i) to (iii). By a reasonable way we mean that firms' beliefs are formed according to Bayes' Law when they observe an equilibrium education level (i.e., an educational level chosen by a family with positive probability in the equilibrium being played) and that these beliefs hold some restrictions for a non-equilibrium educational level (i.e., an educational level that is a deviation from the equilibrium being played).[7]

[7] Conditions (i) and (ii) plus Bayesian consistency constitute the solution concept Perfect Bayesian Equilibrium (PBE). Condition (iv) is a further requirement that we add for tractability. Any equilibrium which verifies (i)-(iii) guarantees the minimum aggregate

Specifically, out of equilibrium beliefs must satisfy a standard minimal restriction: the Intuitive Criterion (Cho and Kreeps, 1987). An educational level e' is equilibrium-dominated for the ij type if this family does worse choosing this level e', no matter how the firms respond, in comparison to her expected equilibrium utility. An equilibrium satisfies the Intuitive Criterion if the firms' belief, when they observe an out-of-equilibrium educational level, e', which is equilibrium-dominated for the type ij, places (if possible) zero probability on this type ij. (This is possible when e' is not dominated for all the types.)

Furthermore, we add a global consistency condition. Suppose that an education level is not chosen by any type of family in an equilibrium but it is chosen with positive probability by some families in an alternative equilibrium. In addition, these families are better off in the latter than in the former equilibrium. In this case, firms' beliefs must assign positive probability only to this set of families.

The intuition behind this condition is that each equilibrium represents a particular "theory" on how the game will be played in the society. The deviation from a particular equilibrium by certain types of families acquiring an unexpected level of education may be interpreted as a confusion on their behalf about what equilibrium is "really" being played. This is a forward induction argument that corresponds to a refinement notion proposed by Mailath et al. (1993), the Undefeated Equilibrium.[8]

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education level. Therefore, we will move straight on to the minimum education level that verifies (i)-(iii).

[8] As usual in signalling games, multiplicity arises and we apply a further refinement to the Intuitive criterion. The Undefeated Equilibrium imposes some restrictions on out-of equilibrium beliefs according to the notion of forward induction supported by extensive literature. This is a non-desirable trait of the model, but the main results (those presented

For tractability, we identify education levels and wages in equilibrium through vectors $(e_{pl}, e_{ph}, e_{rl}, e_{rh})$ and $(w_{pl}, w_{ph}, w_{rl}, w_{rh})$, respectively. We will focus on equilibria in pure strategies and recall that we assume that Condition A.1 holds.

3.2 General analysis

We begin by presenting a result that simplifies the analysis.

Lemma 1 *The education choice made by the different types of families is not increasing with the marginal cost of education.*

Proof. See Appendix A. ■

Therefore, according to assumption $c_e(e,p,l) > c_e(e,p,h) > c_e(e,r,l) > c_e(e,r,h)$, education choices are such that $e_{pl} \leq e_{ph} \leq e_{rl} \leq e_{rh}$. The lower the marginal cost for an individual is, the higher his educational achievement will be.

A direct consequence of Lemma 1 is presented in the following proposition.

Proposition 1 *A separating equilibrium à la Spence,* $(0, e', 0, e')$, *in which firms can distinguish high from low-ability individuals does not exist.*

Intuitively, a separating equilibrium in which firms are able to differentiate low from high-ability workers, does not constitute an equilibrium of this model because any educational level chosen by a poor child of high ability will be imitated by the rich child of low ability. Let us consider that there is an education level $e' > 0$ chosen only by high ability workers, ph and rh,

in the following Section) hold if only the Intuitive Criterion is applied.

that convince the firms that they are talented workers and accordingly, firms will pay the highest wage, $w(e') = H$, to those workers. On the other hand, those with just compulsory education $e = 0$ are considered low-ability individuals and are paid the lowest wage $w(0) = L$. Notice that such a situation is not a PBE because any education level that the poor type of high ability finds rational to attain to get the high wage, will be also found rational to attain by the rich type of low ability, given our assumption about education costs. Therefore this latter type will not be in equilibrium choosing $e = 0$.

Further intuition for Proposition 1 can be found in Figure 1. Let us consider the education $(0, e\prime, 0, e\prime)$ as a potential separating equilibrium. The indifference curves go through the pair $(0, L)$ which represents the option of not getting education and obtaining wage L. The other option is to choose an education level $e\prime$, that convinces firms that they are dealing with a high-ability type, yielding a wage H. Notice that any education level on the horizontal line at level H that type ph finds profitable to attain (for example, e'), will be also found profitable to attain by the rl type; any pair (e, H) with $e \in (0, e'')$ that lies above the ph indifference curve also lies above the indifference curve for the rl type.

Notice the crucial role played by the assumption that wealth is not observable by firms. From the analysis it is easy to see that, if wealth is observable, a separating equilibrium $(0, e\prime, 0, e\prime\prime)$ exists, with $e\prime\prime > e\prime$. The issue is that wealth is private information and poor people may find interest in telling the firms that they are poor. But rich people do not have such interest: the rich and low talented individuals to avoid being identified, and rich and high talented individuals because it implies to achieve a high educational level to convince the firms they are high talented (instead of being considered as poor and be identified as $h-$type just with education

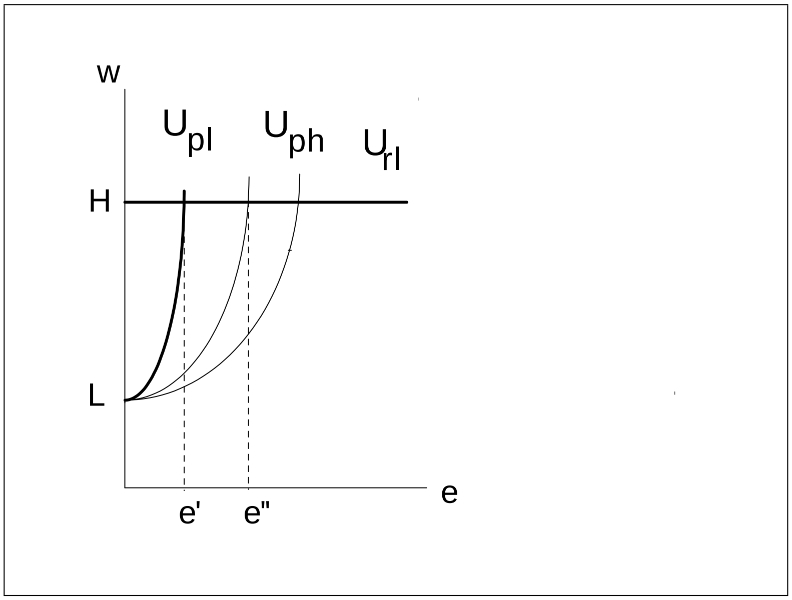

Figure 1: A separating equilibrium à la Spence does not exist

$e\prime$). Therefore, rich people have no incentive to reveal their wealth or, even worse, have incentive to cheat the firm. Since firms do not have the legal possibility to prove the veracity of the information about this aspect of the CV or it could be really costly (besides quite controversial), firms do not use this information as it seems to happen in the real world. Hence, people find no interest in telling the firms wether they are poor or rich.

Another interesting point is that a situation in which poor individuals choose an education level and rich individuals a different one, that is, a vector $(0, 0, e', e')$ can not be an equilibrium. In this case firms' beliefs after observing the education of poor types is that the individual may have high or low ability with equal probability. Accordingly, the firm will pay $\frac{1}{2}H$ $+\frac{1}{2}L$. On the other hand, firms' beliefs after observing the education of rich types is that with equal probability the individual can be of high or low ability and also pay $\frac{1}{2}H + \frac{1}{2}L$. Therefore, it is not rational for rich people to invest in education if the wage will be the same as it would be without investing.

We now analyse the best strategy for the "highest" type, rh.

Proposition 2 *In equilibrium, the education choice of a high ability individual from a rich family is the minimum education level that deters imitation from any other type.*

Proof. See Appendix B. ■

Formally, this proposition states that the education equilibrium choices e^*_{rh} and e^*_{rl}, are such that $U_{rl}(e^*_{rl}, w^*_{rl}) \geq U_{rl}(e^*_{rh}, H)$ where w^*_{rl} is the equilibrium wage for the rl type.

The intuition is clear and can be seen from the simple two-type model, without children's socioeconomic differences. High ability individuals are indistinguishable from low-ability individuals in a pooling equilibrium, so the "highest" type always has an incentive to break away and send a signal that a low-ability individual would never mimic, thereby implying that pooling cannot survive the Intuitive Criterion. The same argument can be used with children's socioeconomic differences. By Lemma 1 we know that the direct competitor of the high-ability and rich individual, rh, is the low-ability individual who is rich. So the above exposed logic applies, except that it is even more costly for the highest type to send a sufficiently large signal that his competitor would never mimic.

This result can also be understood with the help of Figure 2. Let us consider that the rich individual of high ability rh pools in an educational level $\hat{e}$, which implies a wage $\hat{w}$. According to Lemma 1, if the highest type

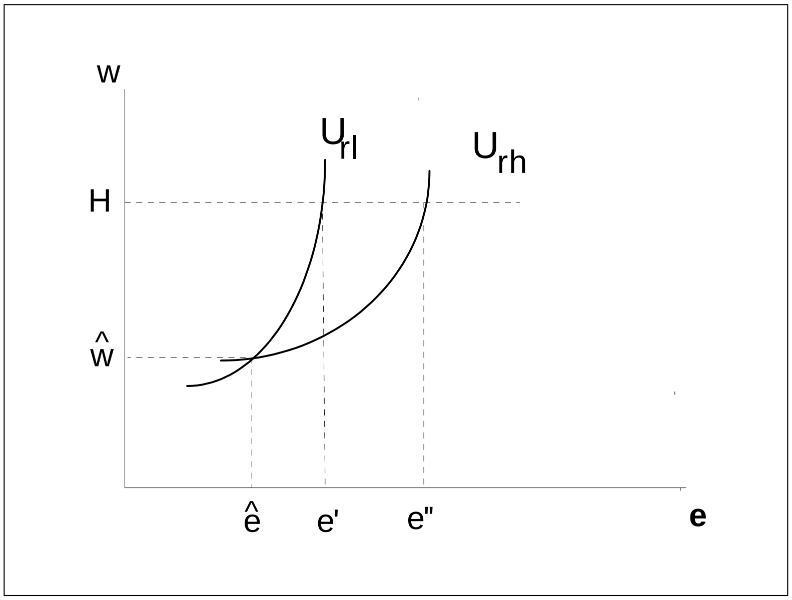

Figure 2: The rich indiviual of high-ability separates from any other type.

rh pools must be at least with the rl type, as education is not increasing with marginal costs. In order for $\hat{e}$ to be an equilibrium of our model, firms must believe that $\mu(rh/e) < 1$ for educational choices between e' and e'', because if $\mu(rh/e) = 1$, then the rh-type will deviate. It is easy to see in the Figure 2 that choices $e > e'$ are equilibrium-dominated for the rl type, because even the highest wage that could be paid to him, namely H, yields a pair (e, w) that lies below the rl type's indifference curve through the equilibrium point $(\hat{e}, \hat{w})$. However, education choices between e' and e'' are not equilibrium dominated for the rh type: if such a choice convinces firms that the worker has high ability, then firms will offer wage H, which will make the highest type, rh, better off than in the indicated pooling equilibrium. Thus, the Intuitive Criterion implies that firms' beliefs must be $\mu(rh/e) = 1$ for $e \in [e\prime, e\prime\prime)$. In this case, the highest type rh will deviate form the pool in $\hat{e}$.

In short, a high ability individual from a rich family never pools in equilibrium. They will attain an educational level that discourages imitation from the other types.

Proposition 3 *In equilibrium, low ability-rich individuals and high ability-poor individuals ("intermediate" types, ph and rl) choose the same education level, that is, $e_{ph} = e_{rl}$.*

Proof. Let us assume (en route to a contradiction) a situation with educational levels $(0, e_{ph}, e_{rl}, e')$, with $e_{rl} < e'$ (according to Proposition 2) and $e_{rl} > e_{ph} \geq 0$. In order for $(0, e_{ph}, e_{rl}, e')$ to be a PBE, firms' beliefs for education equilibrium choices must be (i) if $e_{ph} = 0$, $\mu(pl/0) = \frac{1}{2}$, $\mu(ph/0) = \frac{1}{2}, \mu(rl/e_{rl}) = 1$, $\mu(rh/e') = 1$, and accordingly, wages will be $(\frac{1}{2}(H+L), \frac{1}{2}(H+L), L, H)$ respectively; (ii) if $e_{ph} > 0$, $\mu(pl/0) = 1$, $\mu(ph/e_{ph}) = 1, \mu(rl/e_{rl}) = 1$, $\mu(rh/e') = 1$, and accordingly, wages will be (L, H, L, H) respectively. But notice that, in light of these firms' wage offers, the best response for a rich individual of low-ability is nevertheless to choose e_{ph} rather than e_{rl}: the wage would be higher and the cost of education is lower choosing e_{ph}. The proposed situation $(0, e_{ph}, e_{rl}, e')$, with $e_{rl} > e_{ph}$, is not even a PBE. Therefore, by Lemma 1, $e_{rl} = e_{ph}$ in equilibrium. ■

In short, talented poor children cannot signal their talent because less talented rich individuals make the same educational choice to avoid being identified. However, talented rich children attain an education that prevents imitation. We have formally shown that if condition (A.1) holds education choices in equilibrium are $(0, e, e, e')$ for each type of family, with $0 \leq e < e'$.

Notice that if condition (A.1) does not hold, high-ability individuals have lower educational costs than low ability individuals, regardless of their family's socioeconomic status. This is the basic assumption of Spence's model and, accordingly, the result of the model presented would reproduce the standard result. We will come back to this point at the beginning of Section 5. Nonetheless, we consider that the relationship between marginal cost, which lies under assumption (A.1), cannot be rejected and the implications for the signalling mechanism must be analyzed.

4 The equilibrium of the model

In view of these results, the potential equilibrium of the game is a semiseparating equilibrium in which intermediate types (ph and rl) make the same choice. Particularly, as the next Proposition states, the equilibrium is different depending on the parameters of the model related to the marginal costs of education. Namely, the equilibrium can be what we call a *cuasipooling* equilibrium or a *cuasiseparating* equilibrium. We will restrict our attention to a family of education functions such that $\frac{c_e(e,i,j)}{c_e(e,i',j')}$ is constant for any level of education.[9]

Proposition 4 *The equilibrium of the model is*

(i) a cuasipooling equilibrium, with educational levels $(0,0,0,e^p)$ *and wages* $(\hat{w}^p, \hat{w}^p, \hat{w}^p, H)$, *if*

$$\begin{cases} \frac{c_e(e,p,l)}{c_e(e,r,l)} \leq 2, \textit{ or} \\ \frac{c_e(e,p,l)}{c_e(e,r,l)} > 2 \textit{ and } \delta \in (0,\bar{\delta}]; \end{cases} \tag{S.1}$$

(ii) and a cuasiseparating equilibrium, with educational levels $(0,\hat{e}^s,\hat{e}^s,e^s)$ *and wages* $(L,\hat{w}^s,\hat{w}^s,H)$, *if*

$$\frac{c_e(e,p,l)}{c_e(e,r,l)} > 2 \textit{ and } \delta \in (\bar{\delta},1), \tag{S.2}$$

[9] For instance, a Coob Douglas function satisfies this condition.

where the critical value $\bar{\delta}$ *solves:*

$$\begin{aligned} c(\hat{e}^s, p, l) &= \delta(H - L), \\ c(\hat{e}^s, r, l) &= \tfrac{\delta^2(H-L)}{1+\delta}, \end{aligned}$$

the education equilibrium levels solve:

$$\begin{aligned} c(e^p, r, l) &= \tfrac{H-L}{1+\delta}, \\ c(\hat{e}^s, p, l) &= \delta(H - L), \\ c(e^s, r, l) - c(\hat{e}^s, r, l) &= (1 - \delta)(H - L); \end{aligned}$$

and the equilibrium wages are:

$$\begin{aligned} \hat{w}^p &= \tfrac{1}{1+\delta} L + \tfrac{\delta}{1+\delta} H, \\ \hat{w}^s &= \delta H + (1 - \delta) L. \end{aligned}$$

Proof. See Appendix C. ■

A complete description of cuasipooling and cuasiseparating equilibria can be found in Appendix C, Claim 1 and Claim 2 respectively. Figure 3 presents both situations to give a basic idea.

The cuasipooling equilibrium $(0, 0, 0, e^p)$ is depicted on the left of Figure 3. In this equilibrium the three "lower" types pool at $e = 0$ and get a wage $\hat{w}_p$. The curves labelled U^*_{pl}, U^*_{ph}, U^*_{rl} through point $(0, \hat{w}_p)$ are indifference curves corresponding to these types in equilibrium. The highest type, the talented rich individual, rh, attains just the education level that makes his direct rival, the rl type, indifferent about mimicking him or not. (Note that the rl type equilibrium indifference curve through $(0, \hat{w}^p)$, denoted U^*_{rl}, also passes through point (e^p, H); formally, e^p solves: $U^*_{rl}(0, \hat{w}^p) = U_{rl}(e^p, H)$.) The pair (e^p, H) lies below the indifference curve of the lower types. On

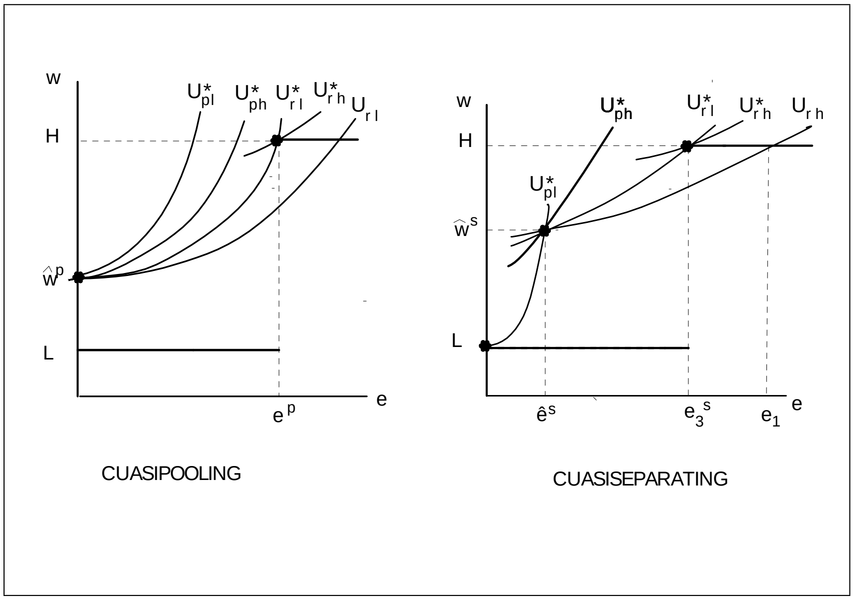

Figure 3: The equilibrium of the model

the other hand, it is impossible to distinguish among " intermediate types" (poor-high ability type ph, and rich- low ability type rl). The reason is that rich, low ability workers (type rl), which cannot imitate the education level that rh uses to separate, choose the same education as the talented poor type, thus avoiding being identified as a low ability worker. The strong lines show the expected wages at out-of-equilibrium education levels, given the beliefs of firms that satisfy the IC.

The cuasiseparating equilibrium $(0, \hat{e}^s, \hat{e}^s, e^s)$ is depicted on the right of Figure 3. In this equilibrium the highest type rh also chooses the education that makes the rl type indifferent about imitating him or not. Formally, e^s solves $U^*_{rl}(\hat{e}^s, \hat{w}^s) = U_{rl}(e^s, H)$. It means to choose an education e_3. It is easy to see that the rl type will never deviate by choosing an educational level $e_3^s < e < e_1$. Even if firms could be convinced that they were dealing with a high ability individual and offer H, this type will be worse off than in equilibrium. Observe that any point along the top horizontal line in bold, for $e^s < e$, lies below the indifference curve U^*_{rl}. Likewise, in this equilibrium talented poor children can not separate from less talented rich individuals.

Our solution concept selects between these two possibilities the equilibrium in which the rich and talented type separates with the smallest cost of separation. The intuition is quite clear. If $e^p < e^s$, that is, the level of education needed by the rh type to completely separate is smaller in the cuasipooling equilibrium (CPE) than in the cuasiseparating equilibrium (CSE), then the latter would be defeated by the former, because if firms observe the level of education e^p they must believe that this curriculum comes from an rh type but not from a rl type as they do in the beliefs that support the CSE.

But this is not true the other way around. Education level e^s is already expected to come from the rh type in the CPE, but this type will not use it because he would get a smaller utility by doing so. In fact, in this case, all types achieve a greater utility in the CPE than in the CSE, that is, the CPE is Pareto dominant.

Whenever the ratio of marginal education costs between the pl and the rl types is small enough or if it is large, when the proportion of poor people in the society is small enough, the solution is the CPE, as Proposition 4 states. This ratio of marginal costs is small when P or the difference between p and r is small. In such a situation the capacity of imitation of the $pl-$type is relatively high. This competition from below pushes up all levels of education in a CSE. Even if this ratio is high (meaning the competition from below from the pl type is not so intense), if there is a small proportion of poor people in the society, then the wage that the rl type gets in CSE pooling with the *ph* type is relatively small. This causes their incentives to imitate the $rh-$type to increase. That is, the competition from below from rl pushes up the level of education needed by rh to separate.

When e^s is smaller than e^p, the solution is the CSE for a similar reason to that described above (although in this case, there is not Pareto dominance between the CPE and the CSE). This situation will appear when the ratio of marginal costs of the rl and pl types is relatively high (because P and the difference $p - r$ are high) and there are enough poor families in the society. This combination reduces the pressure from below of the pl-type and also the pressure of the rl-type given that they obtain a good wage pooling with a relatively high proportion of ph individuals. Notice that the pressure of the rl-type on the rh-type would be stronger in the CPE because the former is pooling with all the poor families (high but also low talented) and the wage is substantially smaller.

Finally, notice that children born from poor parents will have lower wages, on average, than children born from rich parents. The expected wage for poor children in a CPE is $\frac{1}{2}L + \frac{1}{2}\hat{w}^p$, while for rich children it is $\frac{1}{2}H + \frac{1}{2}\hat{w}^p$. Similarly, in CSE poor children obtain $\hat{w}^s$ which is lower than the expected wage of rich children, $\frac{1}{2}H + \frac{1}{2}\hat{w}^s$. Hence, the intergenerational correlation of earnings can also be explained by the presence of investment in *signalling* education. Nowadays, in most Western countries, overeducation in the labour market may lead people to think that additional investment in education (exclusive College degrees) will not be profitable. But from a signalling perspective, it becomes essential to preserve the relative position in the educational ranking. Therefore, rich individuals continue investing in education and they will obtain higher wages.

5 How can poor children signal their talent?

In this section we develop some policy proposals to guarantee equal "signalling" opportunities to individuals. As the reader must be aware, assumption (A.1) is crucial for the preceding results. The following Proposition addresses this issue:

Proposition 5 *If marginal education costs are lower for high-ability than for low-ability individuals, i.e.* $c_e(e,p,l) > c_e(e,r,l) > c_e(e,p,h) > c_e(e,r,h)$ *holds, the equilibrium of the model is the separating equilibrium. This equilibrium can be described by: (i) education levels* $(0, e^f, 0, e^f)$*, where* e^f *verifies* $U^*_{rl}(0, L) = U_{rl}(e^f, H)$*; (ii) firms' belief* $\mu(il/0 \leq e < e^f) = 1$, $\mu(ih/e \geq e^f) = 1$*; and (iii) firms' wage offer* $w(0 \leq e < e^f) = L$, $w(e \geq e^f) = H$.

Proof. *See appendix D.* ■

Thus, it is clear that policies should be oriented to change the relationship between marginal education costs, that is, to reverse the inequality (A.1) and to make the separating equilibrium possible.

It is well known that a reduction in education costs for any educational level cannot help talented poor children to signal their talent. For example, a general subsidy that lowers funding costs for everyone or public provision of education will merely lead to an increase in education equilibrium levels. Considering that a family's educational spending is given by $C^{ij} = \pi^i F^{-1}(e, i, j)$, state or local provision will reduce it: the education cost will be $C = \pi^i F^{-1}(e, i, j) - ge$ where the second component represents the public spending per student (that can also be implemented as grants) which is usually larger for higher educational levels. From Proposition 4, where educational equilibrium levels, e^p, $\hat{e}^s$, and e^s are specified, notice that the left hand side of these expressions will decrease and in order to maintain equality, educational levels will increase. Thus, a general decrease in education costs does not allow poor children to signal their talent, as it does not change either inequality (A.1), $\frac{\pi^p}{F_s(s^{ph},h,p)} > \frac{\pi^r}{F_s(s^{rl},l,r)}$ or the results stated in Section 3.

However, some policies can contribute to making the separating equilibrium possible by reversing condition (A.1): (i) a "large enough" decrease in education costs, but only for poor people (through grants or a reduction of the gross interest rate π^i, for example) and (ii) an increase in educational standards combined with other measures such as assistance to students from less advantageous backgrounds and some changes in the selection of students among schools.

A reduction in educational costs for poor people, by providing grants, for example, can contribute to the reversal of inequality (A.1)[10]; the $c_e(e,p,h)$will be $\frac{\pi^p}{F_s(s,h,p)} - g$. If the reduction is large enough and policy makers manage to reverse condition $c_e(e,p,h) > c_e(e,r,l)$, all talented individuals, regardless of their families' income, will be able to signal their talent. Nonetheless, if the cost reduction is not high enough and the inequality is not reversed, the only effect of such a policy might be to increase the signalling of educational levels. Notice that, considering Proposition 4, in a cuasiseparating equilibrium, educational levels $\hat{e}^s$ and e^s will increase if $c_e(e,p,h)$ decreases. The growing demand for Masters and exclusive degrees by rich families might be a consequence of grant policies in most Western European countries. The more financial possibilities the poor families have, the more expensive the

[10] We consider that the Public Administration has the "monopoly of information" in the sense that they have access to information about families's income, and they use it; this information is not available or not allowed to be used privately.

educational attainment that is required by richer families to separate, will be. Notice that the effectiveness of this policy relates to how accurate Government information concerning family wealth is. In other words, given that the government normally has imperfect information, this policy is not robust to cheating on the side of the families.

A decrease in educational costs for poor people can also be achieved by providing loans at low interest rates for poor people; in fact this policy is currently being implemented in some countries, such the U.K. and Chile, for example. We have assumed that poor people can finance the education of their children at a high price. A decrease in interest rate π^p would facilitate the reversal of inequality (A.1). Notice again that this policy is robust to cheating on the side of the families only if the interest rate for the public loan remains greater for poor families. The next combination of policies that we propose does not depend on the government having perfect information about families' wealth.

Apart from the above financial aids to poor families, other possibilities may play a more important role in the reduction of poor people's educational costs by changing the relative importance of ability and home background on educational achievements. To make matters particularly simple, we will use a Cobb Douglas education production function: $e = s^{\alpha} i^{\gamma} j^{\beta}$, where $0 < \alpha, \gamma, \beta \leq 1$. The parameters α, γ and β give the elasticity of educational achievements with respect to monetary investment, parental background and ability, respectively. Condition $(A.1)$ can be rewritten as $\left(\frac{\pi^p}{\pi^r}\right)^{\alpha} > \left(\frac{p}{r}\right)^{\gamma} \left(\frac{h}{l}\right)^{\beta}$. Thereby, in order to reverse condition $(A.1)$ either ratio $\frac{p}{r}$ must increase, or parameters α, γ must decrease, or β must increase. One possibility is to increase educational standards, which will affect the relative importance of ability in educational achievements and will be reflected, formally, by a decrease in parameter α. Notice that ratio $\frac{\alpha}{\beta}$ is related to the marginal rate of technical substitution MRTS of input s for input j, at a given level $\bar{e}$; specifically, we can see that $\frac{\alpha}{\beta} = \frac{\partial\ MRTS_{js}(\bar{e})}{\partial\left(\frac{j}{s}\right)}$. Formally, a decrease in α leads to a decrease in the slope at any point in space (s, j) of the isoquant associated with an educational level. In other words, in order to achieve an educational level $\bar{e}$, a low ability child will need a higher monetary investment; parents will have to pay additional extra-scholar support (for example, private classes or academies) to enable their children to achieve a given level $\bar{e}$. Therefore, this policy favours poor, high-talented children.

However, the increase in educational standards has another effect: it also changes the substitutability between money and learning capacities due to home background (reflected by the ratio $\frac{\alpha}{\gamma}$). This measure favours the rich but low talented children. Notice that if the increase in standards is large enough and $\left(\frac{h}{l}\right)^{\beta} > \left(\frac{r}{p}\right)^{\gamma}$, condition (A.1)' will be reversed and the effects on poor talented children will be stronger than those on rich, low talented children. Consequently, the former will be able to signal their talent.

However, if $\left(\frac{h}{l}\right)^{\beta} < \left(\frac{r}{p}\right)^{\gamma}$, increasing educational standards will not be effective when it comes to guaranteeing equal signalling opportunities. In this case, this policy should be combined with (i) policies oriented towards reducing disparities in skills linked to home backgrounds (that is, to reduce $\frac{r}{p}$) and (ii) with policies related to segregation of students among schools (that will affect β and γ), which will be discussed below.

Firstly, some policies aimed at reducing the effect of background disparities (measured by the ratio $\frac{r}{p}$) include: target assistance to students from less advantageous backgrounds, individualising learning in order to provide students with appropriate forms of instructions, policies oriented towards reducing technological disparities among families and extra resources for schools in deprived areas (PISA, 2003).

Secondly, some changes in the segregation of students among schools can help talented poor children to signal their talent. If interaction among students holds a positive relationship with performance, segregation of students of the basis of ability or socioeconomic status results in a positive impact for those from a richer background and those who are more talented. Hence, a given difference in home backgrounds (or ability) among students will result in a greater difference in educational achievements. Parameters γ and β give the elasticity of educational achievements with respect to parental background and ability, respectively. Therefore, the increase of segregation based on parental background (and ability) can be captured by an increase in parameter γ (β, respectively). In this framework, in order to help poor talented children segregation on the basis of ability should increase (higher β) and segregation by home backgrounds should be reduced (lower γ).

Some changes in the educational system in Britain in 1965 and evidence from the PISA report seems to support the policies that we propose. The reduction of secondary school selection on the basis of age 11 ability has reduced the role of cognitive ability in determining educational achievements (Galindo-Rueda, Vignoles, 2003). "For various reasons, richer but less able students were able to take most advantage of the change in policy, and thus the achievement of this group increases the most." (Galindo-Rueda, Vignoles, 2003, p.18). In terms of our model, this change in the educational system in Britain can be seen as a decrease in the segregation of students by ability (decrease in β): a low ability is required to attain an educational level and/or there is a higher substitutability between inputs ($\frac{\gamma}{\beta} = \frac{\partial \, MRTS_{si}(\bar{e})}{\partial \left(\frac{s}{i}\right)}$) so that students from a richer family background and low talent can take advantage of these policies, as is actually the case. Nonetheless, the issue of segregation by ability is quite controversial. Highly talented individuals gain and low talented individuals lose with segregation. It is not clear that the former offsets the latter; we only can say that it could help poor and high ability children to signal their talent.

Moreover, the PISA report shows clearly that in the OECD countries social background is determinant for children's educational achievement. For example, in OECD countries, 33% of the variation in students' performance is explained by their attendance at different schools. A phenomenon of this kind may arise both from the financial resources at their disposal and from a concentration of the best students in certain schools. Empirical studies have great of difficulty in detecting any systematic influence of expenditure on education on the performance of students. Consequently, the explanation may rely on segregation among schools: the heterogeneity in the average performance for schools comes partly from the fact that some schools attract the best pupils while others attract the worst. If there is a positive interaction between the pupils' performance and if these performances are themselves positively influenced by parental income, this selection may result in a phenomenon in which the wealthiest people mostly send their children to the same schools (Cahuc and Zylberberg, 2004, p.93).

In short, an increase in educational standards would help poor-talented children if it is combined with measures oriented towards mitigating the effect of disparities in home backgrounds, which affect performance at school, and reducing the segregation among schools based on home background.

6 Conclusions

The aim of this paper has been to explain how financial constraints, innate abilities and learning skills linked to home background affect the signalling possibilities of poor children. This helps to explain the transmission of inequality across generations and why inequality has persisted over time despite the implementation in most Western countries of educational policies aimed at guaranteeing equal opportunities.

We have shown that if education were just a signalling mechanism in the labour market, the difference in wages between poor and rich children cannot be expected to disappear. From a human capital approach of educational investments, the earnings of poor and rich children can be expected to converge (Becker and Tomes, 1979, 1986). These authors argue that when rich families stop investing in human capital, as the marginal rate of returns on human capital and physical capital draws level, poor families will continue investing in education (diminishing returns on human capital are assumed). It will contribute to a convergence in earnings as there will be a convergence in individuals' productivity linked to human capital investments. However, from a signalling perspective, rich parents could find it profitable to finance as much education as their talented children need to signal their ability, which will guarantee them the best position in the labour market. Therefore, when the best wages go to the more educated individuals, the rich try to prevent imitations to the extent that if children from poorer backgrounds have college degrees, for example, they find prestigious and exclusive Institutions to signal their talent. The claim of Becker and Tomes that the greater wealth of rich parents does not always lead to an increase educational investment, and that the higher investment is less effective in increasing their children's earnings, (according to the human capital approach of educational investment) merits more research in the extent that education works as a signalling device.

In this context, public education does not help talented poor children to catch up to rich-talented individuals. Despite the increase in the overall educational achievement in most developed and developing countries (Barro and Lee, 2000), inequality remains unsolved. The paper has shown the role of some policies to guarantee equal signalling opportunities. These policies include an increase in educational standards combined with measures oriented (i) towards reducing the effect on the learning process due to disparities in families' home background (that affect the learning process) and (ii) reducing the segregation of students among schools on the basis of family socioeconomic status.

Finally, the model has shown that in more egalitarian societies, where large differences in educational costs do not exist between rich and poor children, more pressure is placed on the types that try to separate. They will need higher educational levels to prevent imitations from their competitors. But, if there is so much pressure from below, the different types might be better off not signalling their ability. This favours the stability of a situation where only the talented-rich individuals invest in signalling education, i.e., the cuasipooling equilibrium (moreover, this also means that the utility tends to be lower in situations where more types invest in signalling, that is, the cuasiseparating equilibrium.) Hence, we can find a more egalitarian, but less mobile society, in the sense that all poor families have lower wages, as talented poor children do not find it profitable to even separate from the less talented individuals with their same economic status.

APPENDIX

A Proof of Lemma 1

Proof. Consider two workers ij and $i'j'$, with different marginal education costs, specifically, $c_e(i,j) > c_e(i',j')$, where ij and $i'j'$ can correspond to any type verifying $c_e(e,p,l) > c_e(e,p,h) > c_e(e,r,l) > c_e(e,r,h)$. Suppose (en route to a contradiction) that the optimal education levels are e and e', respectively, with $e > e'$. Then it must be the case that worker ij weakly prefers a wage $w(e)$ to $w(e')$:

$$W^i + w(e) - c(e,i,j) - \left[W^i + w(e') - c(e',i,j)\right] \geq 0.$$

Similarly, worker $i'j'$ weakly prefers $(w(e')$ to $w(e)$, that is

$$W^{i'} + w(e') - c(e',i',j') - \left[W^{i'} + w(e) - c(e,i',j')\right] \geq 0.$$

Adding these two inequalities yields

$$c(e,i',j') - c(e',i',j') - \left[c(e,i,j) - c(e',i,j)\right] \geq 0.$$

This is a contradiction as we assumed that $c_e(i,j) > c_e(i',j')$: the increment from e' to e is more expensive for the ij type than for the $i'j'$ type. ■

B Proof of Proposition 2

Proof. En route to contradiction, let us assume that, in equilibrium, the rh-type pools and attains educational level $\hat{e}$ such that firms' beliefs are $\mu(rh/\hat{e}) < 1$, and obtains a wage $\hat{w} > L$. As education does not decrease with the marginal cost of education (by Lemma 1), the $rh-$type must be pooling at least with the $rl-$type. In this situation, the out-of-equilibrium beliefs that sustain such an equilibrium for any $e \in [e\prime, e\prime\prime)$ are $\mu(rh/e) < 1$, where $e\prime$ and $e\prime\prime$ are defined as the education levels that verify $U^*_{rl}(\hat{e}, \hat{w}) = U_{rl}(e', H)$ and $U^*_{rh}(\hat{e}, \hat{w}) = U_{rl}(e\prime\prime, H)$, respectively.

But these beliefs $\mu(rh/e) < 1$, $e \in [e\prime, e\prime\prime)$, do not verify the Intuitive Criterion. According to this refinement, the beliefs for any $e \in [e\prime, e\prime\prime)$ must be $\mu(rh/e) = 1$. The reason is that the only type for which $e \in [e\prime, e\prime\prime)$ is not dominated in equilibrium is the rh type. Formally, $U^*_{rh}(\hat{e}, \hat{w}) < U_{rh}(e, H)$, $e \in [e\prime, e\prime\prime)$ and, furthermore, $U^*_{ij}(\hat{e}, \hat{w}) \geq U_{ij}(e, H), \forall ij \in \{pl, ph, rl\}$. Therefore, if beliefs are modified according to Intuitive Criterion, the best strategy for the rh type is no longer $\hat{e}$ but $e = e'$. A PBE in which rh pools does not survive Intuitive Criterion . ■

C Proof of Proposition 4: equilibrium of the model.

We proceed in four stages. Firstly, we focus on calculating the PBEs that are consistent with conditions (i)-(iii) and that verify the Intuitive Criterion (Claim 1 and 2). Secondly, we compare the education level attained by the "highest type", rh, in each situation (Claim 3). Thirdly, utilities of each type in these PBE are compared (Claim 4). Finally, we apply the undefeated equilibrium concept for out-of-equilibrium beliefs and we obtain the equilibrium of the model for each parameter value (Claim 5).

STAGE 1.

Claim 1 *There exists a PBE (cuasipooling) that satisfies the Intuitive Criterion characterized by: (i) the educational levels are* $(0, 0, 0, e^p)$, *where* e^p *verifies* $U^*_{rl}(0, \hat{w}^p) = U_{rl}(e^p, H)$ *(and solves the equation* $c(e^p, r, l) = \frac{1}{1+\delta}(H-L)$;*(ii) firms' belief after observing equilibrium education levels:* $\mu(pl/0) =$

$\frac{\delta}{1+\delta}$, $\mu(ph/0) = \frac{\delta}{1+\delta}$, $\mu(rl/0) = \frac{1-\delta}{1+\delta}$ *and* $\mu(rh/e^p) = 1$*; and for out-of-equilibrium education levels are:* $\mu(rl/0 < e < e^p) = 1$ *and* $\mu(rh/e \geq e^p) = 1$ *and, finally, (iii) firms' wage offer:* $w(0) = \hat{w}^p = \frac{1}{1+\delta}L + \frac{\delta}{1+\delta}H$, $w(e \geq e^p) = H$ *and* $w(0 < e < e^p) = L$.

Proof. It is clear that the equilibrium described is a PBE: all parties are playing optimally and, on the equilibrium path, beliefs are consistent with Bayes' Law. This PBE also satisfies the Intuitive Criterion: firms do not assign a positive probability for any type for which an education level is dominated in equilibrium. The (iv) "lowest aggregate condition" is also met.

Specifically, workers are playing optimally since the incentive compatibility constrains (ICR) are verified. (These restrictions mean that any type is better off in the specified equilibrium than mimicking another type.) The (ICR) are:

($\cdot$) for the rh type:

$$U_{rh}^*(e^p, H) \geq U_{rh}(0, \hat{w}^p) \Longrightarrow c(e^p, r, h) \leq H - \hat{w}^p, \qquad \text{(p.1)}$$

($\cdot$) for the rl type:

$$U_{rl}^*(0, \hat{w}^p) \geq U_{rl}(e^p, H) \Longrightarrow c(e^p, r, l) \geq H - \hat{w}^p \qquad \text{(p.2)}$$

($\cdot$) for the ph type:

$$U_{ph}^*(0, \hat{w}^p) \geq U_{ph}(e^p, H) \Longrightarrow c(e^p, p, h) \geq H - \hat{w}^p, \qquad \text{(p.3)}$$

($\cdot$) for the pl type:

$$U_{pl}^*(0, \hat{w}^p) \geq U_{pl}(e^p, H) \Longrightarrow c(e^p, p, l) \geq H - \hat{w}^p. \qquad \text{(p.4)}$$

Restriction (p.2) implies (p.3) and (p.4) given the assumptions on education costs. Therefore, for e^p to be an equilibrium education level it must verify (p.1) and (p.2) and the (iv) equilibrium condition; the lowest education level that met these three conditions is the proposed education e^p that solves (p.2) with equality.

Firms' beliefs are consistent with Bayes' Law given the education choices of workers. For out-of-equilibrium education levels, the specified beliefs verify the IC. This refinement requires the following:

$(\cdot)$ $\mu(pl/e \geq e_4^p) = 0$, since $U_{pl}^*(0, \hat{w}^p) \geq U_{pl}(e \geq e_4^p, H)$,

$(\cdot)$ $\mu(ph/e \geq e_3^p) = 0$, since $U_{ph}^*(0, \hat{w}^p) \geq U_{ph}(e \geq e_3^p, H)$, and

$(\cdot)$ $\mu(rl/e \geq e_2^p) = 0$, since $U_{rl}^*(0, \hat{w}^p) \geq U_{rl}(e \geq e_2^p), H)$,

where e_2^p, e_3^p, and e_4^p are the education levels that solve (p.2), (p.3), (p.4) with equality. So, firms do not assign a positive probability to those types after observing these education levels because the equilibrium utility of these workers exceeds the utility of choosing the specified education levels, no matter what the firms believe after observing any of them, that is, these education levels are dominated in equilibrium.

Finally, we have to prove that the rh type chooses the minimum education level consistent with firms' beliefs. IC implies that the firms' beliefs must be $\mu(rh/e^p \leq e \leq e_1^p) = 1$ (e_1^p verifies (p.1) with equality), provided that e is not dominated for the rich, high-ability individual (and is dominated for the others types), which in turn implies that an equilibrium in which the rh type chooses and education level $e > e^p$ cannot satisfy IC because in such an equilibrium the firms must believe that $\mu(rh/e > e^p) < 1$. Therefore, the only equilibrium that satisfies IC is the equilibrium in which the rh type chooses just e^p. ■

Claim 2 *There is a cuasiseparating PBE in which (i) education levels are* $(0, \hat{e}^s, \hat{e}^s, e^s)$, *where* $\hat{e}^s$ *and* e^s *verify* $U^*_{pl}(0, L) = U_{pl}(\hat{e}^s, \hat{w}^s)$ *and* $U^*_{rl}(\hat{e}^s, \hat{w}^s) = U_{rl}(e^s, H)$, *respectively (and solve:*$c(\hat{e}, p, l) = \delta(H - L)$ *and* $c(e^s, r, l) - c(\hat{e}, r, l) = (1 - \delta)(H - L)$*); firms' belief is* $\mu(pl/0 \leq e < \hat{e}^s) = 1$, $\mu(ph/e = \hat{e}^s) = \delta$; $\mu(rl/e = \hat{e}^s) = 1 - \delta; \mu(rl/\hat{e}^s < e < e^s) = 1$ *and* $\mu(rh/e \geq e^s) = 1$; *and firms' wage offer is:* $w(0 \leq e < \hat{e}^s) = L$, $w(\hat{e}^s) = \hat{w}^s = \delta H + (1 - \delta)L$, $w(\hat{e}^s < e < e^s) = L$, $w(e \geq e^s) = H$.

Proof. The equilibrium described is a PBE. Workers are playing optimally given the firms' wage offer; the incentive compatibility constraints are verified. These restrictions are:

$(\cdot)$ for the rh type:

$$U^*_{rh}(e^s, H) \geq U_{rh}(\hat{e}^s, \hat{w}^s) \Longrightarrow c(e^s, r, h) - c(\hat{e}^s, r, h) \leq (1 - \delta)(H - L) \quad \text{(s.1)}$$

$$U^*_{rh}(e^s, H) \geq U_{rh}(0, L) \Longrightarrow c(e^s, r, h) \leq H - L \quad \text{(s.2)}$$

$(\cdot)$ for the rl type:

$$U^*_{rl}(\hat{e}^s, \hat{w}^s) \geq U_{rl}(e^s, H) \Longrightarrow c(e^s, r, l) - c(\hat{e}^s, r, l) \geq (1 - \delta)(H - L) \quad \text{(s.3)}$$

$$U^*_{rl}(\hat{e}^s, \hat{w}^s) \geq U_{rl}(0, L) \Longrightarrow c(\hat{e}^s, r, l) \leq \delta (H - L) \quad \text{(s.4)}$$

$(\cdot)$ for the ph type:

$$U^*_{ph}(\hat{e}^s, \hat{w}^s) \geq U_{rl}(e^s, H) \Longrightarrow c(e^s, p, h) - c(\hat{e}^s, p, h) \geq (1 - \delta)(H - L) \quad \text{(s.5)}$$

$$U^*_{ph}(\hat{e}^s, \hat{w}^s) \geq U_{rl}(0, L) \Longrightarrow c(\hat{e}^s, P, h) \leq \delta (H - L) \quad \text{(s.6)}$$

$(\cdot)$ for the pl type:

$$U^*_{pl}(0, L) \geq U_{pl}(e^s, H) \Longrightarrow c(e^s, p, l) \geq (H - L) \quad \text{(s.7)}$$

$$U^*_{pl}(0, L) \geq U_{pl}(\hat{e}^s, \hat{w}^s) \Longrightarrow c(\hat{e}^s, p, l) \geq \delta (H - L) \quad \text{(s.8)}$$

Notice that if (s.6) holds, (s.4) also holds, and that if (s.3) is fulfilled, ICR (s.5) is also fulfilled. Hence, we look for the education equilibrium levels $\hat{e}^s$ and e^s that verify from (s.1) to (s.8), except for (s.4) and (s.5). On the one hand, education $\hat{e}^s$ must verify (s.8) and (s.6). The minimum education that solves both restrictions is the one that verifies (s.8) with equality.

On the other hand, education e^s has to fulfil the following conditions: (i) $e^s \leq e_i^s$, $i = 1, 2$ and (ii) $e^s \geq e_i^s$, $i = 3, 7$, where e_i^s are the education levels that verify the above restrictions with equality (s.1), (s.2), (s.3) and (s.7). It can be easily proved that $e_7^s < e_3^s$ $\forall \delta \in [0, 1)$, $e_1^s < e_2^s$ $\forall$ $\delta \in (0, 1]$, and $e_3^s < e_1^s$ $\forall \delta \in [0, 1)$. Hence, $e_7^s < e_3^s < e_1^s < e_2^s, \forall \delta \in (0, 1)$. So the minimum educational level e^s that solves all RCI is e_3^s, that is, the proposed education choice for the rh type.

Firms' beliefs are consistent with Bayes's Law, given the workers' choices. For out-of equilibrium education levels, firms' beliefs satisfy the IC. This criterion requires: $\mu(ph/e \geq e_5^s) = 0$ because $U_{ph}^{*s}(\cdot) > U_{ph}^s(e \geq e_5^s, H)$ and $\mu(rl/e \geq e_3^s) = 0$, because $U_{rl}^{*s}(\cdot) > U_{ph}^s(e \geq e_3^s, H)$. Therefore, $\mu(rh/e \geq e_3^s) = 1$. The proof that the only equilibrium that survives IC involves the $rh-$type choosing e^s is similar to the cuasipooling equilibrium. ■

Stage 2.

Next we analyse if the rh-type attains a higher education level in the cuasipooling or in the cuasipooling equilibrium.

Claim 3 *(i) The education choice of the $rh-$type is such that $e^p \leq e^s$ if*

$$\begin{cases} \frac{c_e(e,p,l)}{c_e(e,r,l)} \leq 2 \\ \frac{c_e(e,p,l)}{c_e(e,r,l)} > 2 \text{ and } \delta \in (0, \bar{\delta}]; \end{cases}$$

and (ii) the education choice of the rh-type is such that $e^p > e^s$ *if* $\frac{c_e(e,p,l)}{c_e(e,r,l)} > 2$ *and* $\delta \in (\bar{\delta}, 1)$*.*

Proof. Educational level e^p will be such that $e^p < e^s$ if $c(e^p, r, l) < c(e^s, r, l)$. Considering the equilibrium levels given in Proposition 4, we have that $c(e^p, r, l) < c(e^s, r, l)$ can be rewritten as

$$(H - L)\frac{\delta^2}{1+\delta} < c(\hat{e}^s, r, l). \tag{3}$$

The left hand side, $(H - L)\frac{\delta^2}{1+\delta}$, is increasing and convex in δ and $\frac{\delta^2}{1+\delta}(H - L) \in \left(0, \frac{1}{2}(H - L)\right)$. Let us show that the right hand side is linear on δ: $\frac{d\ c(\hat{e}^s,r,l)}{d\ \delta} = c_e(\hat{e}^s, r, l)\frac{d\ \hat{e}^s}{d\ \delta} = (H - L)\frac{c_e(\hat{e}^s,r,l)}{c_e(\hat{e}^s,p,l)} > 0$ as $\frac{d\ \hat{e}^s}{d\ \delta} = \frac{(H-L)}{c_e(\hat{e}^s,p,l)}$. Moreover, this r.h.s is zero for $\delta = 0$ since $\hat{e}^s = 0$ and for $\delta = 1$ taking into account that it is lineal in δ it can be written as $c(\hat{e}^s(\delta = 1), r, l) = (H - L)\frac{c_e(\hat{e}^s,r,l)}{c_e(\hat{e}^s,p,l)}$, (i) if $\frac{c_e(\hat{e}^s,r,l)}{c_e(\hat{e}^s,p,l)} \geq \frac{1}{2} \Rightarrow c(\hat{e}^s(\delta = 1), r, l) \geq \frac{1}{2}(H - L)$. Thus, $\forall \delta \in (0, 1)$, $e^p < e^s$. But (ii) if $\frac{c_e(\hat{e}^s,r,l)}{c_e(\hat{e}^s,p,l)} < \frac{1}{2}$, there is a $\bar{\delta}$ such that $(H - L)\frac{\delta^2}{1+\delta} = c(\hat{e}^s, r, l)$ where $\hat{e}^s$ solves $c(\hat{e}^s, p, l) = \delta(H - L)$. Then in this case we obtain that (3) holds for some δ values. For $\delta \leq \bar{\delta}$, $e^p \leq e^s$ and for $\delta > \bar{\delta}, e^p > e^s$. ■

STAGE 3.

In this stage the utility levels in these PBE are compared.

Claim 4 *(i) If the education choice of rich individuals of high ability is such that* $e^p \leq e^s$ *the utility of each type of family is higher in the cuasipooling equilibrium than in the cuasiseparating equilibrium, that is,*

$$U_{ij}^{*p} \geq U_{ij}^{*s} \qquad \forall ij;$$

(ii) if $e^p > e^s$ *the utility of a rich family with high ability offspring is higher*

in the cuasiseparating than in the cuasipooling, that is,

$$U_{rh}^{*p} > U_{rh}^{*s},$$

where superscripts p and s denote cuasipooling and cuasiseparating equilibrium.

Proof. Firstly , if $e^p \leq e^s$ we obtain the following results:

- $U_{rh}^{*p}(e^p, w_r) \geq U_{rh}^{*s}(e^s, H)$.
- $U_{rl}^{*p}(0, \hat{w}^p) \geq U_{rl}^{*s}(\hat{e}^s, \hat{w}^s)$. Educational levels e^p and e^s solve the following equations:

  $$\begin{aligned} U_{rl}(e^s, H) &= U_{rl}^{*s}(\hat{e}^s, \hat{w}^s) \\ U_{rl}(e^p, H) &= U_{rl}^{*p}(0, \hat{w}^p). \end{aligned}$$

  If $e^p \leq e^s$, we have that $U_{rl}(e^p, H) \geq U_{rl}(e^s, H)$ and by direct substitution it follows that $U_{rl}^{*p}(0, \hat{w}^p) \geq U_{rl}^{*s}(\hat{e}^s, \hat{w}^s)$.
- $U_{ph}^{*p}(0, \hat{w}^p) > U_{ph}^{*s}(\hat{e}^s, \hat{w}^s)$. Firstly, define e' such that:

  $$U_{rl}^{*p}(0, \hat{w}^p) = U_{rl}(e', \hat{w}^s),$$

  and considering that we have shown $U_{rl}^{*p}(0, \hat{w}^p) \geq U_{rl}^{*s}(\hat{e}^s, \hat{w}^s)$ when $e^p \leq e^s$, then

  $$U_{rl}(e', \hat{w}^s) \geq U_{rl}^{*s}(\hat{e}^s, \hat{w}^s)$$

  which implies $e' \leq \hat{e}^s$.

  Secondly, define $\bar{e}$ such that

  $$U_{ph}^{*p}(0, \hat{w}^p) = U_{ph}(\bar{e}, \hat{w}^s)$$

and, considering the assumption regarding marginal cost, we have $\bar{e} < e'$. Then $\bar{e} < e' \leq \hat{e}^s$. Finally, considering the definition of $\bar{e}$,$(U^{*p}_{ph}(0, \hat{w}^p) = U_{ph}(\bar{e}, \hat{w}^s))$, and that $\bar{e} < e' < \tilde{e}^s$, it follows that:

$$U_{ph}(\bar{e}, \hat{w}^p) > U^{*s}_{ph}(\hat{e}^s, \hat{w}^s).$$

And by substitution of $U^{*p}_{ph}(0, \hat{w}^p) = U_{ph}(\bar{e}, \hat{w}^p)$, we obtain

$$U^{*p}_{ph}(0, \hat{w}^p) > U^{*s}_{ph}(\hat{e}^s, \hat{w}^s).$$

Q.E.D.

- $U^{*p}_{pl}(0, \hat{w}^p) > U^{*s}_{pl}(0, L)$. This is obvious because $\hat{w}^p > L$.

  Secondly, if $e^p > e^s$ it is evident that $U^{*p}_{rh}(e^p, H) < U^{*s}_{rh}(e^s, H)$.

■

STAGE 4.

Claim 5 *The undefeated equilibrium is: (i) the cuasipooling PBE if $e^p \leq e^s$ and (ii) the cuasiseparating PBE if $e^p > e^s$.*

Proof. Case i) $e^p \leq e^s$. The level of education e_p is not chosen in the cuasiseparating equilibrium. As $U_{rh}\ (e_p, w_r) \geq U_{rh}(e_s, w_r)$, beliefs should be $\mu(rh/\ e_p) = 1$, which does not coincide with the beliefs that support CSE. So, CSE is defeated by CPE.

Now imagine education levels $\hat{e}^s$ or e^s are not chosen in CPE. We know by Claim 4 that all types obtain a higher utility in this equilibrium than in CSE (i.e. sending $\hat{e}^s$ and e^s). Therefore, CPE is not defeated by CSE.

Case ii) $e^p > e^s$. CPE is defeated by CSE. Notice that e_s is not chosen in CPE and yields a higher utility to the type rh than choosing e_p. So beliefs after observing e_s should assign probability one to this type. But these beliefs do not support CPE.

On the other hand, the only level of education which is not chosen in CSE but chosen in the CPE is e_p. Given that the utility for rh is smaller in e_p than in e_s in this case (earning w_r in both cases), the beliefs that support the CSE are justified. Therefore, the CSE is not defeated by the CPE.

In short, from claims 1 to 4, the equilibrium of the model is cuasipooling if $\frac{c_e(e,p,l)}{c_e(e,r,l)} \leq 2$ or if $\frac{c_e(e,p,l)}{c_e(e,r,l)} > 2$ and $\delta \in (0, \bar{\delta}]-$which leads to $e^p \leq e^s$ by Claim 3$-$, and cuasiseparating if $\frac{c_e(e,p,l)}{c_e(e,r,l)} > 2$ and $\delta \in (\bar{\delta}, 1)-$which leads to $e^p > e^s$. ■

D The separating equilibrium

Proof. The proof is similar to the proof of Proposition 4 except in what follows concerning the characterization of educational equilibrium level e^f. In this equilibrium, the incentive compatibility constraints are: ■

Proof.

$(\cdot)$ for the rh type:

$$U^*_{rh}(e^f, H) \geq U_{rh}(0, L) \Longrightarrow c(e^f, r, h) \leq H - L \tag{f.1}$$

$(\cdot)$ for the ph type:

$$U^*_{ph}(e^f, H) \geq U_{ph}(0, L) \Longrightarrow c(e^f, p, h) \leq H - L \tag{f.2}$$

$(\cdot)$ for the rl type:

$$U^*_{rl}(0, L) \geq U_{rl}(e^f, H) \Longrightarrow c(e^f, r, l) \geq (H - L) \tag{f.3}$$

$(\cdot)$ for the pl type:

$$U^*_{pl}(0, L) \geq U_{pl}(e^f, H) \Longrightarrow c(e^f, p, l) \geq (H - L)\,. \tag{f.4}$$

Restriction (f.2) implies (f.1) and restriction (f.3) implies (f.4) given the assumption on marginal education costs. Thus, the only educational level that verifies (f.2) and (f.3) is the e^f proposed. This equilibrium satisfies the Intuitive Criterion (see proof of Proposition 2 to check how this refinement is applied). Notice that under this condition on education costs, the Proposition 2 and 3 is not longer true. It breaks the possibility of a pooling equilibrium $(0,0,0,0)$ or a semiseparating equilibrium such as $(0,0,0,e')$ or $(0,e',e',e^H)$. Moreover, Lemma 1 still holds. Hence, if pooling and semiseparating PBE are not equilibria of the game, the separating PBE is the only equilibrium. ■

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