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BARGAINING WITH COMMITMENT UNDER AN UNCERTAIN DEADLINE

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We consider an infinite horizon bargaining game in which a deadline can arise with positive probability and where players possess an endogenous commitment device. We show that for any truncation of the game, the equilibrium agreement can only take place if the deadline arises within this finite horizon. Since the deadline is an uncertain event, the equilibrium exhibits agreements which are delayed with positive probability.

Keywords: Bargaining; endogenous commitment; delays; uncertain deadline.

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1. Introduction

The horizon of a bargaining game is an important factor in determining the outcome of a negotiation process. There exist situations in which a last period is not perceived, i.e., infinite horizon games. On the contrary, there are negotiations in which the final period determines, from the beginning of the game, the strategic behavior of the players. However, in some situations both types of horizon may emerge. For example, many negotiations on environmental issues or labor conditions are conducted under the pressure of a possible intervention of the government by imposing a deadline if the parties do not reach an agreement. Therefore, when players engage in such negotiations, even though they may perceive that there is room for a counterproposal, they can also perceive that at any time the government can impose a deadline after which there will not be scope for an agreement.

On the other hand, commitment is an essential component in many real-life negotiations because it both clearly affects the bargaining power of the negotiator and it may lead to inefficiencies. An (endogenous) commitment, in which players cannot accept a less generous offer than an offer previously rejected (combined with a deadline), can lead to delay in negotiations as it was first shown by Fershtman

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and Seidmann (1993). This assumption can be justified by the existence of representatives. In this sense, a representative would have problems in "explaining" to his principal that he had accepted a worse offer than an offer previously rejected.[1]

Another justification for an uncertain deadline appears in this context. If a principal has a fixed deadline, which is private information, he may prefer not to communicate it to his representative until the very last moment since by knowing a priori the exact date where the cake vanishes, he will be in a weaker bargaining position. A result in this direction has been obtained by Sandholm and Vulkan (1999) in a context of automated negotiations where software agents bargain on behalf of their users. These authors even suggest that "*a user is better off by giving her agent a time discount function instead of a deadline since a deadline puts her agent in a weak bargaining position*". All these arguments imply that from the point of view of a representative the deadline can be an uncertain event.

In the present work, we analyse how the presence of both features, an uncertain deadline and bargainers with endogenous commitments, affect the outcome of the bargaining process. More precisely, we analyse an infinite horizon bargaining game in which there is a positive probability that a deadline appears after any rejection, and where players possess an endogenous commitment.

Although apparently our model resembles a negotiation with probabilistic breakdown, this is not the case. There are two basic differences. First, whereas in the model with a breakdown there is a chance that, after a rejection, the bargaining terminates, in our model after the appearance of the deadline, players remain in the same bargaining relationship, but they face, unexpectedly, the last chance of reaching an agreement in the relationship. Secondly, the payoffs in the model with breakdown are exogenous to the bargaining process, whereas in our work the players's payoffs depend on the history of offers and counterproposals of the game.

We show that, for any truncation of the infinite horizon game, when the players are sufficiently patient and possess an endogenous commitment, they will fail to reach an agreement before this truncation point, unless a deadline appears. However, since the deadline is probabilistic, this will result in delays in equilibrium and, therefore, in inefficiencies.

Finally, note that in Fershtman and Seidmann's framework with an infinite horizon there exists a subgame perfect equilibrium which yields an immediate agreement to the Rubinstein division of the surplus. However, our model shows that this result is not robust to a slight perturbation in the horizon of the game, namely, a small exogenous probability of the appearance of a deadline in each period.

2. The Model

Consider an infinite horizon bargaining game in which a seller (S) and a buyer (B) bargain over the partition of a cake of unit size. At the beginning of each period,

[1]Note that this class of commitment, in contrast with the exogenous commitment is not fixed. Its value depends on the play of the game.

each player gets to be the proposer with probability one half. If an offer is accepted, the game finishes and the players divide the cake according to the accepted offer. By contrast, if an offer is rejected a deadline arises with probability q and players enter in a *deadline stage*. Further, with probability $1-q$ players remain in the *main tree* and the game passes to the next period where again a proposer is chosen randomly.

In the deadline stage it is considered a take-it-or-leave-it procedure in which the proposer is selected with probability one half. Players earn a zero-payoff in case there is a rejection in this stage. More rounds of offers could be assumed before the deadline. However, this would not affect qualitatively the results.

On the other hand, we assume that the players possess an irrevocable endogenous commitment, i.e., a player cannot accept a less generous offer than any offer rejected in a previous period.

More formally, denote by $(1-x_i^t, x_i^t)$, $i \in \{S, B\}$, $x_i^t \in [0,1]$, the share of the proposer and the responder respectively, offered at round $t = 0, 1, 2, \ldots$ when player j, $j \in \{S, B\}$ and $j \neq i$, gets to be the proposer.

In the same way let z_i^t be player i's commitment at period t where[2],

$$z_i^t \equiv \max\left\{0, x_i^0, x_i^1, \ldots, x_i^{t-1}\right\}.$$

We assume that $z_i^0 = 0$, that is, no exogenous precommitments are carried into the game.

Periods in the main tree are discounted by the common discount factor $\delta \in (0,1)$. For simplicity, we assume that when the game enters the deadline stage, players do not discount payoffs. Note that this latter assumption is not essential in this model. The results would still hold by assuming a discount factor in the deadline stage $\delta \leq \delta_D < 1$.

Let $G_t(z_B^t, z_S^t)$ be any subgame which starts in period t and in which the buyer and the seller possess a commitment z_B^t and z_S^t, respectively.

Strategies and subgame perfect equilibria (SPE) for this game are defined in the usual way.

3. The Main Result

We first characterise the SPE payoffs of the deadline stage in the following Claim. It is straightforward to check that there exists an agreement in equilibrium in this stage.

Claim 1. *In any $G_t(z_B^t, z_S^t)$, when players enter the deadline stage, they reach an agreement in equilibrium. The buyer and seller's expected payoffs in this stage will be $\frac{1}{2}(1 + z_B^{t+1} - z_S^{t+1})$ and $\frac{1}{2}(1 - z_B^{t+1} + z_S^{t+1})$ respectively.*

Each player obtains a positive expected payoff in equilibrium when reaching the deadline stage, which depends on the players' current commitments. Since this

[2]Notice that some of its elements can be the null element, depending on the particular realisations of the lotteries which determine the proposer's identity.

event can only occur with probability q in any period, player i's minimum expected payoff in $G_t(z_B^t, z_S^t)$ is given by:

$$\begin{aligned} &\frac{q}{2}\left(1+z_i^{t+1}-z_j^{t+1}\right)\left(1+\delta\left(1-q\right)+\delta^2\left(1-q\right)^2+\ldots\right) \\ &\quad=\frac{q}{2}\left(1+z_i^{t+1}-z_j^{t+1}\right)\left(\frac{1}{1-\delta\left(1-q\right)}\right). \end{aligned} \tag{1}$$

Therefore, a player can guarantee himself a minimum expected payoff in any $G_t(z_B^t, z_S^t)$. This payoff can be obtained by inducing disagreement in the main tree and letting the game reach the deadline stage.

Once analysed the equilibrium in the deadline stage, we now proceed to characterise it in the main tree of the game. The following Lemma establishes a minimum offer that can be accepted by the responder in equilibrium. This will allow us to construct an upper bound in the player's utility derived from reaching an agreement in the main tree.

Lemma 1. *In any period t in which the buyer (seller) possesses a commitment $z_B^t(z_S^t)$, there is no equilibrium in which an offer $x_B^t < \frac{q(1-z_S^t)}{2[1-\delta(1-q)]-q}$ $\left(x_S^t < \frac{q(1-z_B^t)}{2[1-\delta(1-q)]-q}\right)$ is accepted by the buyer (seller).*

Proof. Suppose that the buyer gets to be the proposer in period t. Let x_S^t be the share that the buyer offers to the seller. The seller by rejecting such an offer and offering always $x_B^{t+j} < z_B^t$ can guarantee himself an expected payoff of:

$$\frac{q}{2}\left(1+x_S^t-z_B^t\right)\left(\frac{1}{1-\delta\left(1-q\right)}\right).$$

Then, the seller rejects in equilibrium any offer:

$$x_S^t < \frac{q}{2}\left(1+x_S^t-z_B^t\right)\left(\frac{1}{1-\delta\left(1-q\right)}\right), \text{ that is,}$$

$$x_S^t < \frac{q\left(1-z_B^t\right)}{2\left(1-\delta\left(1-q\right)\right)-q}.$$

In the same way, if S gets to be the proposer in period t, it can be shown that B will reject in equilibrium any offer:

$$x_B^t < \frac{q(1-z_S^t)}{2(1-\delta(1-q))-q}.$$ □

The presence of the commitment increases the offer that a proposer must make to the responder to reach an agreement. This is so because the latter, by rejecting this offer, raises the minimum expected payoff that he can guarantee in (1).

If the upper bound for each player derived from Lemma 1 is below the minimum expected payoff that each player can guarantee in (1) there will not be an agreement in equilibrium in period t. Only if $x_i^{t+s} > z_i^t$, $s > 0$, an agreement can be reached in the main tree. However, as we will see in the following Lemma, the set of offers which can be formulated and rejected in equilibrium are bounded.

Lemma 2. *If $G_t(z^t_B, z^t_S)$ has an equilibrium in which the buyer's offer, $x^t_S \geq z^t_S$, is rejected in the first period of the subgame then,*

$$x^t_S - z^t_S \leq \frac{2(1-\delta)(1-q)}{q}.$$

In the same way,

$$x^t_B - z^t_B \leq \frac{2(1-\delta)(1-q)}{q}.$$

Proof. Suppose that in period t, B and S possess a commitment z^t_B and z^t_S respectively. Assume that the buyer gets to be the proposer in period t. Consider the SPE of the subgame starting in t in which B offers $x^t_S \geq z^t_S$ and this offer is rejected by the seller. Let u_B and u_S be the corresponding equilibrium payoffs of B and S respectively.

Therefore, u_B is derived from a SPE outcome only if:

$$u_B \geq \frac{q}{2}\left(1 + z^t_B - z^t_S\right)\left(\frac{1}{1-\delta\left(1-q\right)}\right).$$

In the same manner, u_S is derived from a SPE outcome in which S rejects the B's offer only if:

$$u_S \geq \frac{q}{2}\left(1 + x^t_S - z^t_B\right)\left(\frac{1}{1-\delta\left(1-q\right)}\right).$$

The most B and S can expect to get together is 1, then, $u_B + u_S \leq 1$.

Hence,

$$\frac{q}{2}\left(1 + z^t_B - z^t_S\right)\left(\frac{1}{1-\delta\left(1-q\right)}\right) + \frac{q}{2}\left(1 + x^t_S - z^t_B\right)\left(\frac{1}{1-\delta\left(1-q\right)}\right) \leq 1,$$

that is,

$$x^t_S - z^t_S \leq \frac{2(1-\delta)(1-q)}{q}.$$

In the same way, when S gets to be the proposer in period t,

$$x^t_B - z^t_B \leq \frac{2(1-\delta)(1-q)}{q}. \qquad \square$$

In Proposition 1 we state our main result. Consider any truncation of the game, for any discount factor higher than a critical one, players can only reach an agreement in the main tree beyond this truncation point. This implies that players can only strike a deal, before reaching this point, in the deadline stage. However, as the deadline only occurs with probability q in any period, there exist delays in equilibrium and, therefore, inefficiencies.

Proposition 1. *For every game $G_0(0,0)$ and $q \leq 3-\sqrt{5}$, there is a $\widehat{\delta}(q,K) \in (0,1)$ such that if $\delta > \widehat{\delta}(q,K)$ in any SPE there is no agreement in the main tree at least until period K.*

Proof. Suppose the first period of the game and choose an arbitrary pair of exogenous precommitments[3] (z_B^0, z_S^0) which satisfies $z_B^0 + z_S^0 < 1$.

We first show that if $\delta > \delta'(q, z_B^0, z_S^0)$ then there cannot be an agreement in equilibrium in the first period of the game.

By Lemma 1 the maximum payoff that the buyer can obtain in equilibrium by making an offer to be accepted is:

$$1 - \frac{q(1-z_B^0)}{2(1-\delta(1-q)) - q}. \tag{2}$$

On the other hand, the buyer can guarantee an expected payoff of:

$$\frac{q}{2}(1 - z_S^0 + z_B^0)\left(\frac{1}{1-\delta(1-q)}\right). \tag{3}$$

Since (2) and (3) are decreasing and increasing in δ respectively, (3) is greater than (2) when δ tends to one and (2) is greater than (3) when δ tends to zero and $q \leq 3-\sqrt{5}$, we can find a $\delta_B(q, z_B^0, z_S^0)$ such that for any $\delta > \delta_B(q, z_B^0, z_S^0)$ there is no offer accepted by the seller in the first period in equilibrium.

Repeating the same argument for the case in which the seller gets to be the proposer, we can find a $\delta_S(q, z_B^0, z_S^0)$ such that for any $\delta > \delta_S(q, z_B^0, z_S^0)$ there is no offer accepted by the buyer in the first period in equilibrium.

Let $\delta'(q, z_B^0, z_S^0) \equiv \max\{\delta_B(q, z_B^0, z_S^0), \delta_S(q, z_B^0, z_S^0)\}$. For any $\delta > \delta'(q, z_B^0, z_S^0)$ in any SPE there is no offer accepted in the first period in equilibrium. It can be checked that for any $t > 0$, $\overline{z}_B \leq z_B^0$, $\overline{z}_S \leq z_S^0$ and $\delta > \delta'(q, z_B^0, z_S^0)$ the game $G_t(q, \overline{z}_B, \overline{z}_S)$ has no equilibrium in which players agree in the first period of the game.

Consider the game $G_0(0,0)$, for $\delta > \delta'(q, z_B^0, z_S^0)$ if planned equilibrium offers are such that $x_B^t \leq z_B^0$ and $x_S^t \leq z_S^0$ for all $t < K$, then there cannot be an agreement in equilibrium in the main tree before $t = K$.

However, by Lemma 2 there is an upper bound in the set of offers that can be formulated and rejected in equilibrium. For instance, if B makes an offer x_S^t which is rejected, then,

$$x_S^t - x_S^{t-1} \leq \frac{2(1-\delta)(1-q)}{q}.$$

In the same way, any equilibrium offer which is rejected, x_B^t, will have the same upper bound on $x_B^t - x_B^{t-1}$.

[3]This is, obviously, a technical device for the proof since we have already assumed that the players do not carry any commitment into our game.

Consider the first K periods of the game $G_0(0,0)$ and apply recursively these bounds. Since, by definition $G_0(0,0)$ starts with $z_B^0 = z_S^0 = 0$. Then, the sequence of offers x_S^t has the property:

$$\max_{0 \leq t < K} x_S^t < K \frac{2(1-\delta)(1-q)}{q}.$$

For any period K we can find a $\delta''(K, q, z_B^0, z_S^0)$ such that:

$$K \frac{2(1-\delta)(1-q)}{q} = \min(z_B^0, z_S^0).$$

For any $\delta > \delta''(K, q, z_B^0, z_S^0)$:

$$K \frac{2(1-\delta)(1-q)}{q} < \min(z_B^0, z_S^0).$$

Then, for any $\delta > \delta''(K, q, z_B^0, z_S^0)$ and $0 \leq t < K$ we obtain that $x_B^t < z_B^0$ and $x_S^t < z_S^0$.

If we define:

$$\widehat{\delta}(K, q) \equiv \max \left\{ \delta' \left(q, z_B^0, z_S^0\right), \delta'' \left(K, q, z_B^0, z_S^0\right) \right\}.$$

If $\delta > \widehat{\delta}(K, q)$, $G_0(0,0)$ has no equilibrium path where either $x_B^t > z_B^0$ or $x_S^t > z_S^0$ for $t < K$.

Consequently, when $\delta > \widehat{\delta}(K, q)$ there cannot exist an agreement in equilibrium in the main tree before period K. □

It is straightforward to check that $\widehat{\delta}(K, q)$ is increasing and decreasing respectively in K and q.

By Proposition 1 players do not reach an agreement in the main tree at least until period K. They will only strike a deal if a deadline arises. This is stated in the following Corollary:

Corollary 1. *If $\delta > \widehat{\delta}(K, q)$, in any $0 \leq t \leq K$, players reach an agreement in the deadline with probability $q(1-q)^t$ in equilibrium.*

Hence, the equilibrium of the game with both an uncertain deadline and endogenous commitments exhibits a novel property. It is required the realisation of a deadline in order to reach an agreement in the first stages.

Furthermore, when the bargaining frictions vanish, that is, players become infinitely patient, a very small probability of the appearance of a deadline yields lengthy delays with very high probability.

In this note we have shown that the delays caused by the presence of endogenous commitments in a finite horizon bargaining game hold in any infinite horizon negotiation in which a deadline can arise with a small but positive probability. This scenario of uncertain deadline seems more appropriate in the kind of situations in which the endogenous commitment assumption is more likely to hold, negotiations in which representatives bargain on behalf of their principals.

In Fershtman and Seidmann's framework but with an infinite horizon there exists a subgame perfect equilibrium agreement to the Rubinstein partition. We have shown that this equilibrium is not robust to the slight perturbation proposed throughout this paper.

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