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Commitment and choice of partner in a negotiation with a deadline

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Abstract. This paper analyses the effects of partially revocable endogenous commitments of a seller in a negotiation with a deadline. In particular, we examine when commitment is a source of strength, a source of inefficiency and when it does not affect the bargaining outcome at all. We show that when commitment possesses a minimum amount of irrevocability this crucially determines the bargaining outcome. In the bilateral bargaining case, commitment becomes a source of inefficiency since it causes a deadline effect. In the choice of partner framework, however, the deadline effect disappears and there is an immediate agreement and, moreover, commitment becomes a source of strength since it increases the seller's equilibrium payoff by triggering off competition between the buyers.

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1 Introduction

The important role of commitment in any bargaining process has long been recognised. Schelling (1960) defined a negotiation as a "*struggle to establish commitments to favourable bargaining positions*". In fact, commitment devices are present in classical models, either explicitly, as in the Nash Variable Threat Game where

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players commit to the disagreement outcome, or implicitly, as in Rubinstein's alternating offer game where, on the one hand, players are committed to implement their proposal if it is accepted by the responder and, on the other hand, committed to the offer they formulate since they are not allowed to modify it for a finite period of time.

More recently, most of the research on commitment has focused mainly on exogenous commitment devices. It is assumed that before players engage in the negotiation phase, they commit to some bargaining positions. However, the literature relies on different key features of this class of commitment to explore its effects on the players' bargaining power. Muthoo (1996) stressed the importance of the costs of revoking a commitment as a determining factor in the outcome of a negotiation. In contrast, Crawford (1982) and, afterwards, Kambe (1999) concentrated on the issue of imperfect information. They considered a class of imperfect commitment in the sense that players become aware that they are committed only after making their respective demands. In any case, an important insight of this literature is that (with complete information) when only one side of the negotiation can commit, the payoff of this party is improved. That is, the commitment is effective or has a value. Moreover, it is obtained that the more irrevocable a commitment is, the more favourable the position of the committed player (see Muthoo 1999).

Nevertheless, this statement is not always true. It relies implicitly on the assumption that the commitment is exogenous to the bargaining process. Fershtman and Seidmann (1993) (henceforth FS) show that the presence of an endogenous commitment in a bilateral negotiation with a deadline with sufficiently patient players, not only does not improve the committed players' payoff but also it results in an inefficiency: there is no agreement until the last period (the deadline effect).

The deadline effect is confirmed by casual observations and experimental evidence. Roth et al. (1988) found that in any negotiation "*there is a striking concentration of agreements reached in the very last seconds*". Although bargaining models with deadlines are readily found in the literature (see Stahl 1972; Harrington 1986), very few explain the occurrence of the deadline effect. Ma and Manove (1993) and Ponsati (1995) are two of the exceptions. Both models obtain agreements near the deadline in two different frameworks. Whereas the former assumes that players have an imperfect control over the timing of offers, the latter considers a concession game with incomplete information.

In contrast, FS obtain the deadline effect in a complete information framework, assuming that a player cannot accept any offer less generous than any offer rejected previously. Therefore, this class of completely irrevocable commitment, in contrast with the exogenous commitment, is not fixed. Its value depends on the play of the game. The endogenous commitment can be justified by the existence of representatives. In this sense, a representative would have problems in "explaining" to his principal that he had accepted a worse offer than an offer previously rejected.[1]

The main goal of this paper is to analyse the robustness of FS's result to two important extensions. On the one hand, instead of assuming completely irrevocable

[1] There exists experimental evidence which confirms that negotiations conducted by representatives result in a substantial increase of the inefficiencies (Schotter et al. 2000).

commitments as FS do, we will consider a much more general class of commitments: partially revocable commitments. On the other hand, we extend the model to a choice of partner scenario, namely, a thin market with one seller and two buyers. We examine whether a committed seller can trigger off competition between the buyers, resulting in effective commitment. Let us discuss these two motivations in turn.

FS do not specify the way by which the completely irrevocable commitment is achieved and we do not discuss here the usefulness of working with a reduced-form model. However, we do believe that many interesting cases are omitted by the complete irrevocability assumption. In real life, commitments are always partial and revocable, although at a finite cost. Therefore, the reduced-form analysis is appropriate only if we work with a wide class of commitment mechanisms, which are captured by different induced costs of revoking such partial commitments.

In the first part of the paper, we generalise the Fershtman and Seidmann result in a bilateral negotiation setting by considering a class of commitment which encompasses all possible cases. In particular, we assume that commitments are partially revocable, that is, they can be revoked up to a certain extent with finite costs but beyond a given point they become irrevocable. We show that in order to obtain the deadline effect it is necessary that the partial revocable commitment possesses a minimum amount of irrevocability. When commitments do not reach this degree of irrevocability there is no delay in equilibrium and the game is equivalent to that with no commitments. In both cases, the commitment is not effective, that is, it does not improve the payoff of the committed player.

In the second part of the paper, we explore the robustness of this result in a choice of partner scenario. Namely, we consider a thin market with one seller[2] and two buyers with different valuations. This class of bargaining games is important because it represents a situation in which one side of the market, the seller, has monopoly power. In this context, the intuition suggests that the seller has a larger bargaining power than in any standard bilateral bargaining game because he has an extra tool: he can threaten to switch to the second buyer in the case where the first buyer does not accept his demand, that is, we would expect some competition between the buyers. However, the results in the existing choice of partner literature (see Binmore 1985; Wilson 1984) generally do not agree with this intuition. They do not capture any competition between the buyers.[3]

Two other different trading procedures have been considered by the literature: random matching (Rubinstein and Wolinsky 1990; Hendon and Tranaes 1991) and auctioning (Binmore 1985; Wilson 1984). Whereas in the former the seller is randomly matched with one of the two buyers, in the latter the buyers simultaneously make offers to the seller. We agree with Chatterjee and Dutta (1998) in that "*...a random matching procedure is an acceptable modelling device in large, anonymous markets...*" but "*... it is less appropriate in thin markets where search costs are*

[2] In the remaining of the paper we will denote the seller and the buyer as he and she, respectively.

[3] An exception arises when the seller can switch to the other buyer after making an offer. In this case, there is a continuum of equilibria and in some of them the seller can obtain a price in excess of what he would otherwise obtain "locked in" with the high valuation buyer (see Osborne and Rubinstein 1990; Chap. 9, p.184).

usually low...". On the other hand, we consider that in the auctioning procedure the mechanism which yields competition between the buyers is merely an auction regardless of the presence of a seller's commitment device.

We demonstrate that in a choice of partner negotiation the presence of commitments with a minimum irrevocability affects the outcome in a very different way as compared to the bilateral case. In particular, if the second buyer's valuation of the good is greater than a critical value, the deadline effect disappears and we obtain an immediate agreement in equilibrium. Furthermore, and more importantly, the seller obtains a price higher than in the bilateral negotiation case (with or without commitment), that is, we capture competition between the buyers.

Notice that the high valuation buyer would prefer to delay the agreement, because in order to have any offer accepted, he must make a more generous offer than in the game without commitments. This is so because the seller, by rejecting this offer, raises the minimum share that he can guarantee himself by delaying the agreement until the last period. For sufficiently high discount factors, the high valuation buyer prefers to make non-serious offers in any period and let the game reach the last period. However, the presence of the low valuation buyer changes the result. This buyer knows that she will surely obtain a zero payoff in the last period. Therefore, she is interested in making positive offers and in case her valuation is above some critical value, her offers will be accepted in equilibrium by the seller. Note that the competition between the buyers is triggered off by the commitment and not by an auction-like mechanism (as is the case in the auctioning procedure).

However, as in the bilateral case, it is required that the partial revocable commitment possesses a minimum degree of irrevocability. Otherwise, the game amounts to the game without commitments, i.e. the presence of a second buyer does not matter and there is no competition between the buyers.

Nevertheless, an important and new insight of our model in the choice of partner scenario is that *it is not necessarily true that more irrevocability in a commitment implies a better bargaining position for the committed player*. Specifically, the seller may obtain a greater payoff with a partially revocable commitment than with a completely irrevocable one. The intuition behind this statement is that the more irrevocable a commitment is, the more the seller is willing to accept an offer in the bargaining phase. It may be the case that the seller requires an offer so high that the second buyer is unable to reach it and, therefore, competition between the buyers would not be triggered off. Hence, the seller prefers a commitment with a partial level of irrevocability to a commitment with a complete one.

It is important to point out that the value of partially revocable commitments remains positive when bargaining is frictionless, that is, the discount factor of the players tends to unity. Again, the best option for the seller is an appropriate choice of partially revocable commitment as opposed to a totally irrevocable one.

The paper is organised as follows. Section 2 presents the model. Section 3 examines a bilateral negotiation with partially revocable commitments. Section 4 analyses the role of partially revocable commitments in a choice of partner negotiation and, finally, Sect. 5 concludes.

2 The model

Suppose that a seller (S), who owns one unit of an indivisible good and has a reservation value of zero, is locked in with two buyers with different valuations of the good. Namely, B_1, with a valuation of the good $v_1 = 1$ and B_2, with a valuation of the good v_2, where $\frac{1}{2} < v_2 < 1$. The price at which the good is exchanged, should trade take place, is determined by a bargaining process between the agents.

We index periods by $t = 0, 1, 2, ...$ and assume that a deadline period, T, exists which means that if no agreement has been reached at the end of period T, then neither player receives a positive share of the pie. It is also assumed that players share the common discount factor $0 < \delta < 1$ and any parameter of the game is common knowledge. The trading procedure is the following: at the beginning of the game, S chooses the buyer he wishes to bargain with in the first period. Then, the chosen buyer makes an offer. If S accepts the offer, the agreement is implemented. Alternatively, he can continue bargaining with the current buyer or switch to the other one in the following period. In any case, only the buyers make offers. The same procedure holds for every period until either an agreement or the deadline is reached. In the last period T and once the seller has chosen a buyer, we assume that a fair lottery determines the identity of the proposer.

Moreover, we make the following crucial assumption: the seller cannot accept an offer less generous than those rejected in any previous period without incurring a cost. This assumption was first considered by Fershtman and Seidmann (1993). These authors assumed that the costs of accepting such an offer were infinite which implied that players were in fact establishing irrevocable commitments. However, in real life commitments are not always completely irrevocable. Schelling (1960) and afterwards Crawford (1982) and Muthoo (1996) argued that commitments are often partial and cost revocable.

Our model encompasses both cases. We assume that when the seller accepts an offer worse than any previously rejected offer, he will incur a finite cost if this offer is higher than a specific critical value. If, however, this offer is lower than this critical value, he faces an infinite cost. Namely, commitments are revocable up to a certain extent and with a finite cost. Beyond a given point, they become irrevocable.

Formally, we define $x_t^i \in [0, v_i]$ as any offer made by buyer i in any period $0 \le t < T$ whenever it is player i's turn to propose an agreement, where $i \in \{1, 2\}$, 1 stands for B_1 and 2 stands for B_2. In the same manner, we denote by x_T^j any offer formulated by player j in the last period T, where $j \in \{S, 1, 2\}$, and S stands for the seller.

On the other hand, we define z_t as the seller's commitment in period $t \le T$, where:

$$z_t = \max\left\{x_0^i, x_1^i, ..., x_{t-1}^i\right\}.$$

Therefore, if the seller accepts any offer x_t^i, he will face a cost for revoking such a commitment which can be defined by the following function,

Definition 1. *Let the cost of revoking function* $C\left(x_t^i, z_t, \varepsilon, \lambda\right)$ *be defined as:*

$$C\left(x_t^i, z_t, \varepsilon, \lambda\right) = \begin{cases} 0 & \textit{if } x_t^i \geq z_t \\ \lambda\left(z_t - x_t^i\right) & \textit{if } (1-\varepsilon) z_t \leq x_t^i < z_t \\ \infty & \textit{if } x_t^i < (1-\varepsilon) z_t \end{cases}$$

where $1 \geq \varepsilon \geq 0$ *and* $0 \leq \lambda \leq 1$.

The parameter ε measures the share of the cake that the seller is ready to give up in relation to the commitment at a finite cost (captured by λ). When ε tends to 0, the commitment becomes completely irrevocable, that is, the seller cannot accept any reduction on the share to which he is committed. This is clearly the Fershtman and Seidmann assumption. In contrast, when ε tends to 1, the commitment becomes completely revocable although at a unitary cost[4] of λ.

Notice that our model incorporates as special cases both class of devices. Nevertheless, our main interest is the case in which $\varepsilon \in (0, 1)$, that is, partially revocable commitments.

If S accepts any offer $z_t (1-\varepsilon) \leq x_t^i < z_t$, he will face a finite cost $\lambda\left(z_t - x_t^i\right)$. Therefore, in this case S can obtain a positive utility by accepting such an offer and revoking the commitment z_t. However, if $x_t^i < z_t (1-\varepsilon)$, S must face an infinite cost for accepting such an offer. Hence, in this case S will never accept any $x_t^i < z_t (1-\varepsilon)$.

The seller's and the buyers's utility from accepting any offer x_t^i in any period $t \leq T$ is given respectively by:

$$U_s\left(x_t^i, z_t, \varepsilon, \lambda, \delta, t\right) = \begin{cases} \delta^t x_t^i & \text{if } x_t^i \geq z_t \\ \delta^t\left(x_t^i - \lambda\left(z_t - x_t^i\right)\right) & \text{if } (1-\varepsilon) z_t \leq x_t^i < z_t \\ -\infty & \text{if } x_t^i < (1-\varepsilon) z_t \end{cases}$$

$$U_{b_i}\left(x_t^i, v_i, \delta, t\right) = \delta^t\left(v_i - x_t^i\right).$$

Strategies and subgame perfect equilibria (SPE) for this game are defined in the usual way.

3 Partially revocable commitments in a bilateral negotiation: The deadline effect

We begin the analysis with the simplest case, where the seller can only sell the good to one buyer, B_1. This case was first considered by Fershtman and Seidmann (1993). These authors proved that when the seller's commitment is completely irrevocable, for sufficiently high discount factors, there is no agreement in equilibrium until the last period (the deadline effect). In this section, we generalise FS's inefficient delays by considering the more general class of partially revocable commitments defined in the previous section. More specifically, we will show that in order to obtain the deadline effect, the only requirement is that the commitments possess

[4] That is, a completely revocable commitment does not imply zero costs of revoking.

a minimum amount of irrevocability. Therefore, the inefficient delays obtained by FS do not hold with totally revocable commitments and, in this case, the game is equivalent to a game without commitments.

In the bilateral case, the trading procedure described in the previous section consists of the buyer making offers in anyone period previous to the deadline and a fair lottery determining the identity of the proposer in the last period.

The existence of a deadline period is crucial to both FS's results and those presented in this study as the consequences of considering more general commitments clearly arise in this period. For this reason, we first characterise the equilibrium in the deadline before going into the general results.

The following Lemma states that if the irrevocable part is not large enough, the seller's utility in the deadline is not affected by the presence of commitment.

Lemma 1. *In the last period* (T) *and for any* z_T*:*

a) If $1 \geq \varepsilon \geq \frac{1}{1+\lambda}$ *the seller's expected utility in equilibrium is given by:*

$$U_s^*(z_T) = \frac{1}{2}.$$

b) If $\varepsilon < \frac{1}{1+\lambda}$*, the seller's expected utility in equilibrium is given by:*

$$U_s^*(z_T, \varepsilon, \lambda) = \frac{1}{2}(1 + Az_T),$$

where $A = 1 - \varepsilon(1+\lambda)$.

Proof. In period T, a fair lottery determines the identity of the proposer. First, if the seller makes an offer he obtains the whole pie in equilibrium, $x_T^s = 1$. Secondly, when the seller is the responder, in equilibrium he only accepts offers such that $U_s\left(x_T^1\right) \geq 0$. This implies that x_T^1 has to be higher than or equal to $\max\{z_T \cdot (1-\varepsilon), x_T'(z_T)\}$, in order to be accepted in equilibrium. The offer that solves $x_T^1 - \lambda\left(z_T - x_T^1\right) = 0$ is $x_T'(z_T) = \frac{\lambda}{1+\lambda} z_T$. When $x_T'(z_T) \geq z_T \cdot (1-\varepsilon)$, $U_s(x_T'(z_T)) = 0$ and the seller does not obtain any additional payoff from being committed. This case arises when $\varepsilon \geq \frac{1}{1+\lambda}$. Then,

$$U_s^*(z_T) = \left[\frac{1}{2} \cdot 1 + \frac{1}{2} U_s(x_T'(z_T))\right] = \frac{1}{2}.$$

On the other hand, when $\varepsilon < \frac{1}{1+\lambda}$, $x_T'(z_T) < z_T \cdot (1-\varepsilon)$. Then,

$$U_s^*(z_T, \varepsilon, \lambda) = \left[\frac{1}{2} \cdot 1 + \frac{1}{2} U_s(z_T \cdot (1-\varepsilon))\right] = \frac{1}{2}[1 + Az_T].$$

□

In the following Lemma we show that if the irrevocable component of the commitment is not large enough, the commitment has no effect on the equilibrium. That is, the deadline effect disappears and the players obtain the same equilibrium payoffs as in the game with no commitments.

Lemma 2. *When $1 \geq \varepsilon \geq \frac{1}{1+\lambda}$, the equilibrium of the bilateral negotiation game coincides with that of the game with no commitments, that is, the game ends with an immediate agreement in which S obtains a payoff of $\frac{\delta^T}{2}$ and the buyer one of $1-\frac{\delta^T}{2}$.*

Proof. When $\varepsilon \geq \frac{1}{1+\lambda}$, by Lemma 1 the seller's expected utility in the deadline for any $0 \leq z_T \leq 1$ is given by $U_s(z_T) = \frac{1}{2}$. Let's define m_t and M_t as the minimum and maximum utility respectively that the seller obtains in equilibrium in any $0 \leq t \leq T$. Then,

$$m_T = M_T = \frac{1}{2},$$

First, notice that in any $t < T$ the seller by rejecting any offer until the deadline will surely obtain a utility of $\frac{\delta^{T-t}}{2}$. Formally,

$$m_t \geq \frac{\delta^{T-t}}{2}, \text{ for any } t < T. \tag{1}$$

Secondly, in $T-1$, since the maximum utility that the seller can obtain by rejecting any offer is no greater than $\frac{\delta}{2}$,

$$M_{T-1} \leq \frac{\delta}{2}. \tag{2}$$

For any $t < T-1$ we need to prove the following Claim.

Claim 1. $M_t \leq \delta M_{t+1}$ *for any* $t < T-1$.

Suppose the contrary, that is, $M_t > \delta M_{t+1}$. Let $\overline{x}_t$ be the share of the pie which yields the seller the utility M_t. Let $m_t^{B_1}$ be the infimum of buyer 1 over the SPE's utilities of period t. We have that $m_t^{B_1} = 1 - \overline{x}_t$.

Assume that B_1 offers $(\overline{x}_t - \tau)$, as the seller's utility is increasing in the share for any given commitment, this share yields the seller the utility $M_t - \xi(\tau)$. For any τ small enough:

$$M_t - \xi(\tau) > \delta M_{t+1}.$$

This implies that S would accept $(\overline{x}_t - \tau)$. However, this contradicts that $m_t^{B_1} = 1 - \overline{x}_t$ be the infimum of B_1 in any SPE since $1 - \overline{x}_t < 1 - \overline{x}_t + \tau$.

Returning to the main statement of the proof. By Claim 1 and Expression (2) we obtain recursively,

$$M_t \leq \frac{\delta^{T-t}}{2}, \text{ for any } t < T. \tag{3}$$

It follows from Expressions (1) and (3) that in any subgame perfect equilibrium the seller's utility is unique and equal to,

$$m_t = M_t = \frac{\delta^{T-t}}{2}.$$

Hence, in the first period the seller's equilibrium utility is uniquely given by $\frac{\delta^T}{2}$. The buyer will always prefer to offer $\frac{\delta^T}{2}$ in the first period since,

$$1-\frac{\delta^T}{2} \geq \max\left\{\delta^t\left(1-\frac{\delta^{T-t}}{2}\right), \frac{\delta^T}{2}(1-z_T)\right\}, \text{for any } 0<t\leq T-1.$$

As a consequence, in equilibrium the game ends with an immediate agreement over the partition $\left(1-\frac{\delta^T}{2}, \frac{\delta^T}{2}\right)$. □

In the rest of the paper we consider partially revocable commitments to have a sufficiently large irrevocable part. Therefore, we impose the following assumption.

Assumption 1. Let ε and λ be related in the following manner:

$$0<\varepsilon<\frac{1}{1+\lambda}.$$

We are now ready to extend FS's deadline effect result to more general commitments. As a previous step, Lemma 3 establishes an upper bound in the buyer's utility in anyone period previous to the deadline.

Lemma 3. *Denote by $G_t(z_t)$ any subgame in any period $t<T$ with the commitment $0\leq z_t\leq 1$. Then, in any $G_t(z_t)$, S rejects in equilibrium with probability 1 any offer $x_t^1<\frac{\delta^{T-t}}{2-\delta A}$.*

Proof. Let $t=T-1$. Suppose that B_1 offers $\widehat{x}_{T-1}=\frac{\delta}{2-\delta A}-\varphi$, where $\varphi\in\left(0,\frac{\delta}{2-\delta A}\right]$. Then, the equilibrium continuation payoff of rejecting $\widehat{x}_{T-1}$ is:

$$U_s(\text{Reject } \widehat{x}_{T-1}) = \max\left\{\frac{\delta}{2}(1+A\widehat{x}_{T-1}), \frac{\delta}{2}(1+Az_{T-1})\right\}.$$

Thus,

$$U_s(\text{Reject } \widehat{x}_{T-1}) \geq \frac{\delta}{2}(1+A\widehat{x}_{T-1}) > \widehat{x}_{T-1} \geq U_s(\widehat{x}_{T-1}),$$
$$\text{since } \widehat{x}_{T-1}<\frac{\delta}{2-\delta A}.$$

In the same way, the following inequality holds,

$$\delta U_s(\text{Reject } \widehat{x}_{T-1}) > \delta\widehat{x}_{T-1} \geq \delta U_s(\widehat{x}_{T-1}). \tag{4}$$

Let $t=T-2$ and suppose that B_1 offers $\widehat{\widehat{x}}_{T-2}=\frac{\delta^2}{2-\delta A}-\psi$, where $\psi\in\left(0,\frac{\delta^2}{2-\delta A}\right]$. Assume that S accepts such an offer. This implies that

$$\widehat{\widehat{x}}_{T-2} \geq U_s\left(\widehat{\widehat{x}}_{T-2}\right) \geq U_s\left(\text{Reject } \widehat{\widehat{x}}_{T-2}\right).$$

Therefore, we can construct a $\widehat{x}_{T-1} < \frac{\delta}{2-\delta A}$ such that $\widehat{\widehat{x}}_{T-2} = \delta \widehat{x}_{T-1}$. Since $U_s\left(\widehat{\widehat{x}}_{T-2}\right) \leq \delta U_s\left(\widehat{x}_{T-1}\right)$,

$$\delta \widehat{x}_{T-1} \geq \delta U_s\left(\widehat{x}_{T-1}\right) \geq U_s\left(\text{Reject } \delta \widehat{x}_{T-1}\right). \tag{5}$$

Notice that,

$$\delta U_s\left(\text{Reject } \widehat{x}_{T-1}\right) = \max\left\{\frac{\delta^2}{2}\left(1 + A\widehat{x}_{T-1}\right), \frac{\delta^2}{2}\left(1 + Az_{T-1}\right)\right\},$$
$$U_s\left(\text{Reject } \delta\widehat{x}_{T-1}\right) = \max\left\{\frac{\delta}{2}\left(1 + A\delta\widehat{x}_{T-1}\right), \frac{\delta}{2}\left(1 + Az_{T-1}\right)\right\}.$$

It can be checked that $U_s\left(\text{Reject } \delta\widehat{x}_{T-1}\right) \geq \delta U_s\left(\text{Reject } \widehat{x}_{T-1}\right)$ and, therefore, by (4)

$$U_s\left(\text{Reject } \delta\widehat{x}_{T-1}\right) \geq \delta U_s\left(\text{Reject } \widehat{x}_{T-1}\right) > \delta\widehat{x}_{T-1} \geq \delta U_s\left(\widehat{x}_{T-1}\right). \tag{6}$$

However, Expression (5) contradicts Expression (6). Then, it can not be an equilibrium that S accepts any $\widehat{\widehat{x}}_{T-2} < \frac{\delta^2}{2-\delta A}$.

In the same way, we can follow the same argument by backward induction for any $t < T - 2$. □

The deadline effect obtained by FS for irrevocable commitments is generalised in the following Proposition for partially revocable commitments. The deadline effect implies that for discount factors sufficiently close to one, there is no agreement in equilibrium until the deadline. The intuition behind this result is that both players have a positive payoff in the last period guaranteed. If B_1 wants to have an offer accepted, he must make a more generous offer than those made in a standard bargaining game. This is so because the seller, when possessing partially revocable commitments, by rejecting this offer, raises the minimum share that he can guarantee himself by delaying the agreement until the last period. Given that the offer that would be accepted is increasing in δ, for a sufficiently high δ, B_1 obtains a worse payoff by making such an offer than by delaying the agreement until the last period making non-serious offers that will certainly be rejected.

Notice that this argument does not hold when the amount of irrevocability is not sufficiently large (i.e. $\varepsilon \geq \frac{1}{1+\lambda}$). The reason is that in this case the seller receives a zero-utility as a responder in the last period for any $0 \leq z_T \leq 1$.

Proposition 1. *(Deadline effect) If S can only sell the good to B_1, then there is a $\widehat{\delta}(T, \varepsilon, \lambda) \in (0, 1)$ such that the unique SPE of the bilateral negotiation game is characterised by delaying the agreement till the last period for every $\delta \in \left(\widehat{\delta}(T, \varepsilon, \lambda), 1\right)$ and the payoff is $\frac{\delta^T}{2}$ for each player.*

Proof. By Lemma 3, B_1 obtains by making an offer that would be accepted in any $t < T$ at most:

$$\left(1 - \frac{\delta^{T-t}}{2 - \delta A}\right) \leq 1 - \frac{\delta^T}{2 - \delta A} \tag{7}$$

On the other hand, by offering $x_t = 0$ in all $t < T$, B_1 can guarantee in the last period a payoff of:

$$\frac{\delta^{T-t}}{2}. \tag{8}$$

Define $\widehat{\delta}\,(T, \varepsilon, \lambda)$ as any value of $\delta \in (0, 1)$ which satisfies

$$1 - \frac{\delta^T}{2 - \delta A} = \frac{\delta^T}{2} \tag{9}$$

for any $\varepsilon < \frac{1}{1+\lambda}$.

At $\delta = 1$, the right hand side (RHS) of (9) is $\frac{1}{2}$ and the left hand side (LHS) is less than $\frac{1}{2}$. Then, since the LHS is monotonic decreasing in δ and the RHS is monotonic increasing, $\widehat{\delta}\,(T, \varepsilon, \lambda) \in (0, 1)$ and it is unique. Furthermore, for all $\delta \in \left(\widehat{\delta}\,(T, \varepsilon, \lambda), 1\right)$

$$1 - \frac{\delta^T}{2 - \delta A} < \frac{\delta^T}{2}.$$

Hence, for every $\delta > \widehat{\delta}\,(T, \varepsilon, \lambda)$, B_1 can guarantee a better payoff by offering $x_t = 0$ in every $t < T$ than by making an offer that would be accepted. □

Denote by $\delta^*(T)$ the critical discount factor necessary for the deadline effect in FS's model. It is easy to check that $1 > \widehat{\delta}\,(T, \varepsilon, \lambda) > \delta^*(T)$, for any $0 < \varepsilon < \frac{1}{1+\lambda}$. Therefore, when commitments have a revocable component the existence of the deadline effect requires a higher discount factor. Thus, intuitively, when commitments have a revocable component the seller demands a lower payoff in order to reach an agreement. As a consequence, the buyer's payoff from making an offer that would be accepted by the seller increases and she will require a higher discount factor in order to reach the deadline in equilibrium. When ε tends to 0, the commitment becomes irrevocable and, then, the critical discount factor which provokes the deadline effect with partially revocable commitments coincides with that obtained by FS. On the other hand, when ε tends to $\frac{1}{1+\lambda}$ the revocable component of the commitment dominates in such a way that the seller's utility in the deadline does not depend on the commitment. Hence, when the buyer makes an offer in any $t < T$, she knows that this offer neither improves the seller's utility in the deadline nor diminishes hers. In this case, there is no deadline effect.

4 Competition between buyers in a choice of partner negotiation

We return in this section to the original setting described in Sect. 2. In this framework, the seller can sell the good to either B_1 or B_2. We restrict our attention, as in the bilateral case, to partially revocable commitments with a minimum amount of irrevocability, that is, Assumption 1 still holds.

The main result is stated below. In particular, it is shown that in a negotiation with a choice of partner, the presence of commitments with a minimum of irrevocability affects the outcome in a very different way as compared to the bilateral case. In particular, if B_2's valuation of the good is above the critical value, the deadline effect disappears and we obtain an immediate agreement in equilibrium. Furthermore, and more importantly, the seller obtains a price higher than in the bilateral negotiation case, that is, we capture competition between the buyers.

4.1 Main result

Before studying the results it is interesting to analyse which buyer the good is sold to if the game reaches the deadline.

Claim 2. *If the equilibrium path reaches the deadline* (T)*,* S *sells the good to* B_1 *with probability* 1.

We state this Claim without any proof as it is suffice to notice that the seller's equilibrium utility of selling the good to B_1 in the last period is $\frac{1}{2}(1 + Az_T)$ whereas with B_2 he obtains $\frac{1}{2}(v_2 + Az_T) < \frac{1}{2}(1 + Az_T)$ for any $v_2 < 1$.

Claim 2 has very interesting consequences. B_2 will obtain a zero payoff if the game reaches the last period. Therefore, she will prefer to make an offer that would be accepted rather than delaying the game until the deadline (contrary to B_1 in the case of high discount factors).

In Lemma 4, we provide a bound in the offers accepted by the seller. This bound will be enough to determine the main result. If B_2's valuation allows this buyer to reach this bound, the competition between the buyers is triggered off and there will be an immediate agreement in equilibrium.

Lemma 4. *Let* $G_t(z_t)$ *be a subgame,* $t < T$*, where* $z_t \le \frac{\delta^{T-t}}{2-\delta A}$*. Then,* S *accepts in equilibrium with probability* 1 *any* $x_t^i \ge \frac{\delta^{T-t}}{2-\delta A}$.

Proof. First, in period $T-1$, if $z_{T-1} \le \frac{\delta}{2-\delta A}$, S accepts any $x_{T-1} \ge \frac{\delta}{2}(1 + x_{T-1}A)$, therefore, he accepts any $x_{T-1} \ge \frac{\delta}{2-\delta A}$. And, if $z_{T-1} > \frac{\delta}{2-\delta A}$, S accepts any $x_{T-1} \ge z_{T-1}$. Then, if we denote by M_{T-1} and $\overline{x}_{T-1}$ as the maximum utility and offer respectively that the seller can obtain in equilibrium in $T-1$,

$$M_{T-1} \le \overline{x}_{T-1} \le \max\left\{\frac{\delta}{2-\delta A}, z_{T-1}\right\}.$$

Hence, the Lemma holds in period $T-1$. Before going into any other period $t < T-1$ we know by Claim 1 of Lemma 2 that $M_t \le \delta M_{t+1}$ for any $t < T-1$. In addition, we require the following Claim,

Claim 3. $\overline{x}_t \le \delta\overline{x}_{t+1}$ *for any* $t < T-1$.

In any $t < T-1$,

$$m_t^{B_1} \ge 1 - \delta\overline{x}_{t+1},$$
$$m_t^{B_1} = 1 - \overline{x}_t.$$

This implies that $1-\overline{x}_t \geq 1-\delta\overline{x}_{t+1}$. This can only be possible if $\overline{x}_t \leq \delta\overline{x}_{t+1}$.

We turn now to the main statement of the proof. Suppose period $t < T-1$. We have to prove that S accepts any offer $x_t^* \geq \frac{\delta^{T-t}}{2-\delta A}$.

First, the seller's utility from accepting such an offer is:

$$x_t^* = U_s\left(x_t^*\right).$$

since by assumption, $z_t \leq \frac{\delta^{T-t}}{2-\delta A}$.

Secondly, the seller's utility from rejecting such an offer is:

$$U_s\left(\text{Reject } x_t^*\right) \leq \delta M_{t+1}.$$

By Claim 1,

$$\delta M_{t+1} \leq \delta^{T-t-1} M_{T-1} \leq \delta^{T-t-1}\overline{x}_{T-1} \leq \delta^{T-t-1} \max\left\{\frac{\delta}{2-\delta A}, z_{T-1}\right\}.$$

By Claim 3,

$$z_{T-1} \leq \max\left\{x_t^*, \delta\overline{x}_{T-1}\right\}.$$

Then,

$$U_s\left(\text{Reject } x_t^*\right) \leq \delta M_{t+1} \leq \delta^{T-t-1} M_{T-1} \leq \delta^{T-t-1}\overline{x}_{T-1} \leq \max \left\{\frac{\delta^{T-t}}{2-\delta A}, \max\left\{\delta^{T-t-1}x_t^*, \delta^{T-t}\overline{x}_{T-1}\right\}\right\} \leq \max\left\{\frac{\delta^{T-t}}{2-\delta A}, \delta^{T-t-1}x_t^*\right\}.$$

Therefore,

$$U_s\left(\text{Reject } x_t^*\right) \leq \max\left\{\frac{\delta^{T-t}}{2-\delta A}, \delta^{T-t-1}x_t^*\right\} \leq x_t^* = U_s\left(x_t^*\right). \qquad \square$$

In the following Proposition we state our main result. If B_2's valuation is sufficiently large, the delay disappears and we obtain an immediate agreement in equilibrium. Furthermore, the seller obtains a price higher than that obtained in the bilateral negotiation case, that is, competition between the buyers is captured.

Proposition 2. *If $\delta > \widehat{\delta}\left(T, \varepsilon, \lambda\right)$ and $v_2 > \overline{v}_2 = \frac{\delta^T}{2-\delta A}$, the equilibrium of the negotiation with choice of partner will be characterised by an immediate agreement in which the seller obtains $U_s = \frac{\delta^T}{2-\delta A}$.*

Proof. We show that when B_2's valuation is sufficiently large she formulates an offer in the first period which is accepted by the seller.

First, by Lemma 4, given that $z_0 = 0 < \frac{\delta^T}{2-\delta A}$, S accepts any $x_0^i \geq \frac{\delta^T}{2-\delta A}$. We prove that for B_2 it is optimal to offer $\widetilde{x}_0^2 = \frac{\delta^T}{2-\delta A}$ in the first period.

It is obvious that B_2 prefers to offer $\widetilde{x}_0^2 = \frac{\delta^T}{2-\delta A}$ than any other $\widehat{x}_0^2 > \widetilde{x}_0^2 = \frac{\delta^T}{2-\delta A}$ since the first offer is accepted by the seller.

We also need to show that such an offer $\widetilde{x}_0^2 = \frac{\delta^T}{2-\delta A}$ is preferred to any continuation subgame payoff.

First, by Claim 2, B_2 never obtains a better payoff if the game reaches the last period.

Secondly, Lemma 3 can be easily extended to the choice of partner case. Then, by this Lemma, making an offer that would be accepted in any period $0 < t < T$ never yields B_2 a payoff greater than

$$\left(v_2 - \frac{\delta^{T-t}}{2-\delta A}\right).$$

Notice that

$$\left(v_2 - \frac{\delta^{T-t}}{2-\delta A}\right) < v_2 - \widetilde{x}_0^2 = v_2 - \frac{\delta^T}{2-\delta A}.$$

On the other hand, notice that if S chooses B_1 to bargain with in the first period, $x_0^1 \leq \frac{\delta^T}{2-\delta A}$. By Lemma 3 making an offer that would be accepted in any period $0 < t < T$ never yields B_1 a payoff greater than

$$\left(1 - \frac{\delta^{T-t}}{2-\delta A}\right) < 1 - \frac{\delta^T}{2-\delta A}. \qquad \square$$

This Proposition has very interesting implications for the choice of partner literature. In particular, it implies that when there is one side of the market with monopoly power, possessing the partial revocable commitment device and facing *n* players from the other side of the market in a negotiation with a deadline, the only requirement is that one of these players has a close enough valuation of the good to the player with the highest valuation. This allows us to obtain competition among players in the long side of the market (contrary to the choice of partner literature) and an immediate agreement (contrary to FS's result). Nevertheless, another inefficiency can arise in equilibrium: the good may not be sold to the buyer with the highest valuation (B_1).

4.2 The value of commitment

The presence of a second buyer with a sufficiently high valuation $1 > v_2 > \overline{v}_2$ has two important effects on our negotiation with a deadline. On the one hand, the deadline effect disappears and there is an immediate agreement. On the other hand, and more importantly, the seller's equilibrium payoff increases not only with respect to the bilateral negotiation scenario with commitments but also with respect to the choice of partner scenario with no commitments.

Commitment is not effective in a negotiation with a deadline with sufficiently patient players in a bilateral negotiation context, because it results in an inefficiency: the deadline effect. However, it does become effective in a choice of partner scenario because it triggers off competition between the buyers.

Notice that when $\varepsilon \to 0$ the commitment becomes irrevocable. In this case, $\overline{v}_2 \to \frac{\delta^T}{2-\delta} > \frac{\delta^T}{2-\delta A}$. That is, the critical valuation v_2 that allows B_2 to compete with B_1 yielding an immediate agreement with the seller, increases, and so does the seller's equilibrium payoff. On the other hand, when $\varepsilon \to \frac{1}{1+\lambda}$, the revocable component of the commitment dominates and the outcome of the game coincides with that of the choice of partner case with no commitments, that is, an immediate agreement in which the seller obtains $\frac{\delta^T}{2}$. This means that the commitments are not effective when they do not entail a minimum amount of irrevocability.

It is important to note that the value of partially revocable commitments

$$\left(\varepsilon < \frac{1}{1+\lambda}\right)$$

still remains positive when the bargaining frictions vanish, that is, when players become infinitely patient. This follows from the next Lemma:

Lemma 5. *For δ sufficiently close to unity and $0 < \varepsilon < \frac{1}{1+\lambda}$, $\overline{v}_2 \to \frac{1}{1+\varepsilon(1+\lambda)}$, which is bounded away from* 1.

Therefore, for a fixed $v_2 > \frac{1}{1+\varepsilon(1+\lambda)}$, we can take a discount factor as close to one as we wish such that the game still yields an immediate agreement in equilibrium.

The most surprising aspect of our results is the following. When commitment devices are available beyond a bilateral bargaining framework, it may not be true that the committed player prefers a completely irrevocable commitment to a partially revocable one. This is clearly in contrast to the commitment literature (see Schelling 1960; Muthoo 1992,1996 among others) which supports the viewpoint that, in bargaining, weakness (the irrevocability of a commitment) is, in fact, a source of strength.

The intuition behind this statement is that the more irrevocable a commitment is, the more the seller requires of an offer for it to be accepted in the bargaining phase. It may be the case, that the seller requires an offer so high that the second buyer is not able to reach it and, therefore, competition is not triggered off. Hence, the seller may prefer a commitment with a partial level of irrevocability to a complete one.

To illustrate this, take a fixed v_2 and call it v_2'. Suppose that S could choose the irrevocability of the commitment (ε) at no cost. Let ε' be the critical amount of irrevocability such that:

$$\frac{\delta^T}{2 - A\delta} = v_2', \text{ for a given } 0 \leq \lambda \leq 1.$$

By choosing any $\varepsilon < \varepsilon'$, $v_2' < \overline{v}_2(\varepsilon)$ and, thus, B_2's valuation will not be high enough as to trigger off the competition between the buyers.

On the other hand, by choosing $\varepsilon \geq \varepsilon'$, $v_2' \geq \overline{v}_2(\varepsilon)$, competition is triggered off and an immediate agreement arises in equilibrium. In such an agreement, the seller obtains a price in excess of the bilateral negotiation.

Therefore, if S could choose the amount of irrevocability of the commitment, the optimal choice would be $\varepsilon^* = \varepsilon'$, which, in general, will be bounded away from zero (the completely irrevocable commitment).

Finally, it is interesting to note that this statement is also true when bargaining frictions disappear.

4.3 The effects of partially revocable commitments when the second buyer has a low valuation

The purpose of this section is to show that even when the second buyer has a low valuation, her presence makes a difference and the model still captures some competition between the buyers.

The intuition behind this argument is that given that B_2's valuation of the good is low, even if she offers a price equal to her valuation, her offers will be rejected by the seller. However, the seller can make use of the second buyer's offers to obtain a better payoff from B_1. Therefore, the presence of B_2 will not avoid the deadline effect because of her low valuation of the good, but, will nevertheless result in a continuum of equilibria in which the seller obtains a larger payoff than in the bilateral negotiation.

To illustrate this Claim, consider a two-period bargaining game. It can be easily verified that if the seller chooses to bargain in the first period with B_1, for any $\delta > \widehat{\delta}(\varepsilon, \lambda)$ this buyer offers $x_0^1 = 0 < \frac{\delta}{2-\delta A}$ which is rejected by the seller. Then, S would obtain

$$U_s(\delta) = \frac{\delta}{2}.$$

On the other hand, if S chooses to bargain in the first period with B_2, assuming that $v_2 < \frac{\delta}{2-\delta A}$ (and $\delta > \widehat{\delta}(\varepsilon, \lambda)$), any B_2's offer is $x_0^2 \leq v_2 < \frac{\delta}{2-\delta A}$. Hence, any second buyers's offer is rejected by the seller with probability 1. As a consequence, since in the second period the good is sold with probability 1 to B_1, B_2 is indifferent to offering any $x_0^2 \in [0, v_2]$. Given that $z_1 = x_0^2$, the seller's utility will be

$$U_s(\varepsilon, \lambda, \delta) = \frac{\delta}{2}(1 + Az_1) = \frac{\delta}{2}\left(1 + Ax_0^2\right) \in \left[\frac{\delta}{2}, \frac{\delta}{2}(1 + Av_2)\right].$$

Hence, it is optimal for the seller to bargain in the first period with B_2 and then to use B_2's offer to obtain a better payoff with B_1 in the deadline. Notice that in this case, some of the seller's payoffs that can be supported in equilibrium, are in excess of what he would obtain in the bilateral negotiation with B_1. That is, the model still captures some competition between the buyers.

5 Concluding remarks

This paper studies the role that endogenous commitment plays in a bargaining game with a deadline. In particular, our results allow us to understand when commitment is a source of strength, a source of inefficiency and when it does not affect the bargaining outcome at all.

First, we show that a negotiation is not affected at all by endogenous commitment when the revocable part of the commitment is sufficiently high. In contrast, when it possesses a minimum amount of irrevocability, it crucially determines the bargaining outcome. However, its effects depend on the bargaining framework being considered. In the bilateral bargaining case, commitment becomes a source of inefficiency since it provokes a deadline effect. In a choice of partner framework, commitment becomes a source of strength since it increases the seller's equilibrium payoff not only with respect to the bilateral negotiation scenario with commitments, but also with respect to the choice of partner negotiation without commitments.

In this paper we have assumed that the costs of revoking any commitment are proportional to the position a player attempts commitment to. Alternatively, it can be considered that the costs of backing down are fixed, that is, independent of that position. However, the main insights of this paper hold qualitatively under both classes of cost of revoking functions. In particular, a fixed cost of revoking function only changes the restrictions on the parameters of the model for which there is a minimum amount of irrevocability, yielding the deadline effect in the bilateral case, and triggering off the competition between the buyers in the choice of partner framework.

The novel aspect of our results is that it is not necessarily true that more irrevocability in a commitment implies a better bargaining position for the committed player. Namely, in the choice of partner scenario, it may be the case that the seller requires an offer so high that the second buyer is unable to reach it and, therefore, the competition would not be triggered off. In such a case, the seller would prefer a commitment with a partial level of irrevocability to a commitment with a complete one.

References

Binmore Binmore, K. (1985) Bargaining and Coalitions. In: Roth A E (ed.) *Game-theoretic models of Bargaining*. Cambridge University Press, Cambridge

Chatterjjee Chatterjee, K., Dutta, B. (1998) Rubinstein Auctions: On Competition for Bargaining Partners. *Games and Economic Behaviour* 23: 119–145

Crawford Crawford, V. (1982) A Theory of Disagreement in Bargaining. *Econometrica* 50: 607–637

De Fraja De Fraja, G., Muthoo, A. (2000) Equilibrium Partner Switching in a Bargaining Model with Asymmetric Information. *International Economic Review* 41: 849–870

Fershtman Fershtman, C., Seidmann, D. (1993) Deadline Effects and Inefficient Delay in Bargaining with Endogenous Commitment. *Journal of Economic Theory* 60: 306–321

Gul Gul, F. (1989) Bargaining Foundations of Shapley Value. *Econometrica* 57: 81–95

Harrington Harrington, J. (1986) A Non-Cooperative Bargaining with Risk Averse Players and an Uncertain Finite Horizon. *Economic Letters* 20: 9–13

Hendon Hendon, E., Tranaes, T. (1991) Sequential Bargaining in a Market with One Seller and Two Different Buyers. *Games and Economic Behaviour* 3: 453–466

Kambe Kambe, S. (1999) Bargaining with Imperfect Commitment. *Games and Economic Behavior* 28: 217–237

Muthoo (1989) Muthoo, A. (1990) Bargaining without Commitment. *Games and Economic Behavior* 2: 291–297

Muthoo (1996) Muthoo, A. (1996) A Bargaining Model Based on the Commitment Tactic. *Journal of Economic Theory* 69: 134–152

Nash Nash, J.F. (1953) Two-Person Cooperative Games. *Econometrica* 21: 128–140

Ma and Manove Ma, C., Manove, M. (1993) Bargaining with Deadlines and Imperfect Player Control. *Econometrica*. 61: 1313–1339

Osborne Osborne, M.J., Rubinstein, A. (1990) *Bargaining and Markets*. Academic Press, San Diego, USA

Ponsati Ponsati, C. (1995) The Deadline Effect: A Theoretical Note. *Economic Letters* 48: 281–285

Roth Roth, A., Murnighan, K., Schoumaker, F. (1988) The Deadline Effect in Bargaining: Some Experimental Evidence. *American Economic Review* 4: 807–823

Rubinstein (1982) Rubinstein, A. (1982) Perfect Equilibrium in a Bargaining Model. *Econometrica* 50: 97–110

Rubinstein Rubinstein, A., Wolinsky, A. (1990) Decentralized Trading, Strategic Behavior and the Walrasian Outcome. *Review of Economic Studies* 57: 63–78

Schelling Schelling, T. (1960) *The Strategy of Conflict*. Harvard University Press, Cambridge Mass., USA

Schotter Schotter, A., Zheng, W., Snyder, B. (2000) Bargaining Through Agents: An Experimental Study of Delegation and Commitment. *Games and Economic Behavior* 30: 248–292

Wilson Wilson, R. (1984) Notes on Market Games with Complete Information. Unpubl. paper. Graduate School of Business, Standford University