

LogMap v1.5.5 – Instructions

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Programmed in C++, OpenGL (graphics) and wxWidgets (GUI).

This free program is released under the GPL license: <http://www.gnu.org/gpl.html>

<http://www.uv.es/oteo/logmap>

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Introduction

LogMap is a program that illustrates aspects of the chaotic dynamics. All the simulations are based on the so-called logistic application. It is defined by the function: $f(x)=r \cdot x \cdot (1-x)$ dependent on one parameter r that takes values in the interval $[0,4]$, and defined in the x interval $[0,1]$. The program allows to study in detail the system defined by the iterations $x(n+1)=f(x(n))$.

Interacting with the graphs

Interaction with the graphs on screen is made by using the mouse.

- By pressing the 'control' key, pressing the mouse left button and dragging one can translate the graphs from one place to another on the screen.
- By pressing the 'shift' key, pressing the mouse left button and dragging one can enlarge or shrink the graphs (zooming).
- By pressing the mouse right button and dragging, a rectangular region unfolds for selecting a region of the graph and expanding it (zooming in a particular area).
- Double clicking over any of the two axis, resets in the direction of the selected axis.
- Double clicking over the left inferior corner of the graph resets both axis.

Calculations

In the menus bar there is the calculations menu. It contains ten options described in the following sections.

Autocorrelation

Autocorrelation is the correlation of a signal with itself. It accounts for the similarity of the values of the trayectory as a function of the temporal distance between themselves. It gives an idea of the 'memory' that the trayectory has of itself.

Options:

- **r – Parameter:** values of the function parameter. Its range is $[0,4]$.
- **Number of iterations** to be used in the calculation.

- **Transient:** number of initial iterations to be discarded.

Bifurcations

The bifurcation diagram is useful because it shows the different behaviours of the iterated function as a function of the r parameter value. The values of x for a number of iterations is represented on the vertical scale, for fixed r which appears on the abscises. The number of effective iterations is particularly important in the neighbourhood of a bifurcation point. It is worth noticing that if the resolution of the calculation of the diagram surpasses that of the screen no more detail will be added.

In the window associated with this calculation we find the following options:

- **Parameter values interval:** interval of r for which the iterations are being calculated.
- **Number of values of the parameter:** amount of values of r that are considered in the interval.
- **Number of iterations:** amount of iterations to be calculated for each value of r .
- **Transient:** amount of iterations to be discarded counting from the first iteration.
- **Cursor bar:** moves a cursor over the graph to select a concrete value of the parameter.

Cobweb

Graphical representation of the iterations.

Its options are:

- **Xo – Initial value:** value over which iteration begins. Its range is $[0,1]$.
- **r – Parameter:** values of the function parameter. Its range is $[0,4]$.
- **Number of iterations:** the amount of iterations to be calculated.
- **Transient:** number of initial iterations to be discarded.
- **Order:** the order of the composite function to be represented.
- **Scrolling sliders associated to Xo and r:** vary these magnitudes continuously.

Harter curves

This theoretical curves represent iterations of $X_0=1/2$ as a function of r . It is possible to draw either only one curve or a number of curves selecting the button 'various' and choosing the number of curve interval from the window controls associated to this calculation.

Histogram: invariant density

Determines the number of times that a small interval of the function definition domain is visited. The histogram is built dividing the x interval $[0,1]$ in cells of the same size and counting how many times they are visited in the process of iteration. In the case of periodic trajectories it is simply a finite set of vertical bars located at the system's attractor. If chaos exists the histogram has a more spread profile.

There are three options for this calculation:

- The **parameter** value.
- **Number of intervals:** amount of intervals in the partition of the function definition interval (computation resolution).
- **Number of iterations:** amount of iterations of the function for a given parameter value.

Power spectrum

Calculates and represents the Fourier power spectrum of a trajectory starting from the initial value X_0 , for a fixed value of the parameter. It is recommended to choose a large enough transient, so that the spectrum shows the genuine properties of the system and not the initial and transient ones. The vertical axis is represented in logarithmic scale.

Options:

- **r – Parameter:** values of the function parameter. Its range is [0,4].
- **Number of iterations:** This number must be a power of 2. The exponent of the power of 2 that determines the number of iterations to be calculated can be chosen.
- **Transient:** number of iterations to be discarded.

Lyapunov

Calculates the Lyapunov exponent. It is a useful tool to determine if the behaviour of the iterated function is periodic or chaotic. When a Lyapunov exponent is positive the system is chaotic for the given parameter value, whereas if it is negative the system is regular. On the graph there are represented the exponents as a function of the parameter value.

Options in this case are:

- **Parameter values interval.**
- **Number of values of the parameter.**
- **Number of iterations** used to carry out the calculation.

Temporal profile

In the case of a periodic system, the number of iterations needed to enter the attractor as a function of the initial value.

For this calculation the options are the following ones:

- **r – Parameter:** values of the function parameter. Its range is [0,4].
- **Number of iterations:** maximum number of iterations to be considered to make the profile. The system must converge before reaching this number. If it does not it can be due to two reasons: either the number of iterations is not enough or the system is chaotic.

Return map

This is a representation in which the vertical axis represents x at the iteration number n and the horizontal axis represents x at the iteration number $n-i$, where i is the delay value.

The options are:

- **r – Parameter:** value of the parameter. Its range is [0,4].
- **Transient:** number of initial iterations to be discarded.
- **Number of iterations** used in the calculation.
- **Delay:** number of iterations backwards respect the n -th iteration.

Trajectories

A number of iterations of the logistic application are calculated and displayed. A window opens in

which one can modify the different parameters that play a role in the iteration and representation.

- **Xo – Initial value:** value over which iteration begins. Its range is [0, 1].
- **Xo – Initial value:** another initial condition, useful to visualize the sensitivity to initial conditions phenomenon.
- **r – Parameter:** values of the function parameter. The system behaviour depends crucially on this value. Its range is [0, 4].
- **Transient:** number of initial iterations to be discarded.
- **Number of iterations:** the amount of iterations to be calculated.
- **Saving in file:** there exists the option of saving in one ASCII file the numerical values of the trajectory. The file is located in the same folder where the program has been launched.
- There exists the possibility to draw the graph either with points or solid **lines**.

Multiple graphs

An interesting option of the program consists in the possibility of displaying on the screen two simultaneous graphs, allowing the study of the system by means of two different calculations. Calculation and representation options remain the same as in the case of one graph on screen. The possible combinations of graphs, accesible from the 'Two graphs' menu, are the following:

- Trajectories and bifurcations.
- Lyapunov and bifurcations.
- Histogram and bifurcations.
- Harter curves and bifurcations.
- Cobweb and bifurcations.
- Cobweb and trajectories.
- Spectrum and bifurcations.
- Autocorrelation and bifurcations.
- Cobweb and temporal profile.

Language

Finally it is worth mentioning that this software is translated into English/Spanish language. The language is changed through the 'Language' menu.