Set-theoretical solutions to the Yang-Baxter equation of finite order

Paola Stefanelli - Università del Salento

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In the last years, the problem of finding set-theoretical solutions of the Yang-Baxter equation [4], shortly *solutions*, has been dealt with from different points of view. Since the late 1990s a large number of works related to bijective solutions has been produced, including the seminal papers of Gateva-Ivanova and Van den Bergh [6], Etingof, Schedler, and Soloviev [5], and Lu, Yan, and Zhu [8]. Recently, the attention has been posed on solutions r that are not necessarily bijective, such as the idempotent ($r^2 = r$) and the cubic ones ($r^3 = r$).

In this seminar, we focus on solutions which are of finite order and not necessarily bijective. In particular, we use the technique of the *matched product solutions* [1, 2] as a unifying tool for treating this kind of solutions that include the involutive and the idempotent ones. Moreover, we show that the matched product of two finite solutions r_S and r_T is of finite order if and only if r_S and r_T are. Finally, we analyse the solution associated with semi-braces, see [3] and [7], that are a generalization of the algebraic structure of braces introduced by Rump in [9]. Specifically, we show in which class of solutions of finite order these maps lie.

References

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