Recent developments of the pentagon equation with an application to the Yang-Baxter equation

Marzia Mazzotta
PhD student from University of Salento

The pentagon equation is widely investigated in Mathematical Physics. Our attention has been posed on the study of the set-theoretical solution of this equation. Specifically, a *set-theoretical solution of the pentagon equation on a set* S, briefly a *PE solution*, is a map s from $S \times S$ into itself that satisfies the relation

$$s_{23}\, s_{13}\, s_{12} = s_{12}\, s_{23},$$

where $s_{12} = s \times id_S$, $s_{23} = id_S \times s$, and $s_{13} = (id_S \times \tau) s_{12} (id_S \times \tau)$, with τ the map given by $\tau(x, y) = (y, x)$. First examples of invertible PE solutions may be found in the pioneering work of Zakrzewski [5], Baaj and Skandalis [1], Kashaev and Sergeev [4]. In particular, in [4] it is proved that the only *invertible* solution s on a group $(G, \cdot)$ is given by $s(x, y) = (x \cdot y, y)$.

In this talk we firstly present the complete description of PE solutions of the form $s\,(x, y) = (x \cdot y,\, \theta_x(y))$ on groups $(G, \cdot)$, where θ_x is a map from G into itself, for every $x \in G$, that has been provided in [2].

Among PE solutions one can find examples of solutions to the well-known Yang-Baxter equation involving particular semigroups. In this context, we present some solutions of the Yang-Baxter equation that are different from those known until now, as recently developed in [3].

References

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