

**Master “Advanced Physics”    Universitat de València**  
**“Advanced Relativity” (Curs 2010–11)**

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**Contents:**

**I.– THE THEORY OF RELATIVITY: FOUNDATIONS.**

1. The Special Theory of Relativity. The Minkowski space-time.
2. Space-like, time-like and light-like subspaces.
3. Local observer. Proper time and proper space. The postulate on the local interpretation of physical measurements.
4. Metric theories of gravitation. The postulate on the space-time geodesics.

**II.– THE GENERAL THEORY OF RELATIVITY.**

1. The Einstein equations. The Ricci tensor and the Weyl tensor.
2. The Cotton tensor and the Bianchi identities.
3. Conformal structures. Conformally flat metrics. Intrinsic characterization.
4. Killing fields and isometries. Maximal symmetry.
5. Spherically symmetric space-times. Invariant characterization. Robertson-Walker metrics.
6. On the classification of gravitational fields. The algebraic types of the Ricci and Weyl tensors.

**III.– RELATIVISTIC HYDRODYNAMICS.**

1. Time-like congruencies: acceleration, vorticity, shear, and expansion.
2. Relativistic fluids: kinematics. Evolution equations of the kinematic coefficients.
3. The energy-momentum tensor of a fluid.
4. Hydrodynamics: basic equations. Perfect fluids.

**IV.– ELECTROMAGNETIC FIELDS**

1. The electromagnetic 2-form (algebra and geometry).
2. Null and non null electromagnetic fields.
3. The Maxwell equations. Relative formulation and involution property.
4. The electromagnetic energy-momentum tensor.

## V.– EVOLUTIVE FORMALISM AND RELATIVISTIC PHYSICS.

1.  $3 + 1$  space-time decompositions. Induced metric and extrinsic curvature.
2. Hydrodynamics:  $3 + 1$  formulation.
3. Maxwell's equations:  $3 + 1$  formulation.
4. Einstein's equations:  $3 + 1$  formulation.
5. Constraint equations. Evolution equations. Involution.

## VI.– NUMERICAL RELATIVITY.

1. Brief historical introduction to Numerical Relativity.
2. Formulations of Einstein's equations adapted to numerical work.  $3+1$  and characteristic formulations.
3. Numerical hydrodynamics and MHD in general relativity.
4. High-resolution shock-capturing schemes.

## VII.– APPLICATIONS OF NUMERICAL RELATIVITY: RELATIVISTIC ASTROPHYSICS.

1. Relativistic rotating stars.
2. Gravitational collapse.
3. Accretion on to compact objects.
4. Mergers of compact binaries.

## VIII.– GRAVITATIONAL RADIATION.

1. Weak gravitational fields. Gravitational waves.
2. Astrophysical sources of gravitational radiation.
3. Detection of gravitational waves.

## Bibliography (basic)

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