

NUMERICAL AND STATISTICAL METHODS IN PHYSICS

A) Numerical Methods

1. Ordinary differential equations: Initial value problems by one step method

- 1.1. Traditional methods
- 1.2. Euler, Euler-Cromer y modified Euler methods
- 1.3. Verlet method
- 1.4. Taylor series method
- 1.5. Runge-Kutta method
- 1.6. Adaptive Runge-Kutta method

2. Ordinary differential equations: Initial value problems by multi-step methods. Stability and errors

- 2.1. Numerov method
- 2.2. Stoer-Burlisch and Richardson extrapolation methods
- 2.3. Explicit methods
- 2.4. Implicit methods
- 2.5. Stability of differential and difference equations.
- 2.6. Truncation and rounding errors
- 2.7. Selection of the integration method

3. Boundary problems with ordinary differential equations

- 3.1 Lineal shooting method
- 3.3 Non-lineal shooting method
- 3.3 Lineal finite-difference method
- 3.4 Non-lineal finite-difference method
- 3.5 Variational methods
- 3.6 Cubic splines
- 3.7 B-splines
- 3.8 Variational methods with B-splines

4. Eigenvalue problems

- 4.1 Introduction. The Schrödinger equation.
- 4.2 Determination of tridiagonal matrices eigenvalues.
- 4.3. Sturm Series
- 4.4 Richardson extrapolation.
- 4.5 Computation of the wave function.
- 4.6 Eigenvalue distributions.

5. Partial differential equations: Parabolic problems

- 5.1 Clasification of partial differential equations.
- 5.2 Heat difusion equation.
- 5.3 Forward difference method.
- 5.4 Backward difference method
- 5.5 von Numann stability criterium
- 5.6 Boundary conditions

- 5.7 Neutron density in a nuclear reactor
- 5.8 The time dependent Schrödinger equation
- 5.9 Non linear partial differential equations with linear time derivative of first order.

6. Partial differential equations: Elliptic and hyperbolic problems

- 6.1 Finite difference solution of the wave equation
- 6.2 Wave equation stability.
- 6.3 Linear advection equation.
- 6.4 Lax and Lax-Wendroff methods
- 6.5 Heat propagation by simultaneous convection and diffusion
- 6.6 Laplace equation
- 6.7 Poisson equation

B) Statistical and Monte Carlo Methods

1. Introductory concepts

- 1.1 Probability.
- 1.2 Random variables.
- 1.3 Computing probabilities
- 1.4 Bayes' Theorem

2. General properties of probability distributions.

- 2.1 The probability density function
- 2.2 The cumulative distribution function
- 2.3 Properties of the probability density functions.
- 2.4 The characteristic function.
- 2.5 Distributions of more than one variable.

3. Error propagation

- 3.1 Linear functions of random variables.
- 3.2 Change of variables.
- 3.3 Several functions. Matrix notation.

4. Probability distributions

- 4.1 The binomial distribution.
- 4.2 The Poisson distribution.
- 4.3 The uniform distribution.
- 4.4 The exponential distribution.
- 4.5 The normal distribution.
- 4.6 The multinormal distribution.

5. The Law of large numbers

- 5.1 Sampling. Inference from the sample.
- 5.2 Law of large numbers. Desigualdad de Chebysev.
- 5.3 Teorema del Límite Central. Significado.
- 5.4 Generado de números aleatorios gaussianos.

6. Monte Carlo Methods. Random number generators

- 6.1 Introduction and definition. Examples.
- 6.2 Generators. General properties.

- 6.3 Truly random numbers.
- 6.4 Pseudo-random numbers.
- 6.5 Linear Congruential Generators. Other Methods.
- 6.6 Statistical tests.

7. Sampling distributions.

- 7.1 Inverse transformation method.
- 7.2 Rejection method.
- 7.3 Composite method.
- 7.4 Sampling discrete distributions. Poisson distribution.
- 7.5 Sampling multi-dimensional distributions.
- 7.6 Multinormal distribution sampling.

8. Monte Carlo Integration.

- 8.1 Introduction.
- 8.2 Hit-miss method.
- 8.3 Crude Monte Carlo method.
- 8.4 Variance reduction techniques. Importance sampling.
- 8.5 Stratified sampling.
- 8.6 Correlation methods. Control variables. Antithetic variables.
- 8.7 Multidimensional integration.

9. Method of de Metropolis

- 9.1 Markov's chains.
- 9.2 Metropolis algorithm.
- 9.3 Examples

C) Simulation of physics processes in Nuclear and Particle Physics. GEANT4

1. Simulation of stochastic systems: radiation transport.

- 1.1 Radiation transport as a stochastic process
- 1.2 Characterization of the source.
- 1.3 Tracing a path
- 1.4 Modelling collision events.
- 1.5 Random walks
- 1.6 Boltzmann equation
- 1.7 Important sampling
- 1.8 Important sampling of integral equations.
- 1.9 Examples.

2. GEANT4 Introduction Toolkit

- 2.1 General introduction: simulation code.
- 2.2 Frame description of Geant4.
- 2.3 GEANT4 Structure.
- 2.4 Software overall view.
- 2.5 Working with GEANT4. An example.

Bibliography

- Numerical Analysis, Burden and Faires, Thomson 2005
- Numerical Recipes in C++, Press, Vetterling, Teukolsky and Flannery, Cambridge University Press 2002
- Alejandro García, Numerical Methods for Physics, Prentice Hall 2000
- Gould, Tobochnik, Christian, Computer Simulation Methods, Addison Wesley 2007

- W.H. Press y otros, *Numerical Recipes*. Cambridge University Press 1992.
- A.G. Frodesen, O. Skjeggstad. *Probability and Statistics in Particle Physics*. Universitetsforlaget 1979.
- W.T. Eadie. *Statistical Methods in Experimental Physics* North-Holland P.C.
- G. Cowan, *Statistical Data Analysis*, Oxford University Press
- R.Y. Rubinstein. *Simulation and the Monte Carlo Method*. Ed. John Wiley and Sons Inc., Nueva York 1981.

- - “Monte Carlo methods, volumen I: Basics”, Malvin H. Kalos and Paula A. Whitlock. Jhon Wilet and sons inc. 1986.
- - “Monte Carlo Methods”. Monographs on Statistics and Applied Probability, J.M. Hammersley and D.C. Handscomb. Ed. Chapman & Hall, 1983.
- - “Monte Carlo Simulation in Statistical Physics”, K. Binder and D.W: Heermann, Ed. Sringer-Verlag, 1992.
- - Colección de manuals de GEANT4, página web:
[http://wwwinfo.cern.ch/asd/geant4/G4UsersDocuments/Overview/html\(index.html\)](http://wwwinfo.cern.ch/asd/geant4/G4UsersDocuments/Overview/html(index.html))