Psicológica (2011), 32, 223-253.

Getting to the source: a questionnaire on the learning and use of arithmetical operations

Àngels Colomé*[1,2], Maria Gracia Bafalluy[1] & Marie-Pascale Noël[3]

[1] *Departament de Psicologia Bàsica, Universitat de Barcelona*
[2] *Institute for Brain, Cognition and Behavior, Universitat de Barcelona*
[3] *Université Catholique de Louvain*

Some current models of mathematical cognition (Dehaene, 1992; Campbell & Clark, 1992) make strong claims about the code in which arithmetical operations are solved, basing themselves on how these operations were originally acquired or are most frequently employed. However, data on acquisition and use are often derived from anecdotic reports and no systematic figures have ever been collected. In this study a questionnaire was devised to investigate how participants learned multiplication tables, as well as the code in which one-digit and multi-digit operations are usually solved. The questionnaire was administered to two groups of university students, one Spanish (Study 1) and the other Belgian (Study 2). The results show that multiplication tables are mainly learnt by oral rehearsal, but adults solve multiplications more frequently by visualizing Arabic digits. This is also their preferred code for calculating additions, subtractions, and divisions. The preference for the Arabic code increases when subjects have to solve multidigit operations.

When dealing with numbers, many different formats can be used to convey the same meaning: Arabic digits, number words, Roman numbers or dots, among others, are all valid ways of representing a precise numerosity. The question, however, is whether some formats are particularly good at performing a given numerical task. On this matter the three main current models on numerical cognition differ (see Figure 1).

---

* Acknowledgements: This research was supported by a grant from the University of Barcelona (research programme for Humanities and Social Sciences, 2009) and PSI2009-10364 from the Spanish Ministry of Science and Technology. The third author is supported by the National Research Fund of Belgium. The authors are grateful to M. Isabel Núñez-Peña for her helpful suggestions about the statistical analyses. They also wish to thank Elisabet Tubau for her comments on a previous version of the document. Corresponding author: Àngels Colomé. Departament de Psicologia Bàsica-Facultat de Psicologia. Universitat de Barcelona. Passeig de la Vall d'Hebron, 171; 08035 BARCELONA (SPAIN). Phone: +34 93 312 51 42. Fax: +34 93 402 13 63. Email: angels.colome@ub.edu

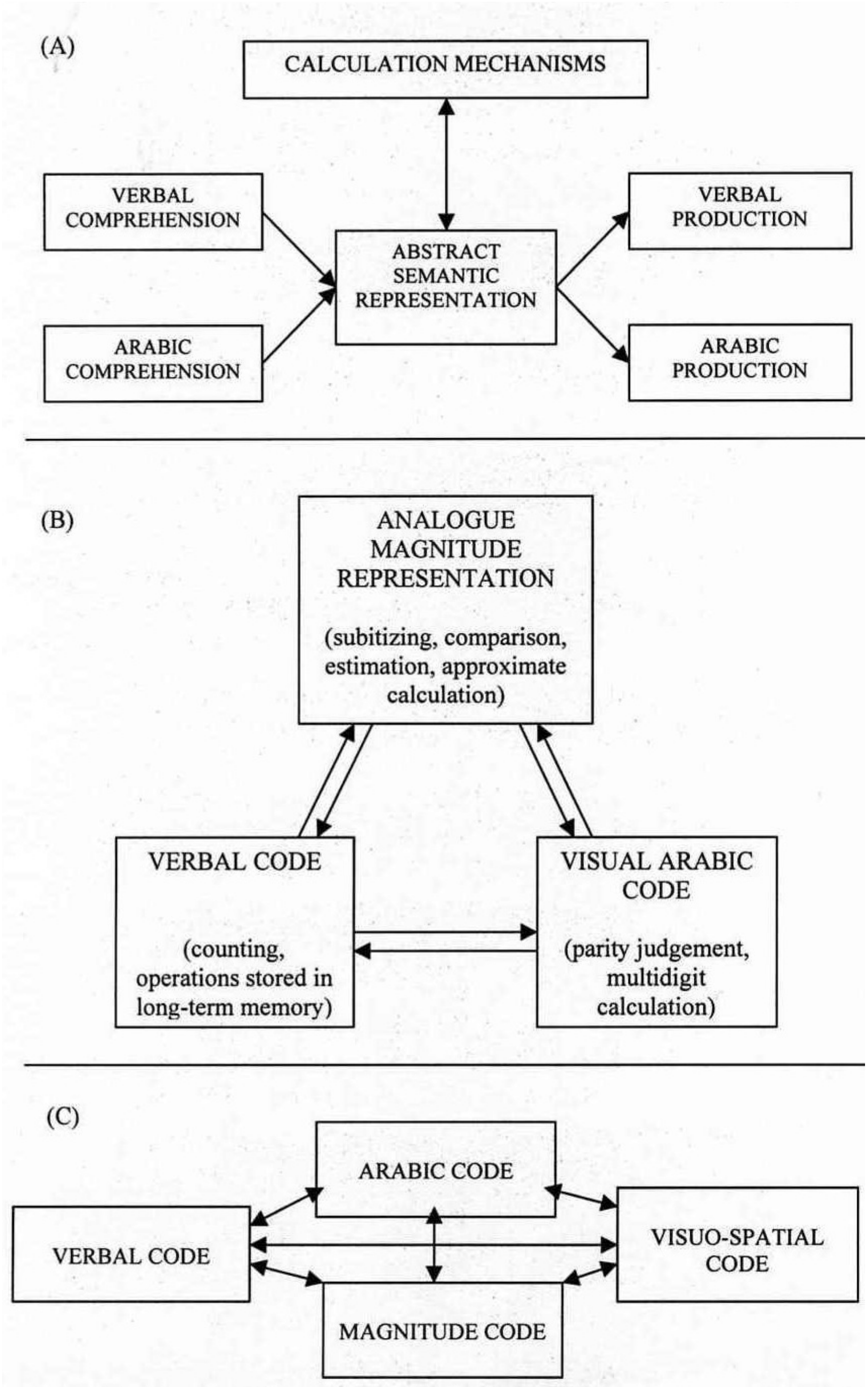

Figure 1. Models on numerical cognition. (A) Abstract-code model by McCloskey (1992); (B) Triple-code model by Dehaene (1992); (C) Multiple encoding hypothesis by Campbell and Clark (1992).

According to McCloskey and colleagues (McCloskey, 1992; McCloskey & Macaruso, 1995) there is no such ideal format. Arabic digits or number words are ways of perceiving or expressing a given quantity, but the central processes — such as calculating, or deciding which of two numbers is the largest — take place in an amodal semantic representation that acts as a bottleneck between input and output stages. Therefore, the only benefit that might derive from receiving numbers in one format rather than another is that certain formats might be perceived, recognised and transcoded to this amodal representation faster than others. For instance, one can imagine that processing a one-digit, Arabic number will be faster than perceiving a word composed of several graphemes. Likewise, this faster transcoding speed might enable an answer to be provided more quickly in one format than in others. However, the central processes, where the numerical manipulation takes place, will never differ across formats.

In contrast, Dehaene and Cohen (Dehaene, 1992; Dehaene & Cohen,1995) claim that there are three kinds of internal representations that deal with numerical processes, namely, an analogue representation of quantity, a visual-Arabic number code[1] and a verbal word frame for numbers. Each of these representations is in charge of specific tasks. Thus, the analogue representation of magnitudes is used for comparison and approximate calculation. The visual-Arabic code is in charge of exact multi-digit calculation. Lastly, the results of some one-digit operations, known as arithmetical facts (mainly multiplications and small additions), are not calculated each time but are retrieved from long-term memory using a verbal code, as they are usually learned by being repeated aloud. When a number is presented in a different format from the one in charge of performing a given task, it has to be transcoded before this task can take place. Thus, for instance, if a multiplication such as 3x4 is presented as Arabic digits, it has to be transcoded into a verbal format before the result can be retrieved.

The third model, proposed by Campbell and collaborators (Campbell & Clark, 1992; Campbell, 1994; Campbell & Epp, 2004), claims that numerals are used in a variety of tasks, which leads to the creation of a network of information in different representational codes. When dealing with a particular task, some of this information is relevant and the rest is not: a series of excitatory and inhibitory connections ensures that the processing is conducted successfully. The way – for instance, the format – in which a given task has been acquired and subsequently practised will

[1]Throughout this paper, “format” will be used to refer to external representations of numbers, and “code” to talk about their internal representation.

strengthen specific encoding-retrieval pathways. In the case of arithmetic, Campbell & Epp (2004) claim that there is a verbal representation of arithmetical facts because at the beginning counting is often used to solve arithmetical operations. Subsequently, calculation is said to take place mainly in Arabic digits (Campbell, 1994; Campbell & Clark, 1992): therefore, this code will also facilitate access to the results of arithmetical operations. In addition, the input format also plays a role: presenting a task in a well-practised format will increase the efficiency of the retrieval process (Campbell & Epp, 2004).

To summarise, two of the main current models make predictions about the suitability of certain formats in performing a given numerical task. Dehaene et al. (1992, 1995) claim that arithmetical facts are learned by verbal repetition, and therefore they remain stored in this way. Campbell et al. (Campbell & Clark, 1992; Campbell, 1994; Campbell & Epp, 2004) agree on the importance of the way we learn to calculate, but also stress the relevance of how we usually perform operations. Several recent studies have investigated the role of the verbal code in the storage of arithmetical facts (e.g. Cohen, Dehaene, Chochon, Lehéricy & Naccache, 2000; Dehaene & Cohen, 1997; Lee & Kang, 2002; LeFevre, Lei, Smith-Chant & Mullins, 2001; Venkatraman, Siong, Chee & Ansari, 2006); most of them have concluded that language plays an important role in storing and accessing this type of information, although some studies suggest that the pass through the verbal code may not be mandatory (e.g. Campbell & Epp, 2004; Kashiwagi, Kashiwagi & Hasegawa, 1987; Rusconi, Galfano, Speriani & Umiltà, 2004). Another important group of studies have compared performance more generally in tasks with different input formats (e.g. Campbell & Alberts, 2009; Campbell & Clark, 1992; Campbell, Parker & Doetzel, 2004; Noël, Fias & Brysbaert, 1997; Metcalfe & Campbell, 2008).

However, this was not the objective of our research: our aim was to take a step back and to determine whether some of the assumptions of the models that these studies are testing are in fact well-founded. For instance, the fact that multiplication tables are frequently associated with the verbal code has its origin in Dehaene et al.'s idea that multiplication is learned by oral rehearsal. To our knowledge, there are no systematic data showing that arithmetical facts are nowadays learned by oral repetition, nor that once fully acquired they continue to be solved verbally (Dehaene, 1992; Dehaene & Cohen, 1995). Neither are there data in support of the notion that Arabic digits is the most common format in calculation, as claimed by Campbell et al. (Campbell & Clark, 1992; Campbell, 1994; Campbell & Epp, 2004).

Dehaene et al. based their argument on anecdotal data from bilingual speakers, suggesting that when they calculate they often revert to the language in which they originally learned these arithmetical facts (Kolers, 1968; Shannon, 1984). In this line, they also conducted a series of experiments (Dehaene, Spelke, Pinel, Stanescu & Tsivkin, 1999; Spelke & Tsivkin, 2001) in which Russian-English bilinguals were taught new arithmetical facts. When tested after the training sessions the participants performed significantly better if operations were presented in the language in which they had learned them, independently of whether it was their dominant language or not. This led the authors to conclude that arithmetical facts had been stored in a language-specific format. However, their participants were tested after two days of intensive training in new arithmetical facts. One might ask, therefore, whether this is comparable to multiplication facts, which are acquired in childhood, repeatedly used subsequently and, more importantly, encountered in various formats. It is not implausible to assume that links between problems and results in other formats will be strengthened by their frequent use, to the detriment of verbal representations that might become less efficient in accessing the fact store due to their infrequent use. This is precisely what Campbell et al. (1992, 1994) would predict. Unfortunately, however, there are no data about the usage frequency of each format that would allow Campbell's hypothesis to be tested.

The research presented here aims to bridge these gaps by collecting data on the frequency of use and preferred formats for calculation in a sample drawn from the population traditionally tested in the experiments used to verify the current models. Specifically, we asked a group of psychology students from the University of Barcelona to fill in a questionnaire in which these aspects were considered. Note that our aim in this study was not to provide a general portrait of how *any* person performs a numerical task. Clearly, individual differences due, for instance, to gender, age or educational level (e.g. Campbell & Xue, 2001; Deloche, Seron, Larroque & Magnien, 1994; Deloche, Souza, Braga & Dellatolas, 1999; Imbo, Vandierendonck & Rosseel, 2007; Jackson & Coney, 2007) would be of relevance here. Instead, we sought to verify whether some of the postulates traditionally assumed in current models, and based on this population, are supported by statistical data. Questionnaires or self-reports have been previously used in mathematical cognition for investigating calculation strategies (Kirk & Ashcraft, 2001; LeFevre, Bisanz, Daley, Buffone, Greenham & Sadesky, 1996; LeFevre, Sadesky & Bisanz, 1996; Romero, Rickard & Bourne, 2006). The capacity of participants to report their behaviour accurately is a slightly controversial issue, as is the influence that self-reports might have in concurrent tasks. However, recent research has shown that self-reports can be reliable reflections of participants' behaviour (e.g. Grabner, Ansari, Koschutnig, Reishofer, Ebner & Neuper, 2009; Smith-Chant & Lefevre, 2003), provided some caution is taken when interpreting their results. One of their possible limitations (Kirk & Ashcraft, 2001) is that information entered into the short-term memory and attended to seems to be more accessible than automatic processes, where only the final product might be available: in the case of arithmetic this means that participants may provide a more accurate account of the format in which they solve operations through procedural strategies than of the code in which operations solved by direct retrieval are stored. In the latter case, it may be that other codes were also activated in the retrieval process, as in the networks proposed by Campbell and colleagues, but that participants' responses did not capture them.

As for this particular questionnaire, another comment should be made: participants were asked to report approximate percentages of the occurrence of certain behaviours or of the use of a given format. We are aware of the difficulty of providing these figures, and they should not be taken as exact quantities but rather as indicators of the prevalence of a given option over others, or of the frequency distribution of different patterns.

At all events, it was important to check that the numbers provided by our participants showed a good level of introspection. In order to test the reliability of our data and its generalization to another subgroup of the same population, we also tested a smaller group of students from the University of Louvain-la-Neuve in Belgium (see Study 2).

Both studies focused on calculation, since this is the task about which the models have made the strongest statements.

The questionnaire was divided into four blocks (see Appendix A). The first contained questions about the age, gender and mother tongue of participants. The data obtained here helped to provide a better description of the sample.

In the second block, participants were asked about how frequently they performed calculations. They also had to report the frequency of each of the four main operations (addition, subtraction, multiplication and division). Frequent use increases the probability of over-learning, i.e. that participants retrieve the result from memory as in the arithmetical facts proposed by Dehaene et al. (Dehaene,1992; Dehaene & Cohen, 1995) instead of calculating it (Imbo & Vandierendonck, 2008).

A third block focused on the format in which participants solved the operations. They were first asked whether there was any format in which they preferred operations to be presented. Alternatives were Arabic digits, oral words and written words. We also asked them in which of these codes they preferred to solve operations. As we said above, Campbell (Campbell & Clark, 1992; Campbell, 1994) considers that input format can determine the internal code used to perform an operation. As for Dehaene's model (Dehaene 1992, Dehaene & Cohen, 1995), it claims that input has to be in the appropriate format to perform a given task; otherwise, stimuli have to be transcoded before the task (in this case, calculation) can take place. This transcoding will presumably take some time, and certain cognitive resources will have to be devoted to it. In asking our participants to pick a preferred format our aim was to verify whether these differences between formats were perceived by them in some way.

Furthermore, we proposed different ways in which operations can be solved and asked them to state the frequency (as a percentage) with which they used them. We distinguished between operations and also between one-digit and multi-digit calculations. This latter distinction was made in line with the differentiation proposed by Dehaene et al. (1992; 1995) between one-digit operations, which might often be directly retrieved from long-term memory, and multi-digit calculations, which must be calculated on each occasion.

The issues we wished to investigate concerned relevant points of the current models. Thus, considering the assumptions made by Dehaene et al. regarding the storage of arithmetical facts, our aim was to answer the following questions: (a) are one-digit multiplications mainly solved verbally? (b) is the verbal code used more frequently in multiplications than in other operations? (c) are there differences in the way (code) in which one-digit and multi-digit operations are solved?

Lastly, a fourth block was devoted to the learning of multiplication facts. Participants were first asked whether they knew the multiplication tables by heart, and were then questioned about their method of learning. Options were oral repetition, exercises with Arabic digits, a combination of both, or others. Although this last block was principally designed to verify that participants knew the multiplication tables by heart, we also took advantage of the bilingualism of the participants in Study 1 (proficient Catalan-Spanish bilinguals) and asked them about the language in which they had learned the multiplication tables. They had to specify whether this was their mother tongue; if it was not, they were asked when they had learned that language[2]. Finally, they were asked to state the language in which they were using multiplication tables currently, and to estimate (as a percentage) their current use of Catalan and Spanish. Our aim was to check whether they continued to use a given language to perform multiplications, and whether this fact depended on the learning language being their mother tongue or more frequently used language.

STUDY 1

Participants and procedure. One hundred and twenty-four students from the Faculty of Psychology of the University of Barcelona took part in this study. As is frequently the case in this population, the vast majority were women (74%). The mean age of participants was 20.36 years (*SD*=2.13).

Barcelona is located in Catalonia, a region in the northeast of Spain where two Romance languages (Catalan and Spanish) coexist. Both of them are used in all sorts of daily activities by most of the local population. In this part of Spain television and radio programmes, as well as written press and books, are available in either language. In the educational context, immersion programmes are implemented in kindergartens to provide access to Catalan to children with monolingual social backgrounds before they begin compulsory education. By the end of primary school, pupils are required to have a good command of both languages. At secondary school and in universities some subjects are taught in Spanish and some in Catalan, although statistics for the Faculty of Psychology show a slight preference for Catalan (Catalan = 53.5%; Spanish = 46.5%; data from year 2004-2005; http://www.ub.edu/sl/ca/socio/docs/dades04-05.pdf).

Thus, all our participants had mastered the two languages. However, in order to know which was their dominant mother tongue we asked them about the language spoken at home with their families. Seventy-one participants (57.25%) reported that they spoke only in Catalan, 49 (39.51%) said they spoke only in Spanish, and 3 of them (2.41%) came from bilingual homes. One participant did not answer this question.

Participants had been recruited for different experiments and were tested in groups of four. Prior to the experiments they were required to fill in the questionnaire designed for this study (see Appendix A). The questionnaire was printed on three pages and written in Catalan.

[2] Some of the participants did not answer this question. In those cases where an answer was provided, variation across participants was too small to allow any relevant conclusions to be drawn. Therefore, this variable was not considered in the analyses.

Participants were given instructions orally and were informed that all their data would be treated anonymously. The experimenter was in the room throughout and answered any questions posed. Filling in the whole questionnaire took between 10 and 15 minutes. Participants received course credits for their collaboration in the whole session.

RESULTS AND DISCUSSION

In what follows we will present the data structured into the blocks described in the introduction. On each occasion the main theoretical conclusions for each block will be discussed.

On the frequency of calculation. Table 1 shows the answers provided by participants when asked about the time they devote to calculations in their daily life. More than half of our participants (63.70%) reported performing calculations everyday and 31.45% of them stated that they used at least one of the four main arithmetical operations three or more times a day. The second largest percentage corresponded to those who said they used calculation at least three times a week (21.77%). In contrast, only a small percentage of the sample (6.45%) reported performing arithmetical operations only on a monthly basis.

A second question explored the percentage of use of each arithmetical operation. Participants reported using mainly additions (*Median (Mdn)*[3] =40% of their operations), followed by subtractions (*Mdn*=25%) and multiplications (*Mdn*=20%), while divisions were reported to be used less often (*Mdn*=10%). The paired Wilcoxon t-tests showed that all these percentages differed significantly (subtractions-additions: Z=-6.65, p<.001; multiplications-additions: Z=-6.87, p<.001; multiplications-subtractions: Z=-3.16, p<.003; divisions-multiplications: Z=-5.58; p<.001).

The fact that addition is the most frequently-used operation fits with the general view in the literature that one-digit additions, together with over-learned multiplications, are the operations most often stored and directly retrieved from long-term memory (Campbell & Xue, 2001; Domahs & Delazer, 2005; Seyler, Kirk & Ashcraft, 2003). As for divisions, it has been proposed (e.g. Domahs & Delazer, 2005; Rusconi, Galfano, Rebonato & Umiltà, 2006) that they would not have a representation in long-term memory but would be calculated either by different procedural strategies or by converting them to multiplications (i.e. '24:3 =' would be

[3] Due to the high variation across participants' answers, medians were considered to be more informative than means, and all the statistics in the paper are based on this measure.

solved by finding the number that multiplied by 3 has 24 as result). A reason for this preference for procedural strategies may be their infrequent use.

To summarise, this first section showed that survey participants were using basic arithmetic calculation regularly. This section also showed that these operations are not equally frequent: additions were significantly more frequent than the other calculations, while divisions were encountered much less often.

Table 1: (A) Frequency of calculation in daily life. Percentage of Spanish and Belgian (between brackets) participants that selected each frequency of use. (B) Frequency of use of each arithmetical operation: median of the percentages reported by the participants.

(A)	On a daily basis		Weekly		Monthly	
Once	15.32%	(12.50%)	1.61%	(12.50%)	0.80%	(0%)
Twice	16.93%	(20.83%)	6.45%	(8.33%)	3.22%	(0%)
Three times or more	31.45%	(45.83%)	21.77%	(0%)	2.41%	(0%)

(B)	Addition		Subtraction		Multiplication		Division	
Median %	40%	(40%)	25%	(20%)	20%	(16.25%)	10%	(10%)

On the format of presentation and solution. A first question of this third block asked participants whether they preferred to have operations presented in a particular format: Arabic digits, and oral and written number words were presented as options. A vast majority of participants (see Table 2) opted for the Arabic format, either alone (85.48%) or in combination with oral number words (2.41%). In contrast, 8.06% of them claimed not to have a preference for any format of presentation. Lastly, only a small percentage of the sample preferred to have operations presented in auditory

(1.61%) or written (2.41%) number words. Therefore, Arabic digits are clearly the favourite format for the presentation of arithmetical operations.

In a second question (see Table 2) we now asked whether there was a code in which participants preferred to solve operations. Again, most of them chose Arabic digits alone (81.40%), or together with oral number words (1.61%). Number words, either auditory or written, were selected by a small proportion of participants (6.45 and 3.22%, respectively). Lastly, 7.25% of participants claimed not to have a preference for any of the formats. Thus, once again, participants seemed to prefer the Arabic format for dealing with arithmetical operations. Interestingly, although 21 of the 124 participants chose different options in these two questions, the answers to both presented significant correlations (R^2=0.16, p<.001), i.e., participants prefer to have operations presented in the format in which they will finally solve them.

Table 2. Format preferences for input presentation and solution of the arithmetical operations. Percentage of Spanish participants (Belgian between brackets) preferring this format (%).

Preferred format	Presentation (%)		Solution (%)	
Arabic digits	85.48	(83.33)	81.40	(50)
Oral number words	1.61	(0)	6.45	(20.83)
Written number words	2.41	(0)	3.22	(0)
Arabic digits + oral Words	2.41	(4.16)	1.61	(8.33)
No preferences	8.06	(12.50)	7.25	(20.83)

A limitation of the last questions was that they did not distinguish between the four operations. However, Dehaene and colleagues (Dehaene, 1992; Dehaene & Cohen, 1995) make a strong statement about the verbal storage of one-digit multiplications and additions, while the code of other operations is much less clear. Neither did the question distinguish between the different ways in which operations can be solved and, more specifically, between retrieval and procedural strategies.

To overcome these limitations, another item from the questionnaire required participants to report the percentage of occasions on which they performed calculations by using the different options indicated. Percentages had to be specified separately for additions, subtractions, multiplications and divisions, as well as for one and multi-digit operations.

Options included (a) saying the number words out loud or mentally, (b) visualising the Arabic digits, (c) writing the Arabic numbers on paper, (d) using a calculator, and (e) others (in this last case, participants had to state which). As can be seen, these options do not correspond to a format but, rather, describe a naturalistic situation in which this format might be used. This was done in order to help participants in their decision-making process. Table 3 shows a summary of the responses given by participants.

The information collected from this item helped us to answer several questions. The first point we wished to address was whether one-digit multiplications are solved mainly verbally, as predicted by Dehaene et al. (Dehaene, 1992; Dehaene & Cohen, 1995). Participants reported that manipulating Arabic digits mentally was their most frequent way of solving this kind of arithmetical operation (*Mdn*=30%). Using number words, a calculator or pen and paper were less frequent options (*Mdn*=10% in all cases) and the difference between the four alternatives was significant according to a Friedman test (χ^2(3, N=124)=34.12, p<.001). Lastly, eight participants (6.45% of the sample) reported using methods other than those listed to solve multiplications: two of them said they used their fingers, three reported using a computer, and three did not specify the method they used. Therefore, visualising Arabic digits mentally was the preferred way of solving one-digit multiplications for our participants. If we take into account the other options that also involved Arabic digits, the preference for this code over the verbal one is even stronger.

We also checked how participants solved the other one-digit arithmetical operations (see Table 3). We thought that even if number words happened not to be the main code in multiplication, they might still be more relevant for this operation than for the rest. However, our data did not support this possibility. For both additions and subtractions, visualising Arabic digits was again the most widely-used option (*Mdn*=50 and 40%, respectively), although number words were the second-ranked option, with a median of 20% of use in both cases. Despite these medians being higher than that obtained for multiplications (*Mdn*=10) a Friedman test showed

Table 3: Medians of the percentages reported by the Spanish participants (Belgian participants between brackets) on their use of the different ways of solving operations.

Way of solving	One-digit operations								Multi-digit operations							
	multiplication		division		addition		subtraction		multiplication		division		addition		subtraction	
number words	10	*(0)*	10	*(0)*	20	*(0)*	20	*(0)*	10	*(10)*	1.25	*(7.5)*	20	*(23)*	12.5	*(15)*
visualizing Arabic digits	30	*(55)*	20	*(50)*	50	*(70)*	40	*(65)*	15	*(15)*	5	*(10)*	30	*(45)*	27.5	*(30)*
pen and paper	10	*(0)*	10	*(0)*	5	*(0)*	10	*(0)*	15	*(13.5)*	10	*(10)*	15	*(12.5)*	15	*(12.5)*
calculator	10	*(0)*	20	*(0)*	0	*(0)*	5	*(0)*	47.5	*(37.5)*	62.5	*(70)*	17.5	*(10)*	20	*(20)*
others	0	*(0)*	0	*(0)*	0	*(0)*	0	*(0)*	0	*(0)*	0	*(0)*	0	*(0)*	0	*(0)*

that the frequency with which participants used the verbal format in the three operations did not differ (χ^2(2, N=124)=1.69, *p*<.50). As for division, mental Arabic digits and (for the first time) a calculator were the most frequently selected options (*Mdn*=20% each). Number words were used less often (*Mdn*=10%) than the former options, and also less often than in the other operations (division-addition: *Z*=-3.08, *p*<.003; division-multiplication: *Z*=-2.60, *p*<.010; division-subtraction: *Z*=-2.98, *p*<.004).

To summarise, the data did not confirm a special link between multiplication facts and number words. On the one hand, visualising Arabic digits was the most frequently selected option for the four operations, including multiplication. On the other hand, although participants reported a frequent use of number words, no differences were obtained between one-digit multiplications, additions and subtractions. Only division, which is seldom encountered, seemed to be less frequently solved through the use of number words.

Finally, a third point we wished to address in this part of the questionnaire was the comparison between one-digit and multiple-digit operations. As stated above, Dehaene and colleagues (Dehaene, 1992; Dehaene & Cohen, 1995) consider that arithmetical facts are stored verbally, while multi-digit calculation is solved by manipulating Arabic digits in a mental sketchpad. Despite the fact that our initial data had already minimised the impact of verbal representations on one-digit arithmetic, we considered that it was still relevant to study which codes were used in multi-digit calculation[4].

A noteworthy finding here was that multiplications and especially divisions were mostly solved with a calculator (*Mdn*=45.70 and 62.50%, respectively). This is an interesting point, since such a huge prevalence of calculator use might work against the ability of our participants to remember the multiplication tables by heart.

Participants also claimed to solve two-digit multiplications by mentally visualising Arabic digits or by using pen and paper (both *Mdn*=15%). Finally, number words (*Mdn*=10%) were used significantly less often than these last two options (mental Arabic digits-number words: *Z*=-2.80, *p*<.006; pen and paper-number words: *Z*=-2.91, *p*<.004). We also

---

[4] When designing the questions on multi-digit arithmetic we were quite aware that the options proposed to participants were somewhat reductionist. One can easily imagine a scenario in which a participant combines different codes in a single operation. However, had we tried to contemplate all possible combinations the number of alternatives would have increased so much that the analysis and the final data would have become impossible to deal with.

compared the use of number words in one- and multi-digit operations, and found a significant decrement in the latter (Z=-3.95, p<.001).

As for divisions, the most frequent options after a calculator were, in this order, pen and paper (*Mdn*=10%), visualising Arabic digits (*Mdn*=5%) and working with number words, which yielded a median of just 1.25% of occasions.

Additions and subtractions, which according to our participants are more frequently encountered, were also less frequently solved with calculators (*Mdn*=17.5% and 20%, respectively) than were multiplications and divisions. Multi-digit additions were mainly solved by manipulating Arabic digits mentally (*Mdn*=30%). Use of number words (*Mdn*=20%) was the second most frequent alternative, followed by a calculator (*Mdn*=17.5%) and pen and paper (*Mdn*=15%). The frequency of use of these three alternatives did not differ statistically (χ^2(2, N=124)=1.08, p<.60).

Lastly, the most frequent method for solving subtractions was again mentally visualising Arabic digits (*Mdn*=27.50%). Other methods employed were a calculator (*Mdn*=20%), pen and paper (*Mdn*=15%) and number words (*Mdn*=12.50%). All the statistical differences between formats were significant, except for the comparisons 'calculator-mentally visualising Arabic digits' (Z=-.98, p<.40) and 'number words-pen and paper' (Z=-.38, p<.99).

Moving from one-digit to multi-digit operations entailed some changes in how participants solved operations. On the one hand, the use of the calculator increased importantly. This was also the case for pen and paper, with the exception of divisions, where their use remained stable. On the other hand, those formats that did not rely on a physical device for the calculation (i.e. visualising Arabic digits mentally or saying the number words aloud or mentally) were less frequently reported. These data fit well with an explanation based on the increasing difficulty and cognitive resources involved in all the steps necessary to solve multi-digit calculations: applying spatial procedures, performing partial operations or holding intermediate results in memory, among others.

On learning and using multiplication facts. The last section of the questionnaire focused on one-digit multiplications. The first thing we wanted to explore was whether this operation was an arithmetical fact for our participants, i.e. whether they had multiplications stored in long-term memory. Eighty-seven per cent[5] of participants claimed they knew all the multiplication tables by heart, 2% said they didn't know any of them, and 11% stated that they only remembered some of them. This latter group was asked which tables they had memorised (see Table 4). All of them reported knowing the two-times and five-times table. The next-ranked tables were the three-, four- and six-times tables (82% of participants). In contrast, the tables that were harder to remember for these participants involved bigger numbers and were, in descending order, the nine-times (45%), seven-times (27%) and eight-times (9%) tables. This last result is consistent with the so-called 'problem size effect' (Ashcraft, 1992; Groen & Parkman, 1972): it is often the case that large problems lead to longer reaction times and more errors than do problems with small operands. This fact has been explained in several ways: one hypothesis is that there is a weaker problem-answer association in large problems because they are less frequently encountered and practised, while another hypothesis is that they are solved through the use of procedural strategies rather than direct retrieval (for a review, see Zbrodoff & Logan, 2005).

Table 4. Multiplication tables remembered by those participants (N=12) who claimed not knowing all the tables by heart.

Multiplication table	Participants who reported knowing it by heart:
2 times table	100%
3 times	81.81%
4 times	81.81%
5 times	100%
6 times	81.81%
7 times	27.27%
8 times	9.09%
9 times	45.45%

[5] Sample size in this section was reduced to 111 participants because the remainder left some of the relevant items unanswered.

Subsequently, participants were asked about the method with which they had been taught the multiplication tables. An important statement of the model by Dehaene and collaborators (1992, 1995) is that multiplications are stored verbally because they were originally learned by oral rehearsal. However, pedagogical methods change over the years and across countries, and we therefore wanted to check whether this was the case for our sample. Our results showed that oral repetition is still the main method of learning. Seventy participants (63% of the population) claimed to have learned one-digit multiplications by repeated oral rehearsal, while a further thirty-four (31%) chose a combination of oral rehearsal and exercises with Arabic digits; when asked about the relative percentage of practice for repetition and exercises, the medians were identical (50% for both methods). One other participant claimed to have combined the former options with "understanding the logic" of the multiplication. Lastly, only six participants (5%) said they had learned the tables simply by using them in exercises in Arabic digit format. Thus, we can confirm that the method used for teaching multiplication facts to our sample was mostly verbal repetition.

However, as argued in the introduction, although this piece of knowledge may have been originally stored in a verbal code, it has subsequently been used in various contexts and formats. This may have led to the creation of new internal representations of these facts in other codes. On the one hand, we saw in the previous section that participants claimed they solved one-digit multiplications mainly by visualising Arabic digits mentally. On the other hand, our participants had the special feature of being very balanced Catalan-Spanish bilinguals. In their 2001 study, Spelke and Tsivkin found that Russian-English participants were better at providing the answer of recently learned arithmetical facts when these were presented in the language of learning, even if this was not their dominant, mother tongue. Would this also be the case for our sample, or would they have a copy of the arithmetical facts in the other language? When asked about the language in which they were currently using the multiplication tables, the results were not as clear-cut. Seventy-six participants (69%) were still using the tables in the language in which they had learned them, but thirty-two of them (29%) reported using the two languages indistinctly (see table 5). Lastly, three participants (2%) said that they used the tables only in the language in which they had *not* learned them. In these three cases, this new language of use was their mother tongue, which led us to wonder if the current use of multiplications might depend on whether the learning language had been the mother tongue of the participant. Ninety-five participants had been taught multiplication tables in their mother tongue. Seventy-one were still using this language, while twenty-four used it and their second language indistinctly. In contrast, only sixteen participants learned multiplication facts in their second language. Of these, five were still using that language, but three had switched to their mother tongue and eight used both of them. A chi-square test showed that these proportions were significantly different ($\chi^2(2)=24.50$, $p<.001$), that is, the probability of changing language in the current use of multiplications depended on whether multiplications had been originally learned in the mother tongue. This is despite the fact that our population is considered one of the most balanced bilingual groups in the world (see, for instance, Costa & Santesteban, 2004).

Table 5. Multiplication tables and bilingualism. Language in which multiplication tables are currently used in relation to the language of learning (mother tongue (L1) or second language (L2)) and the current general use of the training language.

Language in which multiplication tables are currently used			Percentage of current use of the training language in daily life
	Training Language	Number of participants	
Language of learning	L1	71	77.26%
	L2	5	42%
Both languages	L1	24	60.41%
	L2	8	43.12%
Other language	L1	0	
	L2	3	33%

A second factor that may determine the language in which multiplication tables are solved nowadays might be the general percentage of current use of each of the languages: we would expect that if participants were not using the training language very much in any everyday context, then its use when performing multiplications might also have decreased. We applied a Kruskal-Wallis test to compare the percentage of general current use of the training language with respect to the language in which participants claimed to be using the tables (1= training language, 2=other language, 3=depending on the situation). The result was significant ($\chi^2(2)=24.05$, $p<.001$), showing that the likelihood of continuing to use multiplication tables in the training language increased when the percentage of general use of this language in everyday situations was high[6]. Of course the two factors (mother tongue and current most-used language) are highly interrelated: of 111 participants, 89 claimed they used their mother tongue more than their second one, while only nine chose the opposite pattern. A further thirteen reported using both languages with the same frequency.

In addition to the above, we also considered the possibility that the fact of having been taught the multiplication tables in the mother tongue or in a second language might have an effect on our participants' current ability to retrieve these arithmetical facts. However, this was not the case: thirteen of the ninety-five participants who had learned multiplications in their mother tongue reported not knowing all the tables by heart. One of the sixteen participants who learned them in their second language gave the same answer. When compared, the two proportions did not differ ($\chi^2(1)=0.68, p<.50$).

Lastly, we also wondered whether the fact of having learned multiplication tables in the mother tongue would have increased the use of number words for solving products. However, this was not the case ($\chi^2(1)=1.99, p<.20$).

To summarise, our results contrast with the data obtained by Spelke and Tsivkin (2001) (see also Dehaene et al., 1999): more than a third of our participants reported using the multiplication tables in a language other than the training language, or in a combination of the two. This was more often the case when the training language was not their mother tongue, or when it was used less often at present. We believe that the difference between our study and that of Spelke and Tsivkin is that our participants had had life-long practice in these arithmetical facts. Therefore, they had had the chance to find them in different formats, and the most frequent formats (Arabic digits or their mother tongue) had created their own representations, much as Campbell and collaborators (Campbell & Clark, 1992; Campbell, 1994; Campbell & Epp, 2004) would predict.

An important finding is that being taught multiplication tables in a language other than the first, dominant language did not seem to affect their ability to memorise them. This is very relevant if one considers the recent trend in many countries to teach subjects other than language itself in a foreign language (*content and language integrated learning*).

[6] All paired comparisons were also significant, with the exception of the difference between those claiming not to use the training language at all and those using it in combination with another language; this latter comparison approached significance ($Z=-1.87, p<.07$).

STUDY 2

The results from Study 1 coincide with and complement previous experimental data, thanks to the advantages that questionnaires provide compared to experimental situations: that is, the opportunity to explore a wide range of arithmetical operations with a single instrument, and especially tracing changes in the use of operations caused by lifelong practice. However, this questionnaire had been created *ad hoc* for this research, and it was important to test its reliability. We decided to conduct a second study with a new group of Psychology students from the University of Louvain-la-Neuve (Belgium) in order to verify the reliability of our previous results and to check their generalisation outside the Spanish context. In what follows, we summarise the main results obtained in this second study.

Participants and procedure. Twenty-four psychology students from the Université Catholique de Louvain-la-Neuve (Belgium) filled in a translated version of the same questionnaire. All but one of them were women and their mean age was 20.5 years (*SD*=0.78). Louvain-la-Neuve is situated in the French-speaking part of Belgium: all our participants reported being French native speakers and mainly used this language (*Mdn*=99%). The original questionnaire was translated into French and administered collectively, in an auditorium, after a psychology class. Only those students willing to participate stayed behind and filled in the questionnaire.

RESULTS

On the frequency of calculation. Table 1 shows a summary of the data on the frequency of calculation, both in general and for each particular operation. Similarly to the Spanish students, most of the Belgian participants (79.16%) reported performing arithmetical operations daily, and 45.83% of them said they made calculations three or more times a day. Only 20.83% of the participants reported using the four main operations on a weekly basis. In general, Belgian students seem to be calculating slightly more frequently than are their Spanish counterparts. However, the distribution of frequencies across each single operation followed a similar pattern to that obtained in Study 1. Most of the arithmetic calculations corresponded to additions (*Mdn*=40%), followed by subtractions (*Mdn*=20%), multiplications (16.25%) and divisions (10%). Paired Wilcoxon t-tests showed differences between the frequency of additions and subtractions (Z=-3.51, p<.001) and between multiplications and divisions (Z=-3.19, p<.002), but not between subtractions and multiplications (Z=-1.11, p<.30). This lack of significance is the only difference with respect to the Spanish group.

On the format of presentation. Once again, the first question in this section required participants to report the format in which they preferred operations to be presented (see Table 2). Most of them selected the Arabic format, either alone (83.33%) or paired with number words presented orally (4.16%). The remaining participants (12.5%) reported feeling equally comfortable with all the proposed formats (Arabic digits, oral words and written words). None of them picked the verbal format alone.

In a second step they were asked in which code they preferred to solve operations. Arabic digits were again the most selected option (50% alone, and 8.3% paired with oral number words), but the preference for this format was much less marked than in the previous question or in comparison to the Spanish group. As for the remaining participants, 20.83% of them chose oral number words, while a further 20.83% reported having no preferences among the options proposed. This is an odd result, especially when compared with the answers provided by the same participants on the next item, where the same question was addressed to each operation (see Table 3).

The first issue we investigated in this more detailed analysis was whether one-digit multiplications were solved through verbal code. This was not the case: the most frequent way of performing multiplications was again visualising Arabic digits (Mdn=55%), while the medians for number words, pen and paper, calculators or other formats were all zero. The difference between the frequency reported for Arabic digits and those for the other formats was, of course, significant (χ^2(3, N=24)=10.98, p<.02)[7]. Therefore, this population seems to solve multiplication facts mostly through methods other than a verbal one.

We also looked at how the Belgian participants solved other one-digit operations. In all of them, visualising Arabic digits was the only option that reached a median higher than zero. In the case of additions, the use of this format had a median of 70%, which was significantly different from the other alternatives (χ^2(3, N=24)=24.71, p<.001). Paired comparisons for the remaining options never reached significance (all p>.09).

[7] The option "others" was not included in the analysis because none of the participants had selected it. The same occurred for the remaining operations.

Subtractions were the second-ranked operation as regards the use of visualising Arabic digits (*Mdn*=65%). Once again, the difference between this and the remaining alternatives was significant (χ^2(3, N=24)=18.06, p<.001), although the frequency of use of the other options did not differ (χ^2(2, N=24)=1.39, p<.50).

As for divisions, the median of use of visualised Arabic digits (50%) was slightly lower than in the other operations. Nevertheless, it was still the most frequently selected option (χ^2(3, N=24)=9.60, p<.03).

Lastly, although number words were seldom used, we compared the different operations in order to see whether this format was more relevant in some of them. However, the result was not significant (χ^2(3, N=24)=5.93, p<.20).

Hence, just as we found in the Spanish group, visualised Arabic digits was the most selected option for solving all the one-digit operations, multiplications included. The only difference was that the preference for this format was much higher in the Belgians.

Subsequently, we analysed the frequency of each format in multi-digit operations. Once again, the pattern of results paralleled those obtained in the Spanish group, although the frequencies changed slightly across populations. Multiplications (*Mdn*=37.5%) and, especially, divisions (*Mdn*=70%) were mainly solved with the help of calculators.

As in the previous study, multiplications were also performed by visualising them in Arabic digits (*Mdn*=15%) or by using pen and paper (*Mdn*=13.5%), while number words were less widely used (*Mdn*=10%). The frequency of use of these three formats differed significantly from the use of a calculator (χ^2(3, N=24)=8.67, p<.04), but they did not differ among one another (all ps>.40).

This pattern was replicated for divisions: after calculators, the most selected options were, in this order, visualised Arabic digits and pen and paper (both *Mdn*=10%) and number words (*Mdn*=7.5%). Thus, calculators were the most frequently used (χ^2(3, N=24)=20.46, p<.001), while the remaining options did not differ among one another (χ^2(2, N=24)=3.16, p>.20).

The use of a calculator in subtractions was less common than in the former operations (*Mdn*=20%), and was now superseded by the visualisation of Arabic digits (*Mdn*=30%). Other ways of solving multi-digit subtractions were using number words (*Mdn*=15%) and pen and paper (*Mdn*=12.5%). Contrary to what happened in the Spanish group, the analyses showed that these medians were not significantly different (χ^2(3, N=24)=2.87, p<.50, and ps>.09 in all paired comparisons).

Lastly, visualising Arabic digits was the most frequent way of solving multi-digit additions (*Mdn*=45%), although Belgian participants often reported saying number words aloud or mentally (*Mdn*=23%) and using pen and paper (*Mdn*=12.5%). A calculator was seldom used (*Mdn*=10%). The frequency of the four options was significantly different (χ^2(3, N=24)=8.63, p<.04). Paired comparisons showed significant differences between all the contrasts, except for those between visualised Arabic digits and number words (Z=-.60, p<.60), number words and pen and paper (Z=-1.51, p<.20), and pen and paper and a calculator (Z=-.75, p<.50).

To summarise, the patterns observed in the Spanish group were replicated here, both in terms of the preferred formats for each operation and the general order of preference for the remaining options.

On learning and using multiplication facts. Belgian participants were asked about their knowledge of the multiplication tables: unlike the Spaniards, all of them reported knowing the tables by heart.

They then had to report how they had learned them. Sixteen of them (66.6%) claimed to have memorised them by oral rehearsal. Five participants (20.83%) reported having combined oral repetition with exercises in Arabic digits, and three of them (12.5%) claimed to have used exercises alone. Therefore, although other alternatives had also been used for teaching our participants, verbal repetition was again the most frequent method, thus confirming the statements by Dehaene and collaborators (Dehaene, 1992; Dehaene & Cohen, 1995).

DISCUSSION

The aim of this second study was to verify the data obtained in the previous one and investigate the extent to which the findings could be generalised to a population with a different linguistic and educational background. Although there were small variations in the raw data of both populations, their answers did coincide; furthermore, when considering the different formats the alternatives were ranked in similar orders in the two groups. Therefore, this study supports and confirms the data obtained in Study 1.

GENERAL DISCUSSION

This research arose from the need to confirm some of the postulates of the main models regarding number cognition. These postulates have often been taken for granted, without any systematic data being given to support them, probably because these aspects concerned either the way operations had been acquired long before, or the way they have been used since then. Neither of these aspects can be easily investigated through regular experiments. Hence, a different approach was required and we opted for creating a questionnaire to examine these issues. For the first time ever, participants were explicitly asked about the way they had learned multiplications, the way they solved them and the rest of basic operations (one-digit and multidigit), and their preferences regarding the format of presentation.

Our data, replicated across two different populations, confirmed some of the models' postulates. On the one hand, our data showed that visualising Arabic digits was the preferred and most frequently reported format used. This supports the idea of Campbell and colleagues (Campbell & Clark, 1992; Campbell, 1994; Campbell & Epp, 2004), according to whom Arabic digits are the most widely-used format.

On the other hand, a second part of the study confirmed that youngsters continue to learn multiplication tables mainly by oral rehearsal, as posited by Dehaene's model (Dehaene, 1992; Dehaene & Cohen, 1995). However, the subsequent use is not necessarily performed in that original code. Firstly, participants stated a preference for solving one-digit multiplications with Arabic digits. Furthermore, when bilingual participants (study 1) reported using a verbal code this was sometimes the non-training language, especially when multiplications had originally been learned in their second language, or when they did not frequently use the training language in their daily lives. Taken together, these data do not rebut the notion of a verbal representation of multiplication tables in the language in which they were originally learned, but they do cast doubts on the idea that this representation is the only one.

In short, our data seem to indicate that when dealing with arithmetical operations, the important thing is not only how participants learned them but also how they have used them from the learning stage to the current moment. In real contexts the same calculations are encountered in different formats, leading people to adapt their representations to the kind of formats in which calculations have been encountered. In this regard, a questionnaire like ours can be useful, because it allows us to evaluate the use of arithmetical facts after a longer period of time.

As discussed in the introduction, data obtained through questionnaires also have their limitations. On the one hand, certain processes or codes might be less accessible for introspection than others. We ourselves chose not to include a magnitude representation between the alternative methods for solving operations. Semantic or magnitude representations appear in all the numerical models, although their definition changes across them. We decided not to add this alternative because we were mainly interested in the dichotomy between Arabic and verbal codes, but also because we considered that it would be hard for participants to imagine the kind of representation we were talking about. As a consequence, we cannot rule out the possibility that in addition to representations with an external counterpart (such as Arabic or verbal representations) magnitude representations are also activated. Neither can we be sure that representations with codes different from those reported by our participants are not co-activated during the process. However, these considerations do not invalidate our conclusions that Arabic digits play an important role in arithmetic and that there is more than one representation of arithmetical facts.

We were also aware of the difficulty of giving a precise percentage in response to some of our questions regarding the frequency of use. Nevertheless, several details seem to indicate that participants were quite good at this kind of introspection. Firstly, the replication of data patterns across the two populations illustrates their robustness. Secondly, the participants' answers are quite close to the results reported in the experimental literature. For instance, and as we mentioned previously, the percentage of knowledge about the different multiplication tables fits well with the problem-size effect. Other data can easily be related to certain patterns found in neuropsychological patients, such as their frequent loss of divisions, which our questionnaire showed are little practised in adult life (Granà, Hofer & Semenza, 2006). Further experimental work will be necessary in order to confirm the data obtained here and continue to investigate the effects that the frequency and format with which participants perform calculations have on their way of processing them.

RESUMEN

Yendo a las fuentes: un cuestionario sobre el aprendizaje y uso de las operaciones aritméticas. Algunos modelos actuales sobre la cognición matemática (Dehaene, 1992; Campbell & Clark, 1992) hacen sus predicciones sobre el código en que se resuelven las operaciones aritméticas basándose en cómo se aprendieron dichas operaciones o en cómo se utilizan habitualmente. Sin embargo, los datos de adquisición y uso a menudo se han derivado de informes anecdóticos y nunca se han recogido datos de modo sistemático. En este estudio se diseñó un cuestionario para investigar cómo habían aprendido las tablas de multiplicar los participantes, así como el código en que las operaciones de una y de varias cifras se resuelven habitualmente. El cuestionario se pasó a dos grupos de estudiantes universitarios, uno español (Estudio 1) y otro belga (Estudio 2). Los resultados mostraron que las multiplicaciones de una cifra se aprenden básicamente a través de la repetición en voz alta, pero los participantes adultos prefieren resolverlas visualizando mentalmente números arábicos. Éste es el código preferido también para calcular las sumas, restas y divisiones. La preferencia por el código arábico aumenta cuando deben calcularse operaciones de más de una cifra.

REFERENCES

Ashcraft, M.H. (1992). Cognitive arithmetic: a review of data and theory. *Cognition, 44, Special issue: Numerical cognition*, 75-106.

Campbell, J.I.D. (1994): Architectures for numerical cognition. *Cognition*, *53,*1-44.

Campbell, J.I.D. & Alberts, N.M. (2009). Operation-specific effects of numerical surface form on arithmetical strategy. *Journal of Experimental Psychology: Learning, Memory and Cognition., 35*, 999-1011.

Campbell, J.I.D. & Clark, J.M. (1992). Numerical cognition: an encoding-complex perspective. In J.I.D. Campbell (Ed.), *The nature and origins of mathematical skills* (pp. 457-491). Amsterdam: Elsevier Science.

Campbell, J.I.D. & Epp, L. J. (2004). An encoding-complex approach to numerical cognition in Chinese-English bilinguals. *Canadian Journal of Experimental Psychology, 58*, 229-244.

Campbell, J.I.D., Parker, H.R. & Doetzel, N.L. (2004). Interactive effects of numerical surface form and operand parity in cognitive arithmetic. *Journal of Experimental Psychology: Learning, Memory and Cognition, 30*, 51-64.

Campbell, J.I.D. & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General, 130*, 299-315.

Cohen, L., Dehaene, S., Chochon, F., Lehéricy, S. & Naccache, L. (2000): Language and calculation within the parietal lobe: a combined cognitive, anatomical and fMRI study. *Neuropsychologia, 38*, 1426-1440.

Costa, A. & Santesteban, M. (2004). Lexical access in bilingual speech production:

Evidence from language switching in highly proficient bilinguals and L2 learners. *Journal of Memory & Language, 50,* 491-511.

Dehaene, S. (1992): Varieties of numerical abilities. *Cognition*, *44,* 1-42.

Dehaene, S. & Cohen, L. (1995): Towards an anatomical and functional model of number processing. *Mathematical Cognition*, *1,* 83-120.

Dehaene, S. & Cohen, L. (1997): Cerebral pathways for calculation: double dissociations between Gerstmann's acalculia and subcortical acalculia. *Cortex, 33*, 219-250.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R. & Tsivkin, S. (1999): Sources of mathematical thinking: behavioral and brain-imaging evidence. *Science*, *284*, 970-974.

Deloche, G., Seron, X., Larroque, C. & Magnien, C. (1994). Calculation and number processing: Assessment battery: Role of demographic factors. *Journal of Clinical and Experimental Neuropsychology, 16*, 195-208.

Deloche, G., Souza, L., Braga, L.W. & Dellatolas, G. (1999). A calculation and number processing battery for clinical application in illiterates and semi-illiterates. *Cortex, 35*, 503-521.

Domahs, F. & Delazer, M. (2005). Some assumptions and facts about arithmetic facts. *Psychology Science, 47, Special issue: Brain and Number*, 96-111.

Grabner, R.H., Ansari, D., Koschutnig, K. & Reishofer, G. (2009). To retrieve or to calculate? Left angular gyrus mediates the retrieval of arithmetic facts during problem solving. *Neuropsychologia, 47*, 604-608.

Granà, A., Hofer, R. & Semenza, C. (2006). Acalculia from a right hemisphere lesion: Dealing with "where" in multiplication procedures. *Neuropsychologia, 44*, 2972-2986.

Groen, G.J. & Parkman, J.M. (1972). A chronometric analysis of simple addition. *Psychological Review, 79*, 329-343.

Imbo, I, & Vandierendonck, A. (2008). Practice effects on strategy selection and strategy efficiency in simple arithmetic. *Psychological Research, 72*, 528-541.

Imbo, I, Vandierendonck, A., & Rosseel, Y. (2007). The influence of problem features and individual differences on strategic performance in simple arithmetic. *Memory & Cognition, 35*, 454-463.

Jackson, J., & Coney, J. (2007). Simple arithmetic processing: Individual differences in automaticity. *European Journal of Cognitive Psychology, 19*, 141-160.

Kashiwagi, A., Kashiwagi, T. & Hasegawa, T. (1987). Improvement of deficits in mnemonic rhyme for multiplication in Japanese aphasics. *Neuropsychologia, 25*, 443-447.

Kirk, E.P. & Ashcraft, M.H. (2001). Telling stories: the perils and promise of using verbal reports to study math strategies. *Journal of Experimental Psychology: Learning, Memory and Cognition, 27*, 157-175.

Kolers, P.A. (1968). Bilingualism and information processing. *Scientific American, 218*, 78-86.

Lee, K-M. & Kang, S-Y. (2002). Arithmetic operation and working memory: Differential suppression in dual tasks. *Cognition, 83*, B63-B68.

LeFevre, J-A., Lei, Q., Smith-Chant, B. & Mullins, D.B. (2001): Multiplication by eye and by ear for Chinese-speaking and English-speaking adults. *Canadian Journal of Experimental Psychology, 55*, 277-284.

LeFevre, J-A., Bisanz, J., Daley, K.E., Buffone, L., Greenham, S.L. & Sadesky, G.S. (1996). Multiple routes to solution of single-digit multiplication problems. *Journal of Experimental Psychology: General, 125*, 284-306.

LeFevre, J-A., Sadesky, G.S. & Bisanz, J. (1996). Selection of procedures in mental addition: reassessing the problem size effect in adults. *Journal of Experimental Psychology: Learning, Memory and Cognition, 22*, 216-230.

McCloskey, M. (1992): Cognitive mechanisms in numerical processing: evidence from acquired dyscalculia. *Cognition*, *44*, 107-157.

McCloskey, M. & Macaruso, P., (1995), Representing and using numerical information. *American Psychologist, 50*, pp. 351–363.

Metcalfe, A.W.S & Campbell, J.I.D. (2008) . Spoken numbers versus Arabic numerals : differential effects on adults' multiplication and addition. *Canadian Journal of Experimental Psychology*, *62*, 56-61.

Noël, M.P., Fias, W. & Brysbaert, M. (1997). About the influence of the presentation format on arithmetical-fact retrieval processes. *Cognition*, *63*, 335-374.

Romero, S.G., Rickard, T.C. & Bourne, L.E. (2006). Verification of multiplication facts: an investigation using retrospective protocols. *American Journal of Psychology, 119*, 87-120.

Rusconi, E., Galfano, G., Rebonato, E. & Umiltà, C. (2006). Bidirectional links in the network of multiplication facts. *Psychological Research, 70*, 32-42.

Rusconi, E., Galfano, G., Speriani, V. & Umiltà, C. (2004). Capacity and contextual constraints on product activation: evidence from task-irrelevant fact retrieval. *The Quarterly Journal of Experimental Psychology A: Human Experimental Psychology, 57A,* 1485-1511.

Seyler, D.J., Kirk, E.P. & Ashcraft, M.H. (2003). Elementary subtraction. *Journal of Experimental Psychology: Learning, Memory and Cognition, 29*, 1339-1352.

Shannon, B. (1984). The polyglot mismatch and the monolingual tie. Observations regarding the meaning of numbers and the meaning of words. *New ideas in Psychology, 2*, 75-79.

Smith-Chant, B.L. & Le-Fevre, J.A. (2003). Doing as they are told and telling it like it is: self-reports in mental arithmetic. *Memory & Cognition, 31*, 516-528.

Spelke, E.S. & Tsivkin, S. (2001). Language and number: a bilingual training study. *Cognition, 78*, 45-88.

Venkatraman, V., Siong, S.Ch., Cheel, M.W.L. & Ansari, D. (2006): Effect of language switching on arithmetic: a bilingual fMRI study. *Journal of Cognitive Neuroscience, 18,* 64-74.

Zbrodoff, N.J. & Logan, G.D. (2005). What everyone finds: the problem-size effect. In J.I.D. Campbell, (ed.) *Handbook of mathematical cognition,* (pp.331-345), New York: Psychology Press.

APPENDIX A

English translation of the questionnaire used in these studies.

Initials ____________ Age_________ Sex__________

Year ______ Mother tongue________

1. How frequently do you perform calculations (additions, subtractions, multiplications, divisions), either mentally or in writing in your daily life (courses, shopping, work, hobbies)?

Choose the right option (a/b/c or d and specify the frequency (i/ii/iii) for this particular option:

a. Daily:
   i. Once a day:_____
   ii. Twice a day:____
   iii. Three or more times a day: ____
b. Weekly:
   i. Once a week:_____
   ii. Twice a week:_____
   iii. Three or more times a week: _____
c. Monthly:
   i. Once a month: _____
   ii. Twice a month: _____
   iii. Three or more times a month: _____
d. Less than once a month: _____

2. Out of a total of 100%, how many of these calculations are…?
   a. Multiplications ____
   b. Divisions ____
   c. Subtractions ____
   d. Additions ____

3. If you have to work out an operation, in which format do you prefer to have it presented?

a. Arabic digits _____ (e.g. "5")

b. Out loud (oral language) _____

c. Written number words _____(e.g. "five")

d. It does not matter which ______

4. And in which code do you prefer to solve it?

a. Arabic digits _____ (e.g. "5")

b. Out loud (oral language) _____

c. Written number words _____(e.g. "five")

d. It does not matter which ______

5. For each of the operations specified above, state the percentage of occasions with which you solve this operation using the different options indicated in the left-hand column:

	Multiplications		Divisions		Subtractions		Additions	
	1 digit	2 or + digits	1 digit	2 or +	1 digit	2 or +	1 digit	2 or +
Saying the numbers mentally or out loud								
Visualising Arabic digits mentally								
Writing Arabic digits with pen and paper (but without the aid of any other device)								
With a calculator								
Other means (say which)								
The total for each column must be 100%	100%	100%	100%	100%	100%	100%	100%	100%

6. Do you know the multiplication tables by heart?
   a. Yes, all of them ________
   b. No, none _______
   c. Only some of them _______
      Write down which ones you know: ______________

7. Do you remember which method was used to teach you the tables? (if several, write down the corresponding percentage)
   a. Oral repetition in order to memorise them_____________
   b. Exercises with Arabic digits __________
   c. Others (say which)__________

8. In which language did you learn the multiplication tables? ______________

9. Is the language in which you learned the multiplication tables also your mother tongue? YES / NO

10. If it is not your mother tongue, how old were you when you learned it? ______________________

11. In which language do you use the multiplication tables currently?
    a. In the language in which I learned them__________
    b. In another language (say which)____________
    c. It depends on the occasion ________
       Write down which languages and in what percentage:
    __________________ %

    ___________________%

12. General percentage of use of:
    Spanish _____________% Catalan _______________%

(Manuscript received: 21 May 2010; accepted: 14 October 2010)