The Effects of Wage Flexibility on Activity and Employment in the Spanish Economy: Technical Appendix*

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1. The model

The stylised form of the model, with variables in logarithms, is given by:

$$y_t = \phi(z_t^d - p_t) + a z_t^s \quad (1)$$

$$y_t = n_t + z_t^s \quad (2)$$

$$p_t = z_t^p + w_t - z_t^s - \beta u_t \quad (3)$$

$$l_t = \alpha E_{t-1}(w_t - p_t - z_t^s) + z_t^l \quad (4)$$

$$w_t = E_{t-1}(p_t + z_t^s) + z_t^w - \sigma E_{t-1} u_t \quad (5)$$

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$$u_t \equiv l_t - n_t \tag{6}$$

$$z_t^d = z_{t-1}^d + \varepsilon_t^d \tag{7}$$

$$z_t^s = z_{t-1}^s + \varepsilon_t^s \tag{8}$$

$$z_t^l = z_{t-1}^l + \varepsilon_t^l \tag{9}$$

$$z_t^p = \lambda z_t^p + \varepsilon_t^p \tag{10}$$

$$z_t^w = \rho z_{t-1}^w + \varepsilon_t^w \tag{11}$$

where y_t, p_t, w_t, n_t, l_t and u_t denote respectively GDP, prices, nominal wages, employment, labour supply and the unemployment rate.

Equation (1) implies that aggregate demand (y_t) depends on the stance of economic policy in real terms ($z_t^d - p_t$) and on permanent income, approximated by productivity (z_t^s). Equation (2) is the production function with constant returns to scale, omitting capital under the assumption that in the long term it is a constant fraction of GDP. Equation (3) is the price setting rule, which implies the existence of a non-competitive supply in the product market and, consequently, a non-competitive labour demand: prices depend on the unemployment rate in the economy (u_t) and represent a fraction (z_t^p) of the unit labour costs ($w_t - z_t^s$). Equation (4) is the competitive supply of labour (l_t), which depends on demographic factors (z_t^l), and on the difference between real wages ($w_t - p_t$) and productivity.[2] Equation (5) describes the nominal wage function. Wage bargaining takes place at the beginning of the period and implies a non-competitive supply of labour: wages are set such that in real terms they increase according to the expected productivity and a wage shock (z_t^w, representing the bargaining power of workers) but decrease with the unemployment rate. Equation (6) is the identity that defines the unemployment rate.

[2] Intertemporal substitution models of labour (e.g., Lucas and Rapping, 1969) explain the long-term evidence that permanent real wage increases due to increased productivity do not affect the supply of labour.

Equations (7) to (11) describe the dynamic of the (independent, identically distributed and uncorrelated) structural shocks in the model: demand (ε_t^d), productivity (ε_t^s), participation in the labour force (ε_t^l) and rigidities in price and wage formation (ε_t^p and ε_t^w), which we call from now on price and wage shocks.

As shown in the following section, solving the system of equations for the unemployment rate and the share of wages in national income, we obtain:

$$u_t = \frac{1}{\sigma-\beta}\rho z_{t-1}^w + \frac{1}{\sigma-\beta}\lambda z_{t-1}^p + \frac{1}{\sigma-\beta}\varepsilon_t^w + \frac{1}{1-\phi\beta}\left[\phi\varepsilon_t^p - (a+\phi-1)\,\varepsilon_t^s + \varepsilon_t^l - \phi\varepsilon_t^d\right] \tag{12}$$

$$\begin{aligned}[(w_t+n_t)-(p_t+y_t)] \quad = \quad & -\frac{\beta}{\sigma-\beta}\rho z_{t-1}^w - \frac{\sigma}{\sigma-\beta}\lambda z_{t-1}^p - \frac{\beta}{\sigma-\beta}\varepsilon_t^w - \frac{1}{1-\phi\beta}\varepsilon_t^p \\ & + \frac{\beta}{1-\phi\beta}\left[(a+\phi-1)\,\varepsilon_t^s - \varepsilon_t^l + \phi\varepsilon_t^d\right] \end{aligned} \tag{13}$$

which, together with (10) and (11), imply that both variables respond exclusively to price and wage shocks in the medium and long term (i.e., the degree of hysteresis depends on the values of ρ and λ).

2. Model's solution

It is assumed that wages are set at beginning of the period, before all shocks but ε_t^w are observed. Prices are fixed when all the information is revelead. Using equations (3), (11) and (5) in Section 3 we obtain:

$$E_{t-1}u_t = \frac{1}{\sigma-\beta}\left(z_t^w + \lambda z_{t-1}^p\right) \tag{14}$$

which, together with (3), (11) and (4) provides:

$$l_t = -\frac{\alpha\beta}{\sigma-\beta}z_t^w - \frac{\alpha\sigma}{\sigma-\beta}\lambda z_{t-1}^p + z_t^l \tag{15}$$

Equating (1) to (2), and replacing (3), one gets:

$$n_t = -\phi\left(\beta u_t + w_t\right) - \phi z_t^p + (a+\phi-1)\,z_t^s + \phi z_t^d \tag{16}$$

which, together with (13) and (6) yields to:

$$u_t = \frac{1}{1-\phi\beta}\left[\phi w_t - \frac{\alpha\beta}{\sigma-\beta}z_t^w + \phi z_t^p - \frac{\alpha\sigma}{\sigma-\beta}\lambda z_{t-1}^p - (a+\phi-1)\,z_t^s + z_t^l - \phi z_t^d\right] \tag{17}$$

By substituting (7) to (11) in (17), then taking expectations, and finally equating it to (14) we obtain:[3]

$$w_t = \frac{1}{\phi}\left[\frac{1-\phi\beta+\alpha\beta}{\sigma-\beta}z_t^w + \frac{1-\phi\sigma+\alpha\sigma}{\sigma-\beta}\lambda z_{t-1}^p + (a+\phi-1)\,z_{t-1}^s - z_{t-1}^l + \phi z_{t-1}^d\right] \tag{18}$$

which, together with (17) and (7) to (11) result in:

$$u_t = \frac{1}{\sigma-\beta}\rho z_{t-1}^w + \frac{1}{\sigma-\beta}\lambda z_{t-1}^p + \frac{1}{\sigma-\beta}\varepsilon_t^w + \frac{1}{1-\phi\beta}\left[\phi\varepsilon_t^p - (a+\phi-1)\,\varepsilon_t^s + \varepsilon_t^l - \phi\varepsilon_t^d\right] \tag{19}$$

By replacing (17), (19), (8) and (10) in (3), we get:

$$\begin{aligned} p_t \;=\; & \frac{1+\alpha\beta}{\phi\,(\sigma-\beta)}\rho z_{t-1}^w + \frac{1+\alpha\sigma}{\phi\,(\sigma-\beta)}\lambda z_{t-1}^p + \frac{a-1}{\phi}z_{t-1}^s - \frac{1}{\phi}z_{t-1}^l + z_{t-1}^d + \frac{1+\alpha\beta}{\phi\,(\sigma-\beta)}\varepsilon_t^w \\ & + \frac{1}{1-\phi\beta}\varepsilon_t^p - \frac{a\beta-\beta+1}{1-\phi\beta}\varepsilon_t^s + \frac{\beta}{1-\phi\beta}\varepsilon_t^l - \frac{\phi\beta}{1-\phi\beta}\varepsilon_t^d \end{aligned} \tag{20}$$

which can be subtracted from (18) to obtain:

$$\begin{aligned} w_t - p_t \;=\; & -\frac{\beta}{\sigma-\beta}\rho z_{t-1}^w - \frac{\sigma}{\sigma-\beta}\lambda z_{t-1}^p + z_{t-1}^s - \frac{\beta}{\sigma-\beta}\varepsilon_t^w - \frac{1}{1-\phi\beta}\varepsilon_t^p \\ & + \frac{a\beta-\beta+1}{1-\phi\beta}\varepsilon_t^s - \frac{\beta}{1-\phi\beta}\varepsilon_t^l + \frac{\phi\beta}{1-\phi\beta}\varepsilon_t^d \end{aligned} \tag{21}$$

Substituting (15), (19) and (9) in (6) provides:

$$\begin{aligned} n_t \;=\; & -\frac{1+\alpha\beta}{\sigma-\beta}\rho z_{t-1}^w - \frac{1+\alpha\sigma}{\sigma-\beta}\lambda z_{t-1}^p + z_{t-1}^l - \frac{1+\alpha\beta}{\sigma-\beta}\varepsilon_t^w \\ & - \frac{1}{1-\phi\beta}\left[\phi\varepsilon_t^p - (a+\phi-1)\,\varepsilon_t^s + \phi\beta\varepsilon_t^l - \phi\varepsilon_t^d\right] \end{aligned} \tag{22}$$

which, together with (8) and (2) imply:

$$\begin{aligned} y_t \;=\; & -\frac{1+\alpha\beta}{\sigma-\beta}\rho z_{t-1}^w - \frac{1+\alpha\sigma}{\sigma-\beta}\lambda z_{t-1}^p + z_{t-1}^s + z_{t-1}^l - \frac{1+\alpha\beta}{\sigma-\beta}\varepsilon_t^w \\ & - \frac{1}{1-\phi\beta}\left[\phi\varepsilon_t^p - (a+\phi-\phi\beta)\,\varepsilon_t^s + \phi\beta\varepsilon_t^l - \phi\varepsilon_t^d\right] \end{aligned} \tag{23}$$

[3] Note that $E_{t-1}w_t = w_t$, $E_{t-1}z_t^w = z_t^w$ while $E_{t-1}z_{t-1}^p = z_{t-1}^p$.

Finally, using (2), (21) and (8) it is possible to obtain:

$$\begin{aligned}[(w_t+n_t)-(p_t+y_t)] &= -\frac{\beta}{\sigma-\beta}\rho z_{t-1}^{w}-\frac{\sigma}{\sigma-\beta}\lambda z_{t-1}^{p}-\frac{\beta}{\sigma-\beta}\varepsilon_t^{w}-\frac{1}{1-\phi\beta}\varepsilon_t^{p} \\ &+\frac{\beta}{1-\phi\beta}\left[(a+\phi-1)\,\varepsilon_t^{s}-\varepsilon_t^{l}+\phi\varepsilon_t^{d}\right] \end{aligned} \tag{24}$$

Thus, it is clear that y_t, n_t, (y_t-n_t),p_t, w_t, (w_t-p_t) and l_t are $I(1)$ processes, while the order of integration of u_t and $[(w_t+n_t)-(p_t+y_t)]$ depends on ρ and λ.

Hysteresis caused by rigidities in the price and wage setting mechanisms $(\rho=1,\lambda=1)$

If $\rho=1$ and $\lambda=1$, then u_t and $[(w_t+n_t)-(p_t+y_t)]$ are $I(1)$ processes, are affeced in the short run by all shocks, but in the long run only by price and wage shocks ε_t^{w} and ε_t^{p} . In this case, the structural MA representation is given by:

$$z_t^{w}=\frac{1}{1-L}\varepsilon_t^{w};\; z_t^{p}=\frac{1}{1-L}\varepsilon_t^{p};\; z_t^{s}=\frac{1}{1-L}\varepsilon_t^{s};\; z_t^{l}=\frac{1}{1-L}\varepsilon_t^{l};\; z_t^{d}=\frac{1}{1-L}\varepsilon_t^{d} \tag{25}$$

$$\Delta y_t=-\frac{1+\alpha\beta}{\sigma-\beta}\varepsilon_t^{w}-\frac{1+\alpha\sigma}{\sigma-\beta}L\varepsilon_t^{p}+L\varepsilon_t^{s}+L\varepsilon_t^{l}-\frac{1}{1-\phi\beta}(1-L)\left[\phi\varepsilon_t^{p}-(a+\phi-\phi\beta)\,\varepsilon_t^{s}+\phi\beta\varepsilon_t^{l}-\phi\varepsilon_t^{d}\right] \tag{26}$$

$$\Delta n_t=-\frac{1+\alpha\beta}{\sigma-\beta}\varepsilon_t^{w}-\frac{1+\alpha\sigma}{\sigma-\beta}L\varepsilon_t^{p}+L\varepsilon_t^{l}-\frac{1}{1-\phi\beta}(1-L)\left[\phi\varepsilon_t^{p}-(a+\phi-1)\,\varepsilon_t^{s}+\phi\beta\varepsilon_t^{l}-\phi\varepsilon_t^{d}\right] \tag{27}$$

$$\Delta(y_t-n_t)=\varepsilon_t^{s} \tag{28}$$

$$\begin{aligned}\Delta p_t &= \frac{1+\alpha\beta}{\phi(\sigma-\beta)}\varepsilon_t^{w}+\frac{1+\alpha\sigma}{\phi(\sigma-\beta)}L\varepsilon_t^{p}+\frac{a-1}{\phi}L\varepsilon_t^{s}-\frac{1}{\phi}L\varepsilon_t^{l}+L\varepsilon_t^{d} \\ &+\frac{1}{1-\phi\beta}(1-L)\left[\varepsilon_t^{p}-(a\beta-\beta+1)\,\varepsilon_t^{s}+\beta\varepsilon_t^{l}-\phi\beta\varepsilon_t^{d}\right]\end{aligned} \tag{29}$$

$$\Delta w_t=\frac{1}{\phi}\left[\frac{1-\phi\beta+\alpha\beta}{\sigma-\beta}\varepsilon_t^{w}+\frac{1-\phi\sigma+\alpha\sigma}{\sigma-\beta}L\varepsilon_t^{p}+(a+\phi-1)\,L\varepsilon_t^{s}-L\varepsilon_t^{l}+\phi L\varepsilon_t^{d}\right] \tag{30}$$

$$\Delta\left(w_t-p_t\right) = -\frac{\beta}{\sigma-\beta}\varepsilon_t^w-\frac{\sigma}{\sigma-\beta}L\varepsilon_t^p+L\varepsilon_t^s-\frac{1}{1-\phi\beta}\left(1-L\right)\varepsilon_t^p+\frac{a\beta-\beta+1}{1-\phi\beta}\left(1-L\right)\varepsilon_t^s -\frac{\beta}{1-\phi\beta}\left(1-L\right)\varepsilon_t^l+\frac{\phi\beta}{1-\phi\beta}\left(1-L\right)\varepsilon_t^d \quad (31)$$

$$\Delta l_t=-\frac{\alpha\beta}{\sigma-\beta}\varepsilon_t^w-\frac{\alpha\sigma}{\sigma-\beta}L\varepsilon_t^p+\varepsilon_t^l \quad (32)$$

$$\Delta u_t=\frac{1}{\sigma-\beta}\varepsilon_t^w+\frac{1}{\sigma-\beta}L\varepsilon_t^p+\frac{1}{1-\phi\beta}\left(1-L\right)\left[\phi\varepsilon_t^p-\left(a+\phi-1\right)\varepsilon_t^s+\varepsilon_t^l-\phi\varepsilon_t^d\right] \quad (33)$$

$$\Delta\left[\left(w_t+n_t\right)-\left(p_t+y_t\right)\right] = -\frac{\beta}{\sigma-\beta}\varepsilon_t^w-\frac{\sigma}{\sigma-\beta}L\varepsilon_t^p-\frac{1}{1-\phi\beta}\left(1-L\right)\varepsilon_t^p +\frac{\beta}{1-\phi\beta}\left(1-L\right)\left[\left(a+\phi-1\right)\varepsilon_t^s-\varepsilon_t^l+\phi\varepsilon_t^d\right] \quad (34)$$

Long-term solution ($L=1$):

$$\Delta y_t=-\frac{1+\alpha\beta}{\sigma-\beta}\varepsilon_t^w-\frac{1+\alpha\sigma}{\sigma-\beta}\varepsilon_t^p+\varepsilon_t^s+\varepsilon_t^l$$
$$\Delta n_t=-\frac{1+\alpha\beta}{\sigma-\beta}\varepsilon_t^w-\frac{1+\alpha\sigma}{\sigma-\beta}\varepsilon_t^p+\varepsilon_t^l$$
$$\Delta\left(y_t-n_t\right)=\varepsilon_t^s$$
$$\Delta p_t=\frac{1+\alpha\beta}{\phi\left(\sigma-\beta\right)}\varepsilon_t^w+\frac{1+\alpha\sigma}{\phi\left(\sigma-\beta\right)}\varepsilon_t^p+\frac{a-1}{\phi}\varepsilon_t^s-\frac{1}{\phi}\varepsilon_t^l+\varepsilon_t^d$$
$$\Delta w_t=\frac{1}{\phi}\left[\frac{1-\phi\beta+\alpha\beta}{\sigma-\beta}\varepsilon_t^w+\frac{1-\phi\sigma+\alpha\sigma}{\sigma-\beta}\varepsilon_t^p+\left(a+\phi-1\right)\varepsilon_t^s-\varepsilon_t^l+\phi\varepsilon_t^d\right]$$
$$\Delta\left(w_t-p_t\right)=-\frac{\beta}{\sigma-\beta}\varepsilon_t^w-\frac{\sigma}{\sigma-\beta}\varepsilon_t^p+\varepsilon_t^s$$
$$\Delta l_t=-\frac{\alpha\beta}{\sigma-\beta}\varepsilon_t^w-\frac{\alpha\sigma}{\sigma-\beta}\varepsilon_t^p+\varepsilon_t^l$$
$$\Delta u_t=\frac{1}{\sigma-\beta}\varepsilon_t^w+\frac{1}{\sigma-\beta}\varepsilon_t^p$$
$$\Delta\left[\left(w_t+n_t\right)-\left(p_t+y_t\right)\right]=-\frac{\beta}{\sigma-\beta}\varepsilon_t^w-\frac{\sigma}{\sigma-\beta}\varepsilon_t^p$$

Short-term solution ($L = 0$):

$$\Delta y_t = -\frac{1+\alpha\beta}{\sigma-\beta}\varepsilon_t^w - \frac{\phi}{1-\phi\beta}\varepsilon_t^p + \frac{a+\phi-\phi\beta}{1-\phi\beta}\varepsilon_t^s - \frac{\phi\beta}{1-\phi\beta}\varepsilon_t^l + \frac{\phi}{1-\phi\beta}\varepsilon_t^d$$

$$\Delta n_t = -\frac{1+\alpha\beta}{\sigma-\beta}\varepsilon_t^w - \frac{\phi}{1-\phi\beta}\varepsilon_t^p + \frac{a+\phi-1}{1-\phi\beta}\varepsilon_t^s - \frac{\phi\beta}{1-\phi\beta}\varepsilon_t^l + \frac{\phi}{1-\phi\beta}\varepsilon_t^d$$

$$\Delta\left(y_t - n_t\right) = \varepsilon_t^s$$

$$\Delta p_t = \frac{1+\alpha\beta}{\phi\left(\sigma-\beta\right)}\varepsilon_t^w + \frac{1}{1-\phi\beta}\varepsilon_t^p - \frac{a\beta-\beta+1}{1-\phi\beta}\varepsilon_t^s + \frac{\beta}{1-\phi\beta}\varepsilon_t^l - \frac{\phi\beta}{1-\phi\beta}\varepsilon_t^d$$

$$\Delta w_t = \frac{1-\phi\beta+\alpha\beta}{\phi\left(\sigma-\beta\right)}\varepsilon_t^w$$

$$\Delta\left(w_t - p_t\right) = -\frac{\beta}{\sigma-\beta}\varepsilon_t^w - \frac{1}{1-\phi\beta}\varepsilon_t^p + \frac{a\beta-\beta+1}{1-\phi\beta}\varepsilon_t^s - \frac{\beta}{1-\phi\beta}\varepsilon_t^l + \frac{\phi\beta}{1-\phi\beta}\varepsilon_t^d$$

$$\Delta l_t = -\frac{\alpha\beta}{\sigma-\beta}\varepsilon_t^w + \varepsilon_t^l$$

$$\Delta u_t = \frac{1}{\sigma-\beta}\varepsilon_t^w + \frac{\phi}{1-\phi\beta}\varepsilon_t^p - \frac{a+\phi-1}{1-\phi\beta}\varepsilon_t^s + \frac{1}{1-\phi\beta}\varepsilon_t^l - \frac{\phi}{1-\phi\beta}\varepsilon_t^d$$

$$\Delta\left[\left(w_t + n_t\right) - \left(p_t + y_t\right)\right] = -\frac{\beta}{\sigma-\beta}\varepsilon_t^w - \frac{1}{1-\phi\beta}\varepsilon_t^p + \frac{a\beta+\phi\beta-\beta}{1-\phi\beta}\varepsilon_t^s - \frac{\beta}{1-\phi\beta}\varepsilon_t^l + \frac{\phi\beta}{1-\phi\beta}\varepsilon_t^d$$