

**COURSE DATA****DATA SUBJECT****Code:** 34174**Name:** Harmonic analysis**Cycle:** Undergraduate Studies**ECTS Credits:** 6**Academic year:** 2025-26**STUDY (S)**

Degree	Center	Acad. year	Period
1107 - Degree in Mathematics	Facultat de Ciències Matemàtiques	4	Second quarter

**SUBJECT-MATTER**

Degree	Subject-matter	Character
1107 - Degree in Mathematics	Seminar on Mathematical analysis	ELECTIVES

**COORDINATION**

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**SUMMARY**

Harmonic Analysis deals with the representation of functions as series or integrals of simpler ones.

For periodic functions on  $\mathbb{R}$  this leads to the representation of the function as a series of sines and cosines, called Fourier series.

In the case of functions defined on  $\mathbb{R}$  or  $\mathbb{R}^n$ , this leads us to study the Fourier transform.

Some summability methods of convergence of Fourier series and their analogues for Fourier transforms are studied. The notion of convolution between functions in both  $\mathbb{T}$  to  $\mathbb{R}$  is considered and used to approximate and regularize functions as well as to give some summability and convergence



results. The study of Fourier series and Fourier transform of square integrable functions, specially the Plancherel theorem, are also key points in the development of the subject.

## PREVIOUS KNOWLEDGE

### RELATIONSHIP TO OTHER SUBJECTS OF THE SAME DEGREE

There are no specified enrollment restrictions with other subjects of the curriculum.

### OTHER REQUIREMENTS

Linear Algebra and Geometry I and Mathematical Analysis I, II, III, IV.

## COMPETENCES / LEARNING OUTCOMES

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Ability to work in teams.

Capacity for analysis and synthesis.

Capacity for criticism.

Capacity of abstraction and modeling.

Expressing mathematically in a rigorous and clear manner.

Knowing the time and the historical context in which occurred the great contributions of women and men in the development of mathematics.

Learn autonomously.

Possess and understand the mathematical knowledge.

Solve problems that require the use of mathematical tools.

Visualize and interpret the solutions obtained.

## DESCRIPTION OF CONTENTS



**1. Introduction to harmònic analysis**

Partial differential equations: harmònic oscillator, wave equation and heat equation and their relationship with Fourier series.

**2. Fourier series**

Fourier Series. Convergence criteria for Fourier series. Summability of Fourier series

**3. Fourier transform**

Convolution and regularization of functions. Fourier transform on L1. Fourier transform on L2. Plancherel Theorem

**4. Applications**

**WORKLOAD**

**PRESENCIAL ACTIVITIES**

Activity	Hours
Theory	37,50
Other activities	7,50
Classroom practices	15,00
<b>Total hours</b>	<b>60,00</b>

**NON PRESENCIAL ACTIVITIES**

Activity	Hours
Attendance at other activities	0,00
Individual or group project	20,00
Independent study and work	30,00
Preparation of lessons	5,00
Preparation for assessment activities	20,00
Resolution of case studies	15,00
<b>Total hours</b>	<b>90,00</b>

**TEACHING METHODOLOGY**

a. The aim is to gradually introduce and develop the theoretical and practical contents of each topic and the right tools to solve problems.



- b. In the practical sessions we will apply the concepts presented in the lectures to solve problems.
- c. Questions and problems for study will be proposed. This study will be supervised and evaluated. In the practical sessions we will solve and correct exercises.
- d. We will use a symbolic computation software package that helps in the conceptual understanding and visualization. It will also serve as a testing method to provide intuitive knowledge.

## EVALUATION

The evaluation will be carried out by:

- 1) A written exam in which the acquisition of knowledge, writing skills, rigor in demonstrations and the ability to solve questions, problems and exercises will be evaluated. (80% of the grade)
- 2) Participation in the tasks or controls proposed by the teaching staff will be valued (10% of the grade).
- 3) Participation in the seminars will be valued (10% of the grade).

The grades corresponding to the continuous assessment of sections 2 and 3 will be kept in the two calls of the academic year in which they have been taken, since their evaluation will only be possible throughout the quarter and never in the extraordinary call.

## REFERENCES

- Reference b1: Stein, Shakarchi; Fourier Analysis: an Introduction, Princeton Lectures on Analysis, Zaanen.
- Reference b2: Ducandikoetxea; Lecciones sobre las series y las transformadas de Fourier, Apuntes de Managua, 2003.
- Reference b3: Duoandikoetxea; Fourier Analysis, Graduate Studies in Mathematics Volume: 29; 2001
- Reference b4: Grafakos; Classical Fourier Analysis, Springer New York : Imprint: Springer, 2014. 3rd ed. 2014.
- Reference b5: Wolff; Lectures on Harmonic Analysis, Providence, Rhode Island : American Mathematical Society, 2003
- Reference b6: Muscalu-Schlag; Classical and Multilinear Harmonic Analysis, Volume I, Cambridge University Press, 2013

*Complementary references:*

- Reference c1: Katznelson, an introduction to harmonic analysis. Dover Publications, 1976.
- Reference c2: Körner, Fourier analysis, Cambridge University Press, 1988.
- Reference c3: Dym, McKean; Fourier Series and Integrals, Academic Press, 1973. Referencia c4: Zaanen,



A.C.; Continuity, integrations and Fourier theory; Springer-Verlag, 1989.

- Reference c4: Duoandikoetxea; Fourier Analysis, Graduate Studies in Mathematics. Volume: 29; 2001
- Reference c5: Grafakos; Classical Fourier Analysis, Springer New York : Imprint: Springer, 2014.3rd ed. 2014.