COURSE DATA

DATA SUBJECT

Code: 34179
Name: Differential topology
Cycle: Undergraduate Studies
ECTS Credits: 6
Academic year: 2026-27

STUDY (S)

Degree	Center	Acad. year	Period
1107 - Degree in Mathematics	Facultat de Ciències Matemàtiques	4	Second quarter

SUBJECT-MATTER

Degree	Subject-matter	Character
1107 - Degree in Mathematics	Seminar on Topology and differential geometry	ELECTIVES

COORDINATION

NUÑO BALLESTEROS JUAN JOSE

SUMMARY

General Objective:

The main objective of this course is to introduce students to the language and techniques of **Differential Topology**, enabling them to understand and solve some of the fundamental problems in the field.

This course can be seen as a natural continuation of **Topology**, studied in the second year of the Mathematics degree, with a focus on a special class of topological spaces that have proven highly useful across various areas of mathematics: **differentiable manifolds**.

As a **prerequisite**, students are expected to have already been introduced to the basic concepts of differentiable manifolds¿such as the definition of a differentiable structure, tangent space, differentiable maps and their differentials, and submanifolds¿in the course *Classical Differential Geometry*, and to be able to handle these concepts with reasonable fluency, at least in the case of surfaces. The techniques introduced in this course rely on both the **topological** and **differentiable** structures.

We begin with a review of the **topological properties** that a differentiable manifold must satisfy, illustrated with key examples. We then analyze some properties of differentiable maps between manifolds, as an introduction to the classification problem. In particular, we focus on differentiable functions from surfaces to the real line, and on maps between surfaces, analyzing the typical behavior of **stable maps**. This leads us to the study of **Morse functions** on surfaces, both from local and global perspectives.

We will introduce the central technique in Differential Topology: **transversality**, and apply it to various problems, such as the density of Morse functions, **Whitney's Immersion Theorem**, and the **degree of a differentiable map**, along with its properties. An introduction to **Morse Theory** will also be provided, which relates the topological type of a surface to the singularities of a stable (Morse) function defined on it. The course concludes with a study of **stable maps between surfaces** from a local point of view.

Contents:

- Review of basic concepts on **Differentiable Manifolds**
- **Topology of Manifolds**
- **Transversality**
- **Morse Functions**
- **Cellular Complexes and their Homology**
- **Introduction to Morse Theory**
- **Degree of a Differentiable Map**
- **Stable Maps Between Surfaces**

PREVIOUS KNOWLEDGE

RELATIONSHIP TO OTHER SUBJECTS OF THE SAME DEGREE

There are no specified enrollment restrictions with other subjects of the curriculum.

OTHER REQUIREMENTS

COMPETENCES / LEARNING OUTCOMES

1107 - Degree in Mathematics

Ability to work in teams.

Apply the knowledge in the professional world.

Capacity for analysis and synthesis.

Capacity for criticism.

Expressing mathematically in a rigorous and clear manner.

Knowing the time and the historical context in which occurred the great contributions of women and men in the development of mathematics.

Learn autonomously.

Possess and understand the mathematical knowledge.

Solve problems that require the use of mathematical tools.

Visualize and interpret the solutions obtained.

DESCRIPTION OF CONTENTS

1. Manifolds

. Manifolds and smooth mappings
, Tangent space and differential
. The inverse mapping theorem. Immersions
. The regular value theorem. Submersions

2. Manifolds with boundary and orientation

. Manifolds with boundary
. Orientation
. Classification of 1-manifolds

3. Transversality

. Transversality
. Sard's theorem and transversality theorem
. Morse functions
. Whitney's immersion theorem
. Homotopy and stability

4. Partitions of unity and applications

. Partitions of unity
. Embedding manifolds in Euclidean space
. The homotopy transvearsality theorem

5. Intersection theory

. Oriented intersection number
. Mapping degree
. Lefschetz fixed point theory
. Vector fields. Poincaré-Hopf theorem
. Gradient-like vector fields

WORKLOAD

PRESENCIAL ACTIVITIES

Activity	Hours
Theory	37,50
Other activities	7,50
Classroom practices	15,00
Total hours	**60,00**

NON PRESENCIAL ACTIVITIES

Activity	Hours
Attendance at other activities	0,00
Individual or group project	7,00
Independent study and work	18,00
Preparation of lessons	40,00
Preparation for assessment activities	25,00
Resolution of case studies	0,00

	Total hours	90,00

TEACHING METHODOLOGY

The **theoretical component** of the course will be delivered through **lectures**, in which the instructor will introduce the main topics and mathematical methods. Each topic will include sufficient examples to illustrate both the newly introduced concepts and the methods for solving related problems. At the end of each topic, a **set of exercises** will be provided for students to solve independently.

The **practical component** will be organized by dividing students into **permanent groups** of 3 or 4 members. In these sessions, students will work on solving problems related to the material covered in previous theoretical classes, under the supervision of the instructor. Each group will submit their solutions to the assigned exercises for **evaluation**.

Both in theoretical and practical sessions, **computer-based tools** may be used to visualize geometric objects when appropriate.

Periodic seminars will be held, during which students will clarify doubts and discuss with the instructor various aspects of the course content related to **individual assignments** given as part of the assessment. The **presentations of these assignments** will take place during the final seminars of the course.

EVALUATION

Assessment of student learning and acquired competencies will consist of the following components:

1. **Final Exam:**
   A theoretical-practical exam will be administered at the end of the semester, accounting for **50% of the final grade**.

2. **Practical Work:**
   The work completed by each student, as well as its presentation during practical sessions, will be assessed. This component will contribute **10% of the final grade**.

3. **Tutorials and Seminars:**
   The work completed by each student and its presentation during the seminars will be assessed. This component will account for **40% of the final grade**.

Remarks:
Grades obtained in components 2 and 3 will be **retained for both examination periods** within the academic year in which they were completed, as these assessments can only be carried out during the semester and **not during the resit (extraordinary) session**.

REFERENCES

- V. Guillemin y A. Pollack, Differential Topology. Prentice-Hall, Inc., Englewood Cliffs, New Jersey (1974).
- Outerelo Domínguez y J. M. Ruíz Sancho, Topología Diferencial. Addison-Wesley Iberoamericana España S. A. (1998).
- E. Outerelo, J.A. Rojo y J. M. Ruíz, Topología Diferencial, un curso de iniciación. Sanz y Torres S. L. (2014).

Complementary references:

- J. Milnor, Morse Theory. Annals of Mathematics Studies, Princeton University Press (1969).
- E. Outerelo y J. M. Ruíz. Mapping degree theory. Graduate Studies in Mathematics, 108. American Mathematical Society, Real Sociedad Matemática Española (2009).