COURSE DATA

DATA SUBJECT

Code: 34180
Name: Differential geometry
Cycle: Undergraduate Studies
ECTS Credits: 6
Academic year: 2025-26

STUDY (S)

Degree	Center	Acad. year	Period
1107 - Degree in Mathematics	Facultat de Ciències Matemàtiques	4	First quarter

SUBJECT-MATTER

Degree	Subject-matter	Character
1107 - Degree in Mathematics	Seminar on Topology and differential geometry	ELECTIVES

COORDINATION

MONTERDE GARCIA-POZUELO JUAN LUIS

SUMMARY

Introduction to differentiable manifolds, tangent and cotangent manifolds, differentiable applications between manifolds, calculus in differentiable manifolds and Riemann geometry. Vector fields, introduction to tensor calculus, Lie derivative, exterior derivative, metrics, lengths, angles, volumes, Levi-Civita connection; geodesics, curvature, relation of curvature with geometry and topology. Special emphasis on examples, how concepts and theorems are realized in model examples of geometry or physics, with approaches that can be more analytical or more algebraic.

PREVIOUS KNOWLEDGE

RELATIONSHIP TO OTHER SUBJECTS OF THE SAME DEGREE

There are no specified enrollment restrictions with other subjects of the curriculum.

OTHER REQUIREMENTS

The course will start from zero, so it is not necessary to have passed the subject of Classical Differential Geometry (CDG), although you can enjoy this optional subject more if, at least, you have already taken CDG or take it simultaneously.

Another related subject is Analysis III, not all of it, but the part that refers to integration in manifolds, because it explains what the submanifolds of a Euclidean space are, which are immediate examples of Riemannian manifolds.

COMPETENCES / LEARNING OUTCOMES

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Ability to work in teams.

Apply the knowledge in the professional world.

Capacity for analysis and synthesis.

Capacity for criticism.

Expressing mathematically in a rigorous and clear manner.

Knowing the time and the historical context in which occurred the great contributions of women and men in the development of mathematics.

Learn autonomously.

Possess and understand the mathematical knowledge.

Solve problems that require the use of mathematical tools.

Visualize and interpret the solutions obtained.

DESCRIPTION OF CONTENTS

1. Differentiable manifolds

1.0.- Preliminaries.
1.1.- Definition of a differentiable variety and a differentiable application between manifolds.
1.2.- Tangent and cotangent manifolds
1.3.- Submanifolds.

2. Calculus in differentiable manifolds

2.1- Vector fields on a manifold and their integral curves from a conceptual and practical point of view.
2.2.- Introduction to tensor calculus on differentiable manifolds.
2.3.- The Lie derivative.
2.4.- The exterior derivative.

3. Riemannian metrics

3.1.- Motivation: metrics on the plane and the flat torus.
3.2.- Metric on a differentiable manifold.
3.3.- Lengths, angles and volumes.
3.4.- Existence of Riemannian metrics.
3.5.- Examples.

4. Geodesics

4.1.- Well and poorly parameterized geodesics
4.2.- Normal coordinates and spherical geodesic coordinates.
4.3.- Gauss lemma.

5. Curvature

5.1.- Curvature tensor.
5.2.- Sectional curvature.
5.3.- Cartan formalism.
5.4.- Gauss equation of a submanifold.
5.5.- Ricci curvature and scalar curvature.
5.6.- Constant sectional curvature.
5.7.- Einstein spaces.

6. Complete manifolds

6.1.- Distance associated with the Riemann metric.
6.2.- Geodesic completeness.
6.3.- Hopf-Rinow theorem.
6.4.- Completeness of the examples.

WORKLOAD

PRESENCIAL ACTIVITIES

Activity	Hours
Theory	30,00
Other activities	15,00
Classroom practices	15,00
Total hours	**60,00**

NON PRESENCIAL ACTIVITIES

Activity	Hours

Attendance at other activities	0,00
Individual or group project	0,00
Independent study and work	0,00
Preparation of lessons	80,00
Preparation for assessment activities	10,00
Resolution of case studies	0,00
Total hours	**90,00**

TEACHING METHODOLOGY

In-person theoretical classes with non-mandatory attendance. Student participation will be encouraged, trying to correct two defects that students usually have: fear of asking questions and fear of looking ridiculous for having given a false answer. In-person practical classes given by the students themselves. They will consist of a detailed presentation of the examples that they will have previously prepared individually under the guidance of the teacher. Discussion seminars on the examples explained by the students, with questions, suggestions and corrections by the students who have not explained that example and by the teacher.

EVALUATION

Subject evaluation system

Evaluation of the presentation of examples by students in practical classes and seminars. The proportion in which will influence the final grade will be 50%, of which 75% (that is, 37.5% of the total) will correspond to the expos practical classes and 25% (that is, the 12.5% of the total) will correspond to the presentation in the semina percentages correspond to the percentages of practical classes and seminars.

Theoretical-practical exam taking into account the presentation of the examples made by each student. The prop which this test will influence the final grade will be 50%.

REFERENCES

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- N. J. Hicks, Notes on Differential Geometry, Van Nostrand, 1965.

- B. O'Neill, Semi-Riemannian Geometry with applications to relativity, Pure Appl. Math., 103. Academic Press, New York-London, 1983.

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- T. Sakai, Riemannian Geometry, American Math. Soc., 1996

- M. Berger, A Panoramic View of Riemannian Geometry, Springer, 2003

- M. Berger, P. Gauduchon, E. Mazet, Le spectre dune variété riemanniene, Springer, 1971

- Lee, Jeffrey M., Manifolds and differential geometry, American Mathematical Society, 2009, Biblioteca de Ciencias.

- Iva Stavrov, Curvature of Space and Time, with an Introduction to Geometric Analysis, American Mathematical Society , 2021.