COURSE DATA

DATA SUBJECT

Code: 36586
Name: Mathematical Analysis II P-M
Cycle: Undergraduate Studies
ECTS Credits: 12
Academic year: 2026-27

STUDY (S)

Degree	Center	Acad. year	Period
1928 - Double Degree Program Physics-Mathematics	Facultat de Ciències Matemàtiques	2	Annual

SUBJECT-MATTER

Degree	Subject-matter	Character
1928 - Double Degree Program Physics-Mathematics	Segundo Curso (Obligatorio)	COMPULSORY

COORDINATION

MOLL CEBOLLA JOSE SALVADOR

SUMMARY

Mastering differential and integral calculus of functions of several real variables is one of the foundations of mathematics training. One of the objectives of the second degree course is the conceptual understanding and fluency in the use of some basic techniques in this matter.

The course is divided into two parts, each one is studied in a semester. In the first part we deal with Differential Calculus, which is developed for functions between finite dimensional euclidean spaces. The second part of the course is devoted to the study of the Lebesgue integral.

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PREVIOUS KNOWLEDGE

RELATIONSHIP TO OTHER SUBJECTS OF THE SAME DEGREE

There are no specified enrollment restrictions with other subjects of the curriculum.

OTHER REQUIREMENTS

Linear Algebra and Geometry I F-M, Mathematical Analysis I F-M

COMPETENCES / LEARNING OUTCOMES

DESCRIPTION OF CONTENTS

1. Finite dimensional euclidean spaces

1.1 RN as a euclidean, normed and metric space.
Scalar product and euclidean norm in RN. Norm in RN. Distance in RN. Topological concepts. Distance from a point to a set. Distance between sets. Bounded sets.
1.2 Convergence in RN.
Convergent sequences. Sequence characterization of adherent and accumulation points
1.3 Compactness in RN.
Compact, relatively compact and bounded sets.

2. Continuous functions of several variables.

2.1 Functions between finite dimensional euclidean spaces. Limits. Definition of the limit of a function in an accumulation point of the domain. Sequence characterization of the limit. Projections. Coordinate functions. Arithmetic properties of the limits. Continuity in a point and in a set. Linear functions.
2.2 Complex functions. Continuity. Uniform branches of the argument.
2.3 Uniform continuity: Definition, Heine-Cantor's theorem.

3. Differentiation

3.1 Directional derivatives and differential. Uniqueness of the differential. Relationship between continuity, differentiability and existence of directional derivatives. Jacobian matrix and gradient vector.
3.2 Complex differentiation. Cauchy-Riemann equations.
3.3 Chain rule in real and complex functions.
3.4 Mean value theorem and consequences.

4. Higher order derivatives.

4.1 Higher order partial derivatives. CK functions. A sufficient condition for differentiability. Cross derivatives theorems.
4.2 Taylor's formula: Taylor expansion. Bounding Taylor's remainder. Applications.
4.3 Local extrema. Critical points. Sufficient conditions for relative extrema. Hessian matrix.

5. Inverse and implicit function theorems

5.1 Non-null Jacobian functions.
5.2 Inverse function theorem in real and complex variables. Diffeormorphisms.
5.3 Implicit function theorem

6. Conditional extrema and Lagrange multipliers. Applications.

7. Lebesgue integrable functions

7.1 Null sets. Rectangles in RN. Measure of a rectangle. Null sets: Examples.
7.2 Step functions: characteristic function of a set. Step functions. Lebesgue integral of a step function. Properties.
7.3 Upper functions. Integral of upper functions.
7.4 Lebesgue integrable functions. Properties.
7.5 Characterization of Riemann integrable functions. Lebesgue-Vitali theorem. Improper Riemann integral.

8. Convergence theoremes

8.1 Monotone convergence theorem.
8.2 Dominated convergence theorem.
8.3 Fatou's lemma

9. Fubini's theorem and applications

10. Measurable functions and Lebesgue¿s measure

10.1 Measurable functions. Examples and properties.
10.2 Tonelli-Hobson's integrability criterion.
10.3 Measurable sets. Lebesgue's measure in RN: Properties.
10.4 Open sets measurability.
10.5 An example of a non-measurable set.
10.6 Parametric integration.
10.7 Eulerian functions.

11. Integral transforms

11.1 Coordinate transforms.
11.2 Change of variables formula.

12. Lebesgue's outer measure

12.1 Outer measure and regularity.
12.2 Egorov and Luzin's theorems.
12.3 Characterization of measurable functions.
12.4 Vitali's covering theorem.

WORKLOAD

PRESENCIAL ACTIVITIES

Activity	Hours
Theory	60,00
Other activities	15,00
Classroom practices	45,00
Total hours	**120,00**

NON PRESENCIAL ACTIVITIES

Activity	Hours
Attendance at other activities	0,00
Individual or group project	25,00
Independent study and work	55,00
Preparation of lessons	40,00
Preparation for assessment activities	60,00
Resolution of case studies	0,00
Total hours	**180,00**

TEACHING METHODOLOGY

1. The aim is to gradually introduce and develop the theoretical and practical content of each topic and the right tools to solve problems.
2. In the practical sessions we will apply the concepts presented in lectures to solve problems.
3. We will propose questions and problems for study. This study will be supervised and evaluated. In the practical sessions we will solve and correct exercises.
4. We will use a symbolic computation software package that helps in the conceptual understanding and visualization. It will also serve as a testing method to provide intuitive knowledge

EVALUATION

Each student will be asked to demonstrate knowledge of basic concepts, skills and competences of the subject by means of theoretical and practical examinations. Also its capacity to address issues or resolve the problems posed by the teacher will be assessed.

Evaluation will be conducted by:

1. Written theory exams that will measure both the acquisition of knowledge and writing ability and rigor in proofs. Written practice exams will evaluate the ability to solve problems and exercises. There will be two exams throughout the course (middle and end of course). In each exam there will be a theoretical and a practical part which will contribute each fifty percent of the note provided that each note is greater than or equal to three out of ten. The note of each of the partial exams must be greater or equal to four out of ten. Students taking the final exam covering the entire course must obtain a minimum score of 3 out of 10 in both the theory and practical sections in order to pass this Block. Additionally, they must achieve an average score of at least 4 out of 10 when calculating the arithmetic mean of the theory and practical components for each semester. Otherwise, the exam grade will be the lower value between the student's actual grade and 3.9.
2. Participation on the tasks or controls proposed by the teacher will be evaluated (10%), provided that the obtained mark is above a minimum of four points.
3. Participation in the seminars will be evaluated (10%), provided that the obtained mark is above a minimum of four points.

REFERENCES

- Mazón, J. M, Cálculo diferencial: Teoría y problemas, Publicacions de la Universitat de València, 2008.
- Mazón, J. M, La integral de Lebesgue en RN. Teoría y problemas, Publicacions de la Universitat de València, 2017.

Additional Bibliography

- Apostol, T.M., Análisis Matemático, Editorial Reverté, 1977.
- De Burgos, J. ,Cálculo infinitesimal de varias variables. Ed. McGraw-Hill, 1995.
- Del Castillo, F. Análisis Matemático II. Ed. Alhambra, 1989.
- Ortega, J. M. Introducció a l'Anàlisi Matemàtica. Manuals de la Universitat Autònoma de Barcelona, 1993.
- Tao, T. Analysis II, Third Edition, Texts and Readings in Mathematics, Springer, 2016.
- Weir, A.J. Lebesgue Integration and Measure, Cambridge University Press, 1973.