COURSE DATA

DATA SUBJECT

Code: 36591
Name: Hilbert Spaces and Fourier Series
Cycle: Undergraduate Studies
ECTS Credits: 6
Academic year: 2025-26

STUDY (S)

Degree	Center	Acad. year	Period
1928 - Double Degree Program Physics-Mathematics	Facultat de Ciències Matemàtiques	3	Second quarter

SUBJECT-MATTER

Degree	Subject-matter	Character
1928 - Double Degree Program Physics-Mathematics	Tercer Curso (Obligatorio)	COMPULSORY

COORDINATION

GALBIS VERDU ANTONIO

GARCIA FALSET JESUS

SUMMARY

The first part contains an introduction to the theory of Hilbert spaces, with special emphasis on the study of the orthogonal projection and the Fourier series with respect to an orthonormal system. Next we focus on the Fourier series with respect to the trigonometric system of periodic functions on the real line. The Dirichlet and Féjer nuclei are studied and some results are obtained regarding the convergence of the Fourier series. The course is completed with the study of the basic properties of the Fourier transform and Shannon's sampling theorem.

PREVIOUS KNOWLEDGE

RELATIONSHIP TO OTHER SUBJECTS OF THE SAME DEGREE

There are no specified enrollment restrictions with other subjects of the curriculum.

OTHER REQUIREMENTS

Mathematical analysis I F-M, Mathematical analysis II F-M

COMPETENCES / LEARNING OUTCOMES

DESCRIPTION OF CONTENTS

1. Hilbert spaces

Pre-Hilbertian space. Cauchy-Schwarz and Minkowski inequalities. Associated norm. Identity of the parallelogram. Spaces l2 and L2(omega)

2. Optimal approximation

Distance from a point to a closed convex set. Orthogonal projection and orthogonal decomposition. Riesz representation theorem. Stampacchia and Lax-Milgram theorems.

3. Orthonormal systems

The trigonometric system in L2(T). Bessel inequality. Gram-Schmidt method. Maximal orthonormal systems. Parseval identity. Fourier coefficients. Riesz-Fisher theorem.

4. Fourier series in L1(T)

Dirichlet and Fejér kernels. Convergence of the Césaro means in C(T). Riemann-Lebesgue lemma. Completeness of the trigonometric system. Pointwise convergence of a Fourier series: Riemann's localization theorem and convergence to the mean of the jump when there are limits and lateral derivatives.

5. Fourier transform in L1(Rn)

The translation and modulation operators. Fourier transform and convolution. The transform of a Gaussian. Approximates of identity. Dirac delta and Heaviside function. Inversion formula. Shannon's sampling

theorem: bandwidth of a signal and Nyquist frequency.

WORKLOAD

PRESENCIAL ACTIVITIES

Activity	Hours
Theory	30,00
Other activities	7,50
Classroom practices	22,50
Total hours	**60,00**

NON PRESENCIAL ACTIVITIES

Activity	Hours
Attendance at other activities	0,00
Individual or group project	10,00
Independent study and work	0,00
Preparation of lessons	40,00
Preparation for assessment activities	40,00
Resolution of case studies	0,00
Total hours	**90,00**

TEACHING METHODOLOGY

The aim is to gradually introduce and develop the theoretical and practical contents of each topic and the right tools to solve problems.

In the practical sessions we will apply the concepts presented in the lectures to solve problems.

Questions and problems for study will be proposed. This study will be supervised and evaluated. In the practical sessions we will solve and correct exercises.

EVALUATION

Each student will have to demonstrate the knowledge of the basic concepts and the acquisition of the competences of the topic by means of the realization of theoretical-practical exams. Her ability to address the issues or solve the problems proposed by the teaching staff will also be valued.

The evaluation will be carried out by:

1. An exam at the end of the course, which will consist of a theoretical and a practical part that will each account for fifty percent of the grade, and the average will be made provided that each grade exceeds three points out of ten. A theoretical exam will measure both the acquisition of knowledge and the ability to write and rigor in the proofs, as well as the resolution of questions. In the practical exam the ability to solve problems and exercises will be evaluated.
2. Participation in the tasks or controls proposed by the teaching staff will be valued (10% of the grade), provided that the grade of the exams exceeds a minimum of four points.
3. Participation in the seminars (10% of the mark) will be valued, provided that the mark of the exams exceeds a minimum of four points.

REFERENCES

- J. Cerdá ; Intoducció a l'Anàlisi Funcional. Publicacions i Edicions de la Universitat de Barcelona, 2005.
- K. Saxe; Beginning functional analysis. Undergraduate Texts in Mathematics. Springer-Verlag, New York, 2002.
- E.M. Stein, R. Shakarchi; Fourier Analysis: an Introduction, Princeton Lectures on Analysis, 2003.
- J. Duoandikoetxea; Lecciones sobre las series y las transformadas de Fourier. https://www.ugr.es/~acanada/docencia/matematicas/analisisdefourier/Duoandikoetxeafourier.pdf
- G.B. Folland; Fourier analysis and its applications; Brooks/Cole Publishing, 1992.
- M.Carmen Fernández, Antonio Galbis-Verdú; Espacios de Hilbert y análisis de Fourier; PUV (Universitat de València), 2024.

Additional Bibliography

- T.W. Körner, Fourier Analysis, Cambridge University Press, 1988.
- B. Cascales, J.M. Mira, J. Orihuela, M. Raja; Análisis funcional. Ediciones Electolibris, 2012.