COURSE DATA

DATA SUBJECT

Code: 43791
Name: Fixed-income trading models
Cycle: Master's Degree
ECTS Credits: 6
Academic year: 2026-27

STUDY (S)

Degree	Center	Acad. year	Period
2171 - Master's Degree in Actuarial and Financial Sciences	Facultat d'Economia	2	First quarter

SUBJECT-MATTER

Degree	Subject-matter	Character
2171 - Master's Degree in Actuarial and Financial Sciences	Risk control and solvency	COMPULSORY

COORDINATION

TORRO I ENGUIX HIPOLIT

SUMMARY

The objective of the course **Fixed Income Models** is the valuation of derivative assets based on interest rates. The valuation of these assets involves introducing advanced elements of what is known as Modern Financial Mathematics. Specifically, the course focuses on valuation by arbitrage or risk-neutral valuation. To this end, key concepts such as the law of one price and arbitrage opportunities are introduced. Additionally, the theoretical developments in asset valuation require tools such as stochastic processes in discrete time (binomial model) and in continuous time (generalized Wiener processes and Ornstein-Uhlenbeck processes).

The syllabus is structured into four parts:

(I) Advanced topics in interest rate risk analysis and management

(II) Interest rate derivatives

(III) Valuation of interest rate derivatives in discrete time

(IV) Valuation of interest rate derivatives in continuous time

The first part contains a single topic on interest rate risk analysis, which complements the knowledge acquired in Financial Mathematics. This topic is dedicated to the empirical estimation of the Term Structure of Interest Rates (TSIR) and the formation of expectations about interest rates at different maturities.

The second part provides a brief descriptive introduction to interest rate derivatives, offering a comprehensive review of the various contingent assets based on interest rates and their markets. Where possible, arbitrage operations and the resulting valuation of these contracts are introduced. It is worth noting that students will begin to internalize the arbitrage valuation methodology and apply it to contracts with linear payoff functions: FRAs, swaps, and futures. This will serve as a foundation for the next part, as these valuations result from the law of one price through buy-and-hold strategies.

The third part introduces, in Topic 3, the general methodology of arbitrage valuation in discrete time, applicable to any asset. This is the easiest way to understand the methodology, and its general nature allows for the valuation of all derivative assets. With a sufficient theoretical framework, the Cox-Ross-Rubinstein binomial model is introduced, and the Black-Scholes formula is obtained through discrete approximation. In Topic 4, the binomial valuation is applied to interest rate derivatives, emphasizing the importance and impact of the normal and log-normal probability distribution assumptions for interest rates and the adjustment of tree parameters to market conditions. The binomial model is then applied to interest rate derivatives, valuing options and futures on zero-coupon bonds.

The fourth part of the program is devoted to the continuous-time valuation of interest rate derivatives. Given the complexity of the subject, Topic 5 is dedicated to an introductory exposition of the basic tools of stochastic calculus for asset valuation. Specifically, Wiener processes, Itô¿s lemma, and stochastic differential equations are presented with precision, though with a more intuitive than formal approach, including applications to stocks and their derivatives. The concept of market price of risk is then introduced, along with the derivation of the stochastic differential equation valid for the valuation of any derivative asset. This topic concludes with the valuation of interest rate options, which will be addressed using the so-called market model or Black (76) model, widely used among financial professionals. Topic 6 concludes the course, covering the most well-known one-factor models of the instantaneous interest rate. This topic compares the different properties of the various models (mean reversion, level-dependent heteroscedasticity, non-negativity, boundedness of first and second moments, etc.), distinguishing between equilibrium models and no-arbitrage models. Where possible, closed-form valuation formulas for bonds and their derivatives are provided, with commentary on their most relevant aspects.

PREVIOUS KNOWLEDGE

RELATIONSHIP TO OTHER SUBJECTS OF THE SAME DEGREE

There are no specified enrollment restrictions with other subjects of the curriculum.

OTHER REQUIREMENTS

This course is a continuation of Financial Mathematics from the first year of the CAF master's program. Its

field of study is the analysis and management of interest rate risk and the valuation of derivative assets. The valuation of these assets involves the introduction of advanced elements such as stochastic processes. The professional activity of the actuary requires a solid foundation in Financial Economics, and this course contributes to that objective.

COMPETENCES / LEARNING OUTCOMES

2171 - Master's Degree in Actuarial and Financial Sciences

Alcanzar sólidos fundamentos para la toma de decisiones financieras: asignación de recursos en el tiempo bajo incertidumbre, estructura y funcionamiento de los mercados financieros, valoración de activos y selección de carteras.

Poseer las habilidades suficientes para participar en una conversación de negocios y estar capacitado para leer literatura actuarial al menos en dos de los idiomas oficiales de la Unión Europea.

Saber realizar una gestión integral del riesgo y alcanzar los conocimientos suficientes para dar respuesta a los riesgos actuales y a los que puedan surgir resultado del cambiante entorno económico, financiero y social, con vistas a dirigir y gestionar todo tipo de entidades financieras y aseguradoras.

Ser capaces de construir modelos adecuados al entorno económico empresarial a partir de las posibilidades que ofrecen las modernas tecnologías de la información y de la computación.

Ser capaces de gestionar el riesgo como un proceso continuo y en constante desarrollo llevado a término de manera integrada y condicionado a los objetivos estratégicos de la empresa, de forma que se maximice el valor sostenible a largo plazo de cada una de sus actividades y se conjuguen los intereses de todas las partes implicadas.

Ser capaces de interpretar las cuentas y los estados financieros de las empresas aseguradoras y de las instituciones financieras en general.

Students should apply acquired knowledge to solve problems in unfamiliar contexts within their field of study, including multidisciplinary scenarios.

Students should be able to integrate knowledge and address the complexity of making informed judgments based on incomplete or limited information, including reflections on the social and ethical responsibilities associated with the application of their knowledge and judgments.

Students should communicate conclusions and underlying knowledge clearly and unambiguously to both specialized and non-specialized audiences.

Students should demonstrate self-directed learning skills for continued academic growth.

DESCRIPTION OF CONTENTS

FIRST PART: Advanced Topics in Interest Rate Risk Analysis and Management

Topic 1. Analysis and Estimation of the Term Structure of Interest Rates (TSIR)

1.0 Introduction
1.1 Financial calculation: discrete time and continuous time
1.2 Theories on the formation of expectations in the TSIR
1.3 Methods for estimating the TSIR
1.3.1 Non-econometric methods
1.3.2 Econometric methods: spline functions
1.4 The term structure of volatilities (TSV)

Basic bibliography:
Lamothe & Soler (1996), Chapter 5.
Meneu et al. (1992), Chapters 1 and 2.
Sánchez (2001), Chapter 4.
De La Granville (2001), Chapter 11.
Marín, J. M., & Rubio, G. (2001), Chapters 2 and 3.

Additional bibliography:
Augros (1989), Chapter 2
Contreras & Navarro (1993)
Dybvig et al. (1996)
Morini & Calatayud (1999)
Nuñez (1995)
Vasicek & Fong (1982)
De La Granville (2001), Chapter 12
Benito (2001)

SECOND PART: Interest Rate Derivatives

Topic 2. Interest Rate Derivatives

2.1 Introduction: FRAs, Swaps, and Interest Rate Futures: Valuation, settlement, hedging, and arbitrage
2.2 Futures contracts on bonds
2.2.1 Contracts and quotation methods
2.2.2 The cheapest-to-deliver problem
2.2.3 Arbitrage valuation
2.2.4 Hedging with fixed income futures portfolios
2.3 Interest rate option contracts
2.3.1 Options on interest rate futures
2.3.2 OTC options: caps, floors, collars, and swaptions

Basic bibliography:
Hull (1996, 2nd Ed.), Chapters 5, 6, and 17
Alternatively, Hull (2002, 4th Ed.), Chapters 5, 6, and 20
Alternatively, Hull (2009, 6th Ed.), Chapters 4, 6, 7, and 19
Bierwag (1991), Chapter 8

Lamothe & Soler (1996), Chapters 1, 2, 3, 4, and 6

THIRD PART: Valuation of Interest Rate Derivatives in Discrete Time

Topic 3: Fundamentals of Arbitrage Valuation

3.1 One-period binomial market model
3.1.1 Introduction
3.1.2 The law of one price
3.1.3 Arbitrage valuation: the replicating portfolio
3.1.4 Risk-neutral valuation
3.2 General one-period market model
3.2.1 Introduction
3.2.2 Arbitrage opportunities, dominant strategies, and violation of the law of one price
3.2.3 Characterization theorem of arbitrage opportunities
3.2.4 State-price vector and risk-neutral probability measure
3.2.5 Complete and incomplete markets: fundamental theorem of financial economics
3.3 Binomial model (Cox-Ross-Rubinstein model)
3.4 Adjusting the binomial branch width to market volatility

Basic bibliography:
Marín, J. M., & Rubio, G. (2001), Chapter 4
Hull (1996 or 2002), Chapter 10

Topic 4: Binomial Valuation of Interest Rate Derivatives

4.1 Introduction
4.2 Construction of the binomial tree
4.3 Binomial evolution of the instantaneous interest rate
4.3.1 Normality assumption
4.3.2 Log-normality assumption
4.4 Valuation of options on zero-coupon bonds
4.4.1 Put options
4.4.2 Synthetic replication of the option
4.4.3 Call options
4.4.4 Put-call parity relationship
4.5 Futures on zero-coupon bonds
4.5.1 Valuation
4.5.2 Synthetic replication

Basic bibliography:
Jarrow & Turnbull (1996), Chapter 15

FOURTH PART: Valuation of Interest Rate Derivatives in Continuous Time

Topic 5: Price Behavior Models for Financial Assets

5.1 The Markov property in the evolution of financial prices
5.2 Continuous stochastic processes in finance
5.2.1 Wiener process
5.2.2 Generalized Wiener process
5.3 The stochastic process for stocks
5.4 Itô¿s lemma: an intuitive presentation and simple examples
5.5 Derivation of the Black-Scholes-Merton differential equation
5.6 The market price of risk
5.7 Analysis of the Black-Scholes formula
5.8 The standard market model for interest rate derivatives
5.8.1 The Black (76) model or standard market model
5.8.2 European options on bonds
5.8.3 Cash flow calculation in caps, floors, collars, and swaps
5.8.4 European CAP and FLOOR options
5.8.5 European swaption

Basic bibliography:
Hull (2000), Chapters 10, 11, and 20

Topic 6: Introduction to the Valuation of Interest Rate Derivatives with One-Factor Models

6.1 Equilibrium models
6.1.1 Rendleman and Barter model
6.1.2 Vasicek model
6.1.3 Cox, Ingersoll, and Ross model
6.2 No-arbitrage models
6.2.1 Ho and Lee model in continuous time
6.2.2 Hull and White model
6.2.3 Black, Derman, and Toy model in continuous time

Basic bibliography:
Hull (2000), Chapter 21, sections 1 to 10 and 13

WORKLOAD

PRESENCIAL ACTIVITIES

Activity	Hours
Theory	30,00
Classroom practices	30,00
Total hours	**60,00**

NON PRESENCIAL ACTIVITIES

Activity	Hours

Attendance at other activities	10,00
Individual or group project	20,00
Independent study and work	20,00
Preparation of lessons	30,00
Preparation for assessment activities	0,00
Resolution of case studies	10,00
Total hours	**90,00**

TEACHING METHODOLOGY

Theoretical classes will be mainly based on lectures delivered by the instructor, providing financial concepts, their properties, the most relevant results, and the methods commonly used in the financial profession.

The course content is covered in the basic bibliography, so reading it is highly recommended. In fact, the depth at which the various topics are addressed corresponds to that of the basic bibliography itself, which consists of undergraduate textbooks by renowned authors.

Practical classes will be devoted to solving previously assigned exercises. Solving exercises in class complements the theoretical sessions, as "putting numbers" to the variables helps simplify the analysis and highlight the fundamental theoretical aspects.

Throughout the course, the instructor will propose a series of tasks or practical exercises that students may complete individually or in groups of up to four people. Experience from previous years shows that serious preparation of these practical exercises is the best guarantee for successfully tackling the course.

EVALUATION

Assessment System

In both official exam sessions, a written test with both theoretical and practical components will be administered. This test will consist of a section with theoretical-applied questions and another section with practical exercises to be solved.

The tasks assigned throughout the course may account for up to 20% of the final grade, constituting a non-recoverable continuous assessment. The remaining 80% of the final grade will be obtained from the official exam.

REFERENCES

Garbade, K. D. (1998), Fixed Income Analytics, MIT Press.

Grandville La, O. de (2001), Bond Pricing and Porfolio Management, MIT Press.

Hull, J. C. (1996, 2202 ó 2009), Introducción a los mercados de futuros y opciones, Prentice Hall.

Hull, J. C. (2000 ó 2006), Options, futures, and other Derivatives, Fourth Edition, Prentice Hall.

Jarrow R. (1996): Modelling Fixed Income Securities and interest Rate Options, McGraw-Hill.

Jarrow, R. y S. Turnbull (1996), Derivatives Securities, South-Western College Publishing.

Lamothe, P. y J.A. Soler (1996), Swaps y otros derivados OTC en tipos de interés, McGraw-Hill.

Marín, J. M., y Rubio, G., (2001), Economía Financiera, Antoni Bosch Editor.

Meneu, V., E. Navarro y M. T. Barreira (1992), Análisis y gestión del riesgo de interés, Ariel.

Sánchez, J. L. (2007) (Ed.), Curso de bolsa y mercados financieros, Ariel.

Bibliografía Complementaria

Augros, J. C. (1987), Finance. Options et obligations convertibles, Economica.

Augros, J. C. (1989), Les options sur taux dinterêt, Economica.

Benito, S. (2001): "Factores Comunes en los Niveles y la Volatilidad de los Tipos de Interés Cupón Cero de la Deuda Pública en España". Tesis Doctoral, Universidad Complutense de Madrid.

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Borrell, M. y A. Roa (1990), Los mercados de futuros financieros. Ariel.

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Dybvig, P.H.; J.E. Ingersoll, Jr. And S.A. Ross (1996), Long Forward and Zero-Coupon Rates Can Never Fall, Journal of Business, Vol. 69, no. 1, págs. 1-25.

Fabozzi, F. J. (1996), Measuring and Controlling Interest Rate Risk, FJF Associates.

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Fabozzi, F.J., (Ed.) (1997), Advances in fixed income valuation, modelling and risk management, FJF Associates.

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Fernández, P. (2000), Opciones, futuros e instrumentos derivados, Ediciones Deusto.

Ferruz, L., M. P. Portillo y J. L. Sarto (2001), Dirección financiera del riesgo de interés, Pirámide.

Ho, T. S. Y. y S. B. Lee (1986), Term structure movements and pricing of interest rate claims, Journal of Finance, 41, págs. 1011-1029.

Morini, S. y F.P. Calatayud (1999), Un análisis de los modelos de estimación de la estructura temporal de tipos de interés, Ponencia presentada en el VII Foro de Finanzas celebrado