



## COURSE DATA

### DATA SUBJECT

**Code:** 44076  
**Name:** Fundamentals of advanced mathematics  
**Cycle:** Master's Degree / Doctorate  
**ECTS Credits:** 6  
**Academic year:** 2025-26

### STUDY (S)

Degree	Center	Acad. year	Period
2183 - Master's Degree in Mathematical Research	Facultat de Ciències Matemàtiques	1	First quarter
2903 - Doble M.U. Prof.Educ.Secund (esp. matem.) e Invest.Matem.	Facultat de Formació del Professorat	1	First quarter
3138 - PhD in Mathematics	Escola de Doctorat		First quarter
3138 - PhD in Mathematics	Escola de Doctorat		First quarter

### SUBJECT-MATTER

Degree	Subject-matter	Character
2183 - Master's Degree in Mathematical Research	Fundamentals of advanced mathematics	COMPULSORY
2903 - Doble M.U. Prof.Educ.Secund (esp. matem.) e Invest.Matem.		
3138 - PhD in Mathematics		
3138 - PhD in Mathematics		

### COORDINATION

RUEDA SEGADO MARIA PILAR

FALCO BENAVENT FRANCISCO JAVIER

## SUMMARY

The course focuses on Measure Theory and its applications. After developing the notions of measure on a sigma-algebra and the construction processes of measures from outer measures, specific cases such as Lebesgue measure in  $\mathbb{R}^n$  and Borel-Stieltjes measures on intervals will be constructed.

The student's familiar concepts of measurable and integrable functions will be reviewed, along with classical theorems (monotone convergence, dominated convergence of Lebesgue, Fubini's theorem) in the general context of abstract measures. The key result of the course will be the Radon-Nikodym theorem and some of its applications.



## PREVIOUS KNOWLEDGE

### RELATIONSHIP TO OTHER SUBJECTS OF THE SAME DEGREE

There are no specified enrollment restrictions with other subjects of the curriculum.

### OTHER REQUIREMENTS

The student should be familiar with the basic tools of integration in one and several variables.

## COMPETENCES / LEARNING OUTCOMES

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Que los estudiantes comprendan los conceptos y las demostraciones rigurosas de teoremas fundamentales de alguna de las áreas específicas de las Matemáticas.

Que los estudiantes sean capaces de aplicar los resultados y técnicas aprendidas para la resolución de problemas complejos de alguna de las áreas de las Matemáticas, en contextos académicos o profesionales.

Que los estudiantes sean capaces de comprender de manera autónoma artículos de investigación o innovación en alguna de las áreas de las Matemáticas.

Que los estudiantes sean capaces de construir, interpretar, analizar y validar modelos matemáticos avanzados que simulen situaciones reales.

Que los estudiantes tengan capacidad para elaborar y desarrollar razonamientos lógico-matemáticos e identificar errores en razonamientos incorrectos.

Students should apply acquired knowledge to solve problems in unfamiliar contexts within their field of study, including multidisciplinary scenarios.

Students should be able to integrate knowledge and address the complexity of making informed judgments based on incomplete or limited information, including reflections on the social and ethical responsibilities associated with the application of their knowledge and judgments.

## DESCRIPTION OF CONTENTS

### 1. Abstract measure

1. Outer measures. Extension of measures. Fourier-Stieltjes measures. Measurable and non-measurable sets.



## 2. Measurable functions and integrable functions

1. Measurable functions. Some types of convergence. Integrable functions.

## 3. Product measure and Fubini's theorem

1. Product measure. Fubini's theorem. Applications.

## 4. The Radon-Nikodym theorem

1. Complex and real measures. The Radon-Nikodym theorem. Applications.

### WORKLOAD

#### PRESENCIAL ACTIVITIES

Activity	Hours
Theory	60,00
<b>Total hours</b>	<b>60,00</b>

#### NON PRESENCIAL ACTIVITIES

Activity	Hours
Attendance at other activities	0,00
Individual or group project	30,00
Independent study and work	60,00
Preparation of lessons	0,00
Preparation for assessment activities	0,00
Resolution of case studies	0,00
<b>Total hours</b>	<b>90,00</b>

### TEACHING METHODOLOGY

Combination of lectures and presentations by students on selected topics. In face-to-face classes, the theoretical content of each topic and the appropriate tools for problem-solving will be introduced and developed progressively. A series of results, questions, and problems will be proposed for study, applying the concepts covered in the theoretical classes. Students will be required to present their solutions.

### EVALUATION

The course will be assessed through the presentation of problems and questions related to the subject,



proposed individually, or through the student's presentation of part of the course content on the board. There will also be assignments, completed either individually or in groups, along with their corresponding presentations in class.

## REFERENCES

- Bartle, R. The elements of integration and Lebesgue measure. Wiley classics Library. Edition 1995.
- M. de Guzmán; B Rubio. Integración, Teoría y Técnicas. Ed. Alhambra, 1979.
- M. Valdivia Ureña, Análisis Matemático V. UNED. Edición 2002.
- Mukherjea, A.; Pothoven, K. Real and functional analysis. Part A. Real analysis. Second edition. Mathematical Concepts and Methods in Science and Engineering, 27. Plenum Press, New York, 1984.
- George, C. Exercises et problems of integration. Gauthier-Villars, Paris, 1980.
- W. Rudin, Analisis real y complejo. Mac Graw-Hill, 1988.