COURSE DATA

DATA SUBJECT

Code: 44078
Name: Seminar on mathematical analysis
Cycle: Master's Degree
ECTS Credits: 3
Academic year: 2025-26

STUDY (S)

Degree	Center	Acad. year	Period
2183 - Master's Degree in Mathematical Research	Facultat de Ciències Matemàtiques	1	First quarter
2903 - Doble M.U. Prof.Educ.Secund (esp. matem.) e Invest.Matem.	Facultat de Formació del Professorat	2	First quarter

SUBJECT-MATTER

Degree	Subject-matter	Character
2183 - Master's Degree in Mathematical Research 2903 - Doble M.U. Prof.Educ.Secund (esp. matem.) e Invest.Matem.	Specialty in fundamental mathematics	ELECTIVES

COORDINATION

GARCIA FALSET JESUS

SEGURA DE LEON SERGIO

SUMMARY

The field where research in Mathematical Analysis develops, in most cases, revolves around Banach spaces. These are introduced in undergraduate studies, and it is necessary to complete the basic principles which include the Hahn-Banach theorem, the closed graph theorem, and the principle of uniform boundedness. Another essential tool is the so-called weak topology. Fundamental examples of Banach spaces will be developed.

The determination of critical points, potentially extremal, of real functionals defined in certain spaces of functions lies at the root of many problems in economics, mechanics, hydrodynamics, elasticity, etc. Classical Variational Calculus studies integral-type functionals, originating from specific physical problems posed in the 17th century.

The objective of Variational Calculus is the study of the possible existence of extremal points of integral-

type functionals, as well as the effective calculation or approximation thereof. This falls within a broad area of Nonlinear Functional Analysis.

The goals of the course at hand are:

Understanding some of the basic techniques of the subject accessible from undergraduate courses, with special emphasis on the Euler-Lagrange theorem and its prerequisites.

Familiarity with solving some classical examples (brachistochrone, shortest line problems, etc.). Ability to model other simple physical problems.

Understanding some connections of variational inequalities problems with other classic existence theorems in Analysis.

PREVIOUS KNOWLEDGE

RELATIONSHIP TO OTHER SUBJECTS OF THE SAME DEGREE

There are no specified enrollment restrictions with other subjects of the curriculum.

OTHER REQUIREMENTS

As prerequisites for taking the course, it will be assumed that the student is familiar with the basic concepts of Functional Analysis, as well as the content of the mandatory and core subjects in Analysis taught in a Mathematics degree. These include differential and integral calculus of functions of several variables, and the limits of sequences and series.

COMPETENCES / LEARNING OUTCOMES

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Capacidad de integrar conocimientos y formular juicios.

Que los estudiantes comprendan los conceptos y las demostraciones rigurosas de teoremas fundamentales de alguna de las áreas específicas de las Matemáticas.

Que los estudiantes comprendan los conceptos y las demostraciones rigurosas de teoremas fundamentales de áreas transversales de las Matemáticas.

Que los estudiantes posean la capacidad para enunciar y verificar proposiciones en alguna de las áreas de las Matemáticas y para transmitir los conocimientos matemáticos adquiridos, oralmente y por escrito.

Que los estudiantes sean capaces de aplicar los resultados y técnicas aprendidas para la resolución de problemas complejos de alguna de las áreas de las Matemáticas, en contextos académicos o profesionales.

Que los estudiantes sean capaces de comprender de manera autónoma artículos de investigación o

innovación en alguna de las áreas de las Matemáticas.

Que los estudiantes tengan capacidad para elaborar y desarrollar razonamientos lógico-matemáticos e identificar errores en razonamientos incorrectos.

Students should apply acquired knowledge to solve problems in unfamiliar contexts within their field of study, including multidisciplinary scenarios.

Students should be able to integrate knowledge and address the complexity of making informed judgments based on incomplete or limited information, including reflections on the social and ethical responsibilities associated with the application of their knowledge and judgments.

Students should demonstrate self-directed learning skills for continued academic growth.

Students should possess and understand foundational knowledge that enables original thinking and research in the field.

DESCRIPTION OF CONTENTS

1. Overview of Banach Spaces

Basic notions of normed and Banach spaces are introduced or reviewed. The most relevant examples of the theory such as spaces of sequences, integrable functions, and differentiable functions are described.

2. Basic Principles of Functional Analysis

The three fundamental principles are demonstrated: the theorems of Hahn-Banach, closed graph, and Banach-Steinhaus. Their most important consequences are presented.

3. Weak Topology

he weak topology on a Banach space is defined and its most important properties are studied, with special attention to weakly compact sets.

4. Complements of Differential Calculus

The notions of Gateaux and Frechet differentials are introduced, their relationship will be studied, and applications of these concepts will also be provided.

5. Euler-Lagrange Equations. Application. Variational Lemmas

Necessary conditions for minimizing an integral functional will be derived, showing that these lead to the Euler-Lagrange equations. Classic problems such as the Brachistochrone and the determination of geodesics on a sphere will be studied.

6. Constrained Extremes: Euler-Lagrange Theorem

The study of constrained optimization problems in functional spaces is considered. We will apply the Euler-Lagrange theorem to study isoperimetric problems, as well as certain optimization problems that can be reformulated as problems of variational inequalities.

WORKLOAD

PRESENCIAL ACTIVITIES

Activity	Hours
Theory	30,00
Total hours	**30,00**

NON PRESENCIAL ACTIVITIES

Activity	Hours
Attendance at other activities	0,00
Individual or group project	15,00
Independent study and work	30,00
Preparation of lessons	0,00
Preparation for assessment activities	0,00
Resolution of case studies	0,00
Total hours	**45,00**

TEACHING METHODOLOGY

Traditional exposition combined with the student's completion of practices primarily consisting of solving thematic exercises and problems.

EVALUATION

Each student's solution to a personalized collection of exercises will be evaluated, as well as the oral presentation of some of these exercises.

REFERENCES

- E. Giusti, Direct Methods in the Calculus of Variations, World. Scientific, 2003.
- J. L. Troutman, Variational Calculus with Elementary Convexity, Springer-Verlag, 1983
- Conway, John B. A course in functional analysis. Second edition. Graduate Texts in Mathematics, 96. Springer-Verlag, New York, 1990.
- Megginson, Robert E. An introduction to Banach space theory. Graduate Texts in Mathematics, 183. Springer-Verlag, New York, 1998
- E. Zeidler, Applied Functional Análisis, Main Principles and their applicatoions, Col. Applied Mathematical Sciences, vols. 108 y 109, Springer Verlag, 1995.
- E. Zeidler, Nonlinear Functional Analysis and its Applicacions III, Variational Methods and Optimization, Springer Verlag, 1984.
- Jameson, G. J. O. Topology and normed spaces. Chapman and Hall, London; Halsted Press [John Wiley & Sons], New York, 1974