# COURSE DATA

## DATA SUBJECT

**Code:** 44086
**Name:** Geometric principles of computer design
**Cycle:** Master's Degree
**ECTS Credits:** 3
**Academic year:** 2026-27

## STUDY (S)

| Degree | Center | Acad. year | Period |
|---|---|---|---|
| 2183 - Master's Degree in Mathematical Research | Facultat de Ciències Matemàtiques | 1 | First quarter |

## SUBJECT-MATTER

| Degree | Subject-matter | Character |
|---|---|---|
| 2183 - Master's Degree in Mathematical Research | Specialty in applied mathematics | ELECTIVES |

## COORDINATION

MARTINEZ CAMPOS CEDRIC

MONTERDE GARCIA-POZUELO JUAN LUIS

# SUMMARY

In the field of computer-aided curve and surface design, classical differential geometry provides powerful methods for efficiently generating objects with particular properties.

This course introduces the student to one of the techniques for generating curves and surfaces and examines some of the applications of differential geometry in this context.

# PREVIOUS KNOWLEDGE

## RELATIONSHIP TO OTHER SUBJECTS OF THE SAME DEGREE

There are no specified enrollment restrictions with other subjects of the curriculum.

## OTHER REQUIREMENTS

They are not necessary, although the software package "Mathematica" will be used, and the necessary

definitions of classical differential geometry will be reviewed.

## COMPETENCES / LEARNING OUTCOMES

### 2183 - Master's Degree in Mathematical Research

Que los estudiantes sean capaces de aplicar los resultados y técnicas aprendidas para la resolución de problemas complejos de alguna de las áreas de las Matemáticas, en contextos académicos o profesionales.

Que los estudiantes sean capaces de comprender de manera autónoma artículos de investigación o innovación en alguna de las áreas de las Matemáticas.

Que los estudiantes sean capaces de construir, interpretar, analizar y validar modelos matemáticos avanzados que simulen situaciones reales.

Que los estudiantes sean capaces de diseñar, desarrollar e implementar programas informáticos eficientes para abordar problemas relacionados con las Matemáticas y sus aplicaciones.

Que los estudiantes sean capaces de validar e interpretar los resultados obtenidos, comparando con visualizaciones, medidas experimentales y/o requisitos funcionales del correspondiente sistema físico.

Que los estudiantes sepan aplicar los conocimientos y habilidades adquiridas planificando el tiempo y los recursos disponibles.

Que los estudiantes sepan elegir y utilizar herramientas informáticas adecuadas para abordar problemas relacionados con las Matemáticas y sus aplicaciones.

Students should apply acquired knowledge to solve problems in unfamiliar contexts within their field of study, including multidisciplinary scenarios.

Students should be able to integrate knowledge and address the complexity of making informed judgments based on incomplete or limited information, including reflections on the social and ethical responsibilities associated with the application of their knowledge and judgments.

Students should demonstrate self-directed learning skills for continued academic growth.

## DESCRIPTION OF CONTENTS

- . Definition
- . Properties
- . Relationship with the power basis
- . Degree elevation
- . Bernstein's proof of the Weierstrass convergence theorem
- . Generating functions of Bernstein polynomials
- . The envelope of Bernstein polynomials

## 1. Bernstein Polynomials

- . Definition
- . Properties
- . Relationship with the power basis
- . Degree elevation- . Exercises

## 2. Bézier Curves

- . De Casteljau algorithm
- . Bézier curves using Bernstein polynomials
- . Properties of Bézier curves
- . Derivatives of Bézier curves
- . Subdivisions of Bézier curves
- . Change of basis
- . Exercises

## 3. Fitting Using Bézier Curves

- Fitting points using Bézier curves
- Fitting a semicircle
- Choosing parameter values
- Shape fitting
- Exercises

## 4. Rational Bézier Curves

- Introduction
- The standard rational parameterization of the circle
- Definition
- Effect on the rational Bézier curve of a change in weight
- Salkowski curves
- Exercises

## 5. Bézier Surfaces

- Definition and initial properties
- Properties of Bézier surfaces
- Partial derivatives of a Bézier surface
- The normal vector
- The tangent plane along the boundary
- Rational Bézier surfaces
- Richmond and Henneberg surfaces
- Exercises

## 6. Two Applications to Curve Design

- Moving trihedra associated with a curve in space
- Trihedra along a curve that minimize rotation
- Exercises
- Pythagorean hodograph polynomial curves: Definition
- Pythagorean hodograph polynomial curves: Construction
- Pythagorean hodograph polynomial curves with an associated rational Frenet trihedron
- Exercises

## 7. Harmonic and Biharmonic Bézier Surfaces

- Harmonic polynomial surfaces: explicit solutions
- Harmonic Bézier surfaces: explicit solutions
- Low degrees
- Harmonic biquadratic Bézier surfaces
- Harmonic bicubic Bézier surfaces
- Biharmonic polynomial surfaces: explicit solutions
- Biharmonic Bézier surfaces: explicit solutions
- Low degrees
- Biharmonic biquadratic Bézier surfaces
- Biharmonic bicubic Bézier surfaces
- Exercises

# WORKLOAD

### PRESENCIAL ACTIVITIES

| Activity | Hours |
|---|---|
| Theory | 30,00 |
| **Total hours** | **30,00** |

### NON PRESENCIAL ACTIVITIES

| Activity | Hours |
|---|---|
| Attendance at other activities | 0,00 |
| Individual or group project | 20,00 |
| Independent study and work | 20,00 |
| Preparation of lessons | 0,00 |
| Preparation for assessment activities | 0,00 |
| Resolution of case studies | 5,00 |
| **Total hours** | **45,00** |

# TEACHING METHODOLOGY

Master classes held simultaneously online synchronously for students with attendance exemption.

Classes in computer lab.

## EVALUATION

Submission of assignments and exercise collections.

## REFERENCES


- G. Farin, J. Hoschek and M.-S. Kim, eds. Handbook of Computer Aided Geometric Design, Ed. North-Holland Elsevier (2002)
- Rida T. Farouki, Pythagorean-Hodograph Curves. Algebra and Geometry inseparable, Springer, Berlin (2008).G. Farin,
- Gray, A., Modern Differential Geometry of Curves and Surfaces with Mathematica, Second edition, CRC Press (1998).