Psicológica (2012), 33, 711-733.

A Functional Model for the Integration of Gains and Losses under Risk: Implications for the Measurement of Subjective Value

Ricardo G. Viegas[1,*], Armando M. Oliveira[1], Ana Garriga-Trillo[2], Alba Grieco[1]

University of Coimbra, Portugal[1]; UNED, Madrid, Spain[2]

In order to be treated quantitatively, subjective gains and losses (utilities/disutilities) must be psychologically measured. If legitimate comparisons are sought between them, measurement must be at least interval level, with a common unit. If comparisons of absolute magnitudes across gains and losses are further sought, as in standard definitions of loss aversion, a common known zero must be added to the common unit requirement. These measurement issues are typically glossed over in complex models of decision under risk. This paper illustrates how Functional Measurement (FM) affords ways of addressing them, given some conditions. It establishes a relative ratio model for the integration of gains and losses in a mixed gamble situation with independent outcome probabilities. It subsequently documents how this model yields functional estimates of gains and losses on a common unit scale with a known zero. The psychological significance of the found integration model is discussed, and some of its implications for measurement further explored across two studies.

The need to measure subjective value has been around since the introduction of Expected Utility Theory by Bernoulli (1738/1954), and that of measuring subjective probability since the introduction of Subjective Expected Utility Theory by Savage (1954). Obtaining valid subjective measures has thereby become a critical endeavor in testing decision theories, and a difficult one to attain, moreover, in the absence of suitable psychological theories of measurement (Anderson, 1991; Anderson, 1996, chap. 10).

---

* Acknowledgments: this work was supported by grant SFRH/BD/45349/2008 from the Portuguese Foundation for Science and Technology. Correspondence address: Ricardo G. Viegas, University of Coimbra, Portugal. Email: rviegas@fpce.uc.pt

One complicating aspect in this regard concerns attitudes toward risk. Under expected utility theory, curvature of the utility function is the only source of risk attitudes. To take an example, risk aversion would be entirely determined by a concave utility function, with the intensity of the former directly reflecting the degree of curvature of the latter. Rabin (2000) has demonstrated in the meantime that the theory is unable to provide a realistic account of empirically observed degrees of risk aversion, and pointed out the need for additional sources of risk attitude. A similar stance was taken by many authors in face of the evidence of systematic violations of expected utility (Brooks & Zank, 2005; Köberlling & Wakker, 2005) and of inconsistencies in utility measurement based on the theory assumptions (Hershey & Schoemaker, 1985; Abdellaoui, Bleichrodt, & Paraschiv, 2007). As they moved away towards alternative theories of decision under risk, additional components of risk attitude were called upon, chiefly among them Loss Aversion and Probability Weighting (Köberlling & Wakker, 2005).

Loss Aversion is currently seen as a major cause of risk aversion and as capable of explaining a variety of previously anomalous field data (Abdellaoui et al., 2007). It refers generally to the notion that people are more sensitive to losses than to perfectly commensurate gains (for a somewhat distinct intuition, see Brooks & Zank, 2005). It was modeled under Prospect Theory (Kahneman & Tversky 1979; Tversky & Kahneman, 1992) as a kink of the value function at the reference point, resulting in greater steepness for losses than for gains. The three components of attitude toward risk allowed for in Prospect Theory are the utility function, the probability weighting function, and loss aversion. The latter is thus not to be confused with the intrinsic curvature of utility, which is a distinct component (Fox & Poldrack, 2009; Köberlling & Wakker, 2005; Tversky & Kahneman, 1992). Despite their formal independence, however, both notions are closely intertwined from the practical standpoint of measurement, as made clear in the formal definition of loss aversion (Kahneman & Tversky, 1979):

$$-v(-x) > v(x), for\ all\ x > 0. \quad (1)$$

Comparing $-v(-x)$ and $v(x)$ amounts to be able to measure subjective gains and losses on a scale having a known zero and a common unit. The additional specification «for all $x > 0$» amounts to measuring utility completely across gains and losses, and rests on the same demanding measurement conditions (a loss aversion coefficient can then be derived as the mean or median of the ratios $-v(-x) / v(x)$ across the relevant range of x:

Abdellaoui et al., 2008). Thus, both loss aversion and the full characterization of "basic utility" require ratio level measurement of subjective values across gains and losses.

One problem is that existing preference-based methods cannot handle gains and losses simultaneously under complex models of risky decision, such as prospect theory (Abdellaoui et al., 2007; Abdellaoui et al., 2008). The reason is that they do not take account either of loss aversion or of probability weighting, which may be different for losses and for gains. This applies to the probability, certainty, and lottery equivalence methods (Hershey & Shoemaker, 1985; McCord & de Neufville, 1986; see also Wakker & Deneffe, 1996; Abdellaoui et al., 2008), and explains why so much of what is supposedly known on risk attitudes is based on strictly positive prospects. Imposing a priori parametric assumptions to the probability weighting function, the utility function, or both, has thus often appeared as the only practicable path for measuring utility (Tversky & Kahneman, 1992).

Methods have been offered which seemingly account for probability weighting separately in the domains of loss and of gain (Fenemma & Van Assen, 1988; Wakker & Deneffe, 1996). However, because they cannot deal with both domains at a time, they do not make room for loss aversion and do not afford a full determination of utility. Separate examination of risk behavior in pure gain and pure loss lotteries would be supported, but this yields different results from directly assessing the effects of the integration of gains and losses in mixed lotteries (Hershey, Kunreuther, & Schoemaker, 1982; Abdellaoui et al., 2007)

In face of the reported shortcomings, a method to completely measure utility under Prospect Theory has more recently been proposed by Abdellaoui and collaborators (Abdellaoui, 2000; Abdellaoui et al., 2007, Abdellaoui et al., 2008). The hallmark of the approach is that it doesn't have to constrain the shape of either the utility or the probability weighting function, from which it derives the designation of «parameter-free method». However, it rests on the assumed validity of some initial trade-offs (Abdellaoui, 2000) and, most of all, on the additive composition assumed in Prospect Theory for multiple outcomes.

In the present work, the methodology of Information Integration Theory (IIT) and Functional Measurement (FM) is used to investigate the integration of gains and losses in a mixed gamble situation involving two non-null outcomes with independent probabilities ($p + q < 1$). Independent probabilities were adopted so as to allow the expression of possible differences in the weighting of probabilities associated with gains and with losses. Gains, losses, probabilities of gain and probabilities of loss could thus be manipulated as independent informers in the task, and their integration studied in independence from arbitrary assumptions on how they should combine, or about the shape of their mapping into subjective scales. As is typical of FM methodology, subjective metrics of utility and probability will be derivable upon condition that a cognitive integration model is empirically established for the task. The properties of such metrics will be similarly dependent upon the model and not decidable a priori, which further signals the fundamentally empirical, inductive nature of the approach.

STUDY 1

METHOD

Participants. 21 undergraduate students at the University of Coimbra (aged 18 to 24) participated in the experiment in exchange for course credits, all of whom were naïve regarding the purposes of the experiment.

Stimuli. Stimuli were schematic depictions of a one-roulette spinner game. In each trial, a disk was presented, divided along its vertical diameter. The left half was assigned to losses (signaled by a minus sign), and the right half to gains (plus sign). These two sectors were colored to different extents in red and green, respectively, causing the probabilities that a spinning arrow determined a loss (P_L) or a gain (P_G) to vary independently from each other, with a complementary probability ($1 - P_L - P_G$) of a null event. Variable money amounts were associated with the loss and gain sectors, corresponding to the two possible non-null outcomes in each trial: value of loss (V_L) and value of gain (V_G).

This gamble situation is distinct from 2-roulette, duplex games (Slovic & Lichtenstein, 1968), which comprise, in addition to the possibility of either gaining or losing, the further one of simultaneously gaining and losing. It was envisioned to favor direct comparisons between gains and losses by approaching a sheer "gain-or-lose" situation, though with independent probabilities of gaining and losing. This latter feature separates it, on the other hand, from standard 1-roulette gambles with complementary probabilities. The way this feature was implemented makes that no sure (P = 1) non-null outcomes are possible. As a downside, it also makes uncertain how the colored areas in the disk map onto the 0-1 range of probabilities ("low", "medium" and "high" probabilities can only have a relative meaning in this setting).

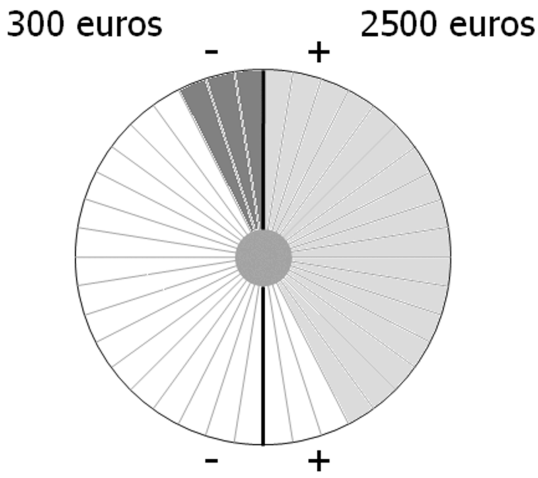

Figure 1. Illustration of a two (non-null)-outcome mixed prospect with independent probabilities of loss and gain, as presented to the participants.

The following notational conventions will be used throughout the paper: P_L, P_G, V_L and V_G will stand for Probability of Loss, Probability of Gain, Value of Loss and Value of Gain**,** respectively. P will stand for Probability and V for Value. G and L will denote Expected Gain (combinations of Probability and Value in the gain domain) and Expected Loss (combinations of Probability and Value in the loss domain). Subscript i will index the variable levels of these factors. Italicized versions of these notations will be used to represent their subjective, psychological counterparts.

Design and procedure. Three colored portions of each sector (0.15, 0.50, 0.85) were factorially combined with three monetary values of gain and loss (€ 25/-25 , 300/-300, 2500/-2500) to produce a 9 (G) × 9 (L) overall design, with a 3 (P) x 3 (V) subdesign embedded in each molar factor. This layered structure allows for alternatively redescribing it as a 3 (P_G) × 3 (V_G) × 3 (P_L) × 3 (V_L) four-factor design. Participants judged on a bipolar graphic scale (left-anchor: “very unsatisfied”; right-anchor: “very satisfied”) the satisfaction each game would bring them in case they were forced to play it. Games were never actually played. Careful instructions and a block of 10 training trials were provided beforehand.

RESULTS

Cognitive algebra. Fig. 2 displays the factorial diagram corresponding to the G x L overall design. Visual inspection reveals a barrel-shaped trend in the patterns (cigar-like), consistent with a relative ratio model of the form:

$$r = G/(G + L), \tag{3}$$

where *r* represents the psychological response. Relative ratio rules can actually give rise to more than one kind of graphical patterns. Typical barrels occur when three or more equivalent and geometrically spaced levels are used in both factors. Linear fans occur when levels in one factor are consistently larger (smaller) than levels in the other (Anderson, 1981, p. 77). As a first, qualitative check on the model, three ancillary 3 × 3 experiments were thus performed with specific levels of G and L selected among those of the focal experiment. In agreement with the hypothesized model, barrel patterns occurred with matched geometrically spaced levels, and linear fans were observed when levels of G (respectively L) were made consistently inferior to levels of L (respectively G).

Statistical analysis buttressed the visual inspection. A repeated measures ANOVA performed over the raw data of the G × L design disclosed significant main effects of G, $F(8, 160) = 127.7$, $p < .001$, and L, $F(8, 160) = 129.3$, $p < .001$, as well as a significant G × L interaction, $F(64,1280) = 11.3$, $p < .001$. This interaction rested mainly on highly significant linear × quadratic and quadratic × linear components, $F(1, 20) = 69.4$ and 47.4 respectively, $p < .001$, as could be expected from the signaled barrel trend in the data.

The possibility of redescribing the 9 (G) × 9 (L) design as a 3 (P_G) × 3 (V_G) × 3 (P_L) × 3 (V_L) design motivated a second round of graphical and statistical analysis, focusing now on the P × V embedded designs. Two-way plots of the four stimuli variables are displayed in Figure 3. A multiplicative integration of *V* and *P* in both the gain and loss domains is suggested by the linear fans observed in plots A and B. This was supported in the associated ANOVA by significant $P_G \times V_G$ and $P_L \times V_L$ interactions, $F(4, 80) = 18.1$ and 18.3 respectively, $p < .001$, concentrated moreover in their bilinear components, $F(1, 20) = 32.3$ and 42.7 respectively, $p < .001$.

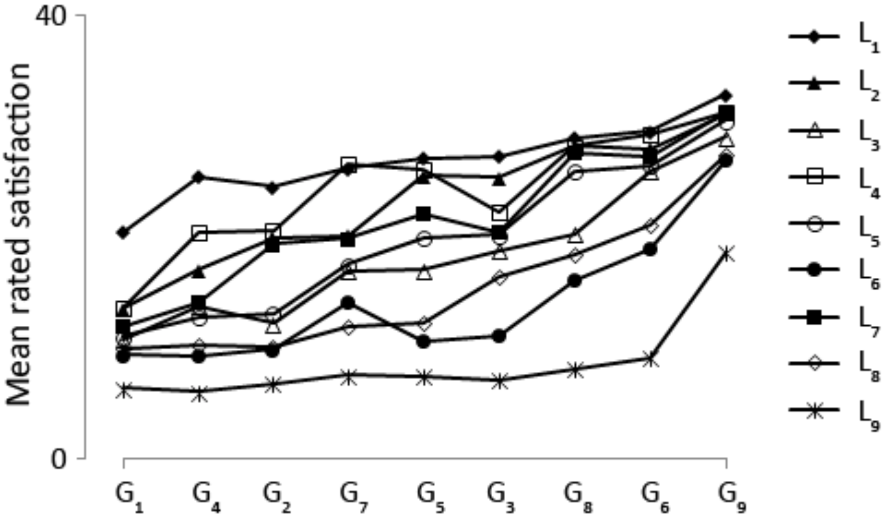

Figure 2. Factorial plot associated with the 9 (G) x 9 (L) overall design. Increasing marginal means of G (Expected Gain) are used in the abscissa, and L (Expected Loss) is the curve parameter.

The slanted barrel patterns in plots C and D are on their turn consistent with a relative ratio operation between the probabilities of gain and loss (P_G and P_L). Statistical results concurred with the visual inspection by revealing a significant interaction term, $F(4, 80) = 14.7$, $p < .001$, resting entirely in its linear × quadratic and quadratic × linear components, $F(1, 20) = 51.8$ and 16.6 respectively, $p \leq .001$. The same holds for plots E and F, concerning the relations between V_G and V_L. The noticeable barrel-shapes indicate a relative ratio operation, well supported by a significant interaction, $F(4, 80) = 29.4$, $p < .001$, concentrated once again in the linear × quadratic and quadratic × linear components, $F(1, 20) = 70.1$ and 49.8 respectively, $p < .001$.

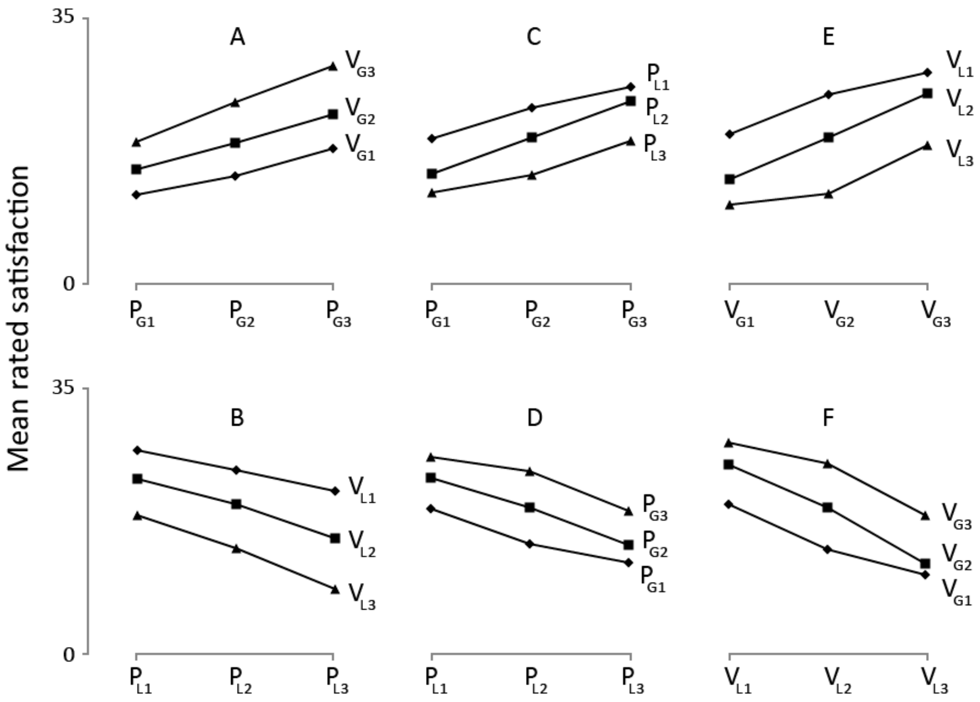

Figure 3. Two-way factorial plots for the 3 (P_G) × 3 (V_G) × 3 (P_L) × 3 (V_L) design. Plot A: P_G × V_G. Plot B: P_L × V_L. Plot C: P_G × P_L. Plot D: P_L × P_G. Plot E: V_G × V_L. Plot F: V_L × V_G. V_G and V_L stand for Value of Gain and Value of Loss, respectively, P_G and P_L for Probability of Gain and Probability of Loss.

Taking altogether, the complete algebraic structure of the model should thus write as:

$$r = (P_G \times V_G)/(P_G \times V_G + P_L \times V_L), \quad (4)$$

A foremost concern in the IIT/FM framework is the linearity of the response scale, without which no sensible interpretation can be made of the data patterns (Anderson, 1981; 1982). On this regard, the finding of a multiplying model between *P* and *V*, replicated in the gain and loss domains and well in line with previous findings in the FM literature (e.g., Anderson & Shanteau, 1970; Shanteau, 1974; 1975), can be viewed as support for the linearity of the response. The inner consistency between the clear-cut barrel patterns found in the V_G × V_L and P_G x P_L plots and the overall barreled shape of the G x L plots can similarly be argued to support the validity of the response scale.

Functional estimates of subjective expected values of gain and loss: Cognitive algebraic models implicitly carry subjective metrics of the stimuli, which can be derived by functional measurement (Anderson, 1981; 1982). These subjective values are the parameters of the empirically established model. For additive-type models, as for the multiplying model, the marginal means of the rows and columns of the design provide legitimate functional metrics (Anderson, 1982). Being nonlinear, however, the relative ratio model requires iterative estimation procedures.

One useful consideration is that the relative ratio rule may be viewed as an instance of the averaging model of IIT in applying to two competing informers (Anderson, 1981, p. 77; 1996, pp. 59-60). The averaging equation then writes as:

$$r = (w_{Ai}A_i + w_{Bi}B_i)/(w_{Ai} + w_{Bi}), \tag{5}$$

with w_{Ai} and w_{Bi} representing the weight (importance) of informers A_i and B_i, and A_i and B_i their scale values. By setting $A_i = 1$ and $B_i = 0$ to signal the polarization of the response, the strength of the judgment/decision in one or the other direction boils down to the relative ratio of their weights (Anderson, 1996, p. 59):

$$r = w_{Ai}/(w_{Ai} + w_{Bi}). \tag{6}$$

This authorizes using the AVERAGE program (Zalinski & Anderson, 1987) for simultaneously testing the goodness-of-fit and estimating the parameters of the relative ratio model. Since weights are given by AVERAGE on ratio scales with an arbitrary unit (see Anderson, 1982, Section 2.3.2), the ensuing G_i and L_i estimates will share a common unit and a common zero, a measurement condition enabling their direct comparison. Estimations were accordingly performed with AVERAGE, for each participant, using the differential weighting averaging model with scale values of loss and gain set to 0 and 1 respectively. Following standard practice in IIT, goodness-of-fit was assessed via a repeated-measures ANOVA over the residuals left by the model (see the "replications method" in Anderson, 1982, Section 4.4). No significant main effects were present, while the G × L interaction was still significant, $F(64, 1280) = 1.5$, $p = 0.025$. This was due to high-order components of the interaction term (linear

× order 6, cubic × quadratic, order 6 × order 7), taken as expressing the sensitivity of the test to small deviations from predictions (Anderson, 1982, p. 196) rather than challenging the basic adequacy of the model.

An additional set of estimations was performed with the Solver function of Excel, using minimization of the root-mean-square-deviation as a criterion (mean obtained RMSD = .039). As with AVERAGE, data were previously normalized to the full range of the response scale. The obtained estimates showed noticeable convergence with those of AVERAGE, which were just slightly higher. The Pearson correlation between the two series of estimations was 0.99 ($p < .001$) and, with one exception for the higher value of L, no significant differences were found among them. These results on the one hand lend credence to the adjusted parameters and on the other hand endorse the use of Solver for estimation with the relative ratio model.

Table 1 presents some of the derived G_i and L_i estimates, which can be lawfully compared among them. The values in the table agree with the notion that "losses loom larger than gains", and thereby illustrate loss aversion at the level of the expected value of matched gains and losses. Loss aversion was more formally quantified, following *Inequality 1*, as the mean of the ratios between the functional values of losses (L_i) and those of their commensurate gains (G_i). Thus computed, a result > 1 signals loss aversion, a result < 1 gain seeking (see Abdellaoui et al., 2007), and a result = 1 a neutral attitude towards losses and gains.

Table 1. Functional estimates of G (Expected Gains) and L (Expected Losses)

	AVERAGE		SOLVER TOOL	
Estimates	Gain (G)	Loss (L)	Gain (G)	Loss (L)
Lowest	4.9	|-5.9|	4.3	|-4.6|
Highest	35.8	|-46.8|	34	|-44.6|

The mean computed value of loss aversion was 1.19 using the AVERAGE estimates, and 1.13 using the Solver estimates, which is rather close to a neutral attitude. As estimations proceeded on an individual basis, an LA index was calculated for each participant. 86% had an index > 1, and 14% an index < 1. Loss aversion was thus prevalent, even if some of the participants actually qualified as gain seeking subjects (the opposite of loss averse: see Abdellaoui et al., 2007).

Functional estimates of value and probability. The layered structure of the model, which also includes a multiplying operation between probabilities and values, makes room for the derivation of additional functional estimates, this time for V_i and P_i. By virtue of the linear-fan theorem of IIT (Anderson, 1981, p. 41), the column and row means of a multiplying model offer legitimate functional metrics of the stimuli variables. One problem in making a straightforward use of this theorem in the present situation is that the marginal means of the $P_G \times V_G \times P_L \times V_L$ design do not directly provide the sough metrics, since the overall relative ratio rule is nonlinear. Yet, the G_i and L_i values obtained at the previous stage may legitimately be used, in the terms of the model ($G = V_G \times P_G$; $L = V_L \times P_L$), for deriving marginal means which are valid functional measures of V_i and P_i .

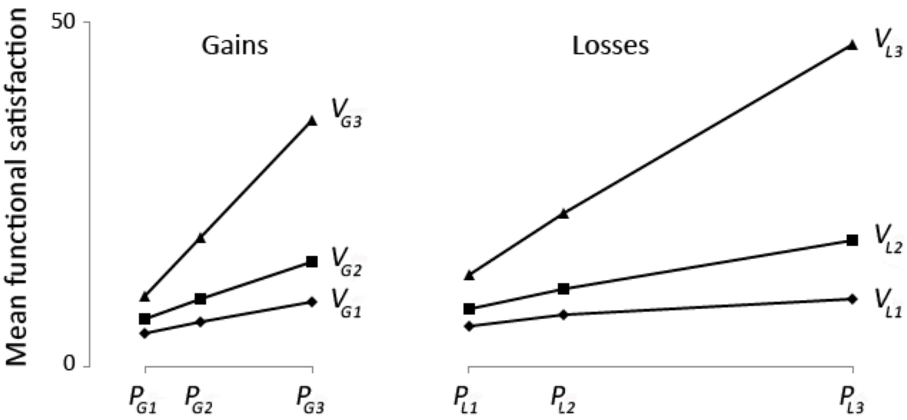

Figure 4. Factorial plots of Probability × Value in the gain and loss domains, with mean functional satisfaction in the ordinate (i.e., mean G_i and L_i functional estimates in place of the raw data means). Spacing on the abscissa is in functional coordinates. For convenience, functional satisfaction for losses is represented in absolute value. G_i and L_i in the parenthesis stand for the functional value of expected gains and expected losses, respectively.

Figure 4 illustrates exactly the replotting of $P_G \times V_G$ and $P_L \times V_L$ with mean G_i and L_i values in place of the raw data on the ordinates. This new dependent variable will be designated “mean functional satisfaction”, as distinct from “mean rated satisfaction”. Like before, the patterns go on revealing the typical fan-like signature of multiplying models (Anderson, 1981; 1982). Statistically, a multiplying model implies that the bilinear component of the interaction should be significant and leave no significant residuals behind. This was verified with the FM routine of the CALSTAT program (Weiss, 2006), which disclosed $F < 1$ for residuals in both cases. As entailed by the linear-fan theorem, then, the marginal means of rows and columns as computed from the G_i and L_i values provide valid functional scales of V_i and P_i.

Of note, however, is that functional estimates of V_i and P_i are not at the same level of measurement as G_i and L_i estimates. The latter form a ratio scale, with a common known zero and a common unit across the gain and loss domains. The former are on linear, equal-interval scales, without a common unit across domains due to the multiplication operation. Limitations stemming out of these measurement properties and ways to at least partly overcome them will be addressed in Study 2 below. Also of note is that measurement at the interval level actually ensures meaningful psychophysical curves when plotting V_i and P_i against probabilities and monetary outcomes. An overall characterization of those curves as linear or nonlinear, or as concave or convex, might thus legitimately be pursued from there. Considering however that only a few data points were available for that purpose in the present experiment (3 per curve), this was also left to be more specifically address in Study 2.

DISCUSSION

Study 1 empirically documented the participants' use of a compound relative ratio rule for the integration of gains and losses in the present task. This is a critical step to enable the use of functional measurement, which depends on cognitive algebra as its base and frame (Anderson, 1981; 1996), for deriving psychological measures of both gains and losses.

The established model conjoins an overall relative ratio rule, operating on expected gains and expected losses, and an embedded multiplying rule, operating upon probabilities and values. This latter rule is consistent with what has been defended by most descriptive models for the combination of value and probability (Edwards, 1955; Kahneman & Tversky, 1979) and is also in line with numerous results in the FM literature (Anderson & Shanteau, 1970; Shanteau, 1974; Schlottmann & Anderson, 1994; Schlottmann, 2001). The relative ratio rule is at odds with the adding assumption for the combination of multiple outcomes, and it generally concurs with frequent reports of additivity violation in the FM literature (e.g., Shanteau, 1974; 1975; Troutman & Shanteau, 1976; Lynch & Cohen, 1978; Schlottmann, 2000; 2001).

Based on the compound structure of the model, two sorts of estimates could be derived using the methodology of functional measurement. One sort concerns the subjective values of expected loss and expected gain, which were obtained from the relative ratio rule on a common unit scale, with a common known zero, across the loss and gain domains. Direct comparisons between them were thus legitimate, which allowed computing meaningful indices of loss aversion according to its standard definition. Another sort of estimates were derived from the embedded multiplying rules and concerned the subjective values of probability and monetary outcomes. These latter were on linear scales and allowed for meaningful psychophysical curves of value and probability. However, lack of a common unit across gains and losses drastically limited the possibilities of comparison among them.

STUDY 2

Study 2 includes two replications of Study 1 aimed at exploiting further the capabilities for measurement of the established model. A shortcoming of Study 1 was the reduced number of monetary outcomes and probability values in each domain (gain, loss), which was deemed not suitable for the fitting of psychophysical functions. Study 2 had the goal of circumventing this limitation and allowing for the characterization of psychophysical functions of value and probability, with an additional view to their possible comparison across domains.

METHOD

Participants. 30 naïve undergraduate students at the University of Coimbra (aged 18 to 25) participated in the experiments in exchange for course credits.

Stimuli. Identical to those of Study 1, except for the range and number of the monetary outcomes and the probability levels employed.

Design and procedure. Study 2 consisted of two experiments. In one of them, the Value Experiment, 5 levels of gain and loss were used (+/-15, 150, 500, 2000, and 7000 €), and only 2 levels of probability (0.25 and 0.85 of the area in each sector). 5 levels of probability (0.05, 0.275, 0.5, 0.725, and 0.95) and only 2 levels of gain and loss (+/-150, 2000 €) were used conversely in the other, the Probability Experiment. The factorial combination of Probability × Value in each domain (gain, loss) thus originated an overall 10 (G) x 10 (L) design. Procedures, including instructions and the response required, were as in Study 1. Participants performed in both experiments, which were counterbalanced for order.

RESULTS

Cognitive algebra. The findings in Study 2 closely mimicked those in Study 1, offering good support for the compound relative ratio model. Similar barrel-shaped patterns were apparent in the G × L plots in both experiments, and clear-cut linear fans in the $P_G \times V_G$ and the $P_L \times V_L$ plots. Noticeable barrels occurred as well in the $V_G \times V_L$ plot of the Value Experiment, and in the $P_G \times P_L$ graph of the Probability Experiment, in accordance with the proposed model.

Statistical analysis (repeated measurements ANOVAs) also disclosed the same trends as in Study 1. Alongside with significant main effects of G and L ($p < .001$), significant linear × quadratic and quadratic × linear components of the G × L interaction ($p < .001$) were found in both experiments. The same happened in the Value Experiment and in the Probability Experiment, respectively, with the $V_G \times V_L$ and the $P_G \times P_L$ interactions, both concentrated in the linear × quadratic and quadratic × linear components ($p < .001$). The linear fans observed in the $P_G \times V_G$ and $P_L \times V_L$ factorial diagrams were well supported by significant interactions concentrated in the bilinear component ($p < .001$) in both experiments.

Functional estimates. Study 2 was foremost concerned with estimating P_i and V_i parameters from the embedded $P \times V$ multiplying model, with the goal of assessing typical psychophysical assumptions about the value and probability functions. However, as illustrated in Study 1, functional estimates of G_i and L_i are required for that, which were obtained by using Solver to fit the relative ratio model to data and estimate its parameters (mean RMSD values were 0.045 in the Value Experiment and 0.043 in the Probability Experiment).

Functional measures of monetary value and probability could then be derived as in Study 1, which were at the interval level and lacking a common unit across the loss and gain domains. This latter feature has the signaled consequence of limiting comparisons across domains to the overall profile of psychophysical curves. Another limitation of these measures, arising from the lack of a known zero, is that parameters of the functions that best fit the data cannot be properly interpreted (e.g., exponents of adjusted power functions do not have a precise meaning unless measurement has a known zero). Taking measurement up to the ratio level would thus seem desirable for further characterization of the value and probability functions.

This very possibility is afforded in principle by the multiplying model, which essentially embodies a true zero (Anderson, 1982, pp. 82-83). The practical problem, however, is the need to estimate the zero point in the response scale, corresponding to the C_0 parameter in the full algebraic expression of the multiplying model:

$$R = C_0 + C_1(s_{Ai}s_{Bi}), \tag{7}$$

where R represents the observable response, s_{Ai} and s_{Bi} the subjective counterparts of stimuli A_i and B_i, and C_0 and C_1 constants in the linear transformation of the psychological multiplication onto R. Once C_0 is estimated, $R - C_0$ provides a ratio-scale measure of $s_{Ai} \times s_{Bi}$, and thus the basis for measuring s_{Ai} and s_{Bi} at the ratio level.

To the end of estimating C_0, we adopted in this study the procedure set out in Masin (2004), resting on the use of differences with minimum relative error (for details on the procedure and its algebraic derivation see Masin, 2004; for an applied illustration in a different domain, see Teixeira & Oliveira, 2008). 10 estimates of subjective value/utility (5 for gains, 5 for losses) were thus derived from the Value Experiment, after estimation and subtraction of C_0. Likewise, 10 estimates of subjective probability (5 for gains, 5 for losses) were derived from the Probability Experiment. These ratio level measures were the ones used for plotting the psychophysical functions of value and probability (Figures 5 and 6), as well as in all associated analysis.

As it happens, the best adjusted curvature parameters (e.g., power exponents) do now hold a meaning independent of the measurement units (absorbed by the scaling constants). This enables comparisons across gains and losses, thus partially solving for the lack of a common unit across domains. However, that the solution is only partial can be seen in that comparisons of height or steepness of the curves remain barred by the absence of a shared unit of measurement.

Psychophysical functions. Figure 5 presents the psychophysical plots obtained with mean V_i functional estimates in the ordinate, designated as “mean functional monetary value”. Pronounced non-linearity of the curves, concave for gains and convex for losses, is the most salient feature. The best fitted functions (in the least-squares sense) to the aggregated data were power functions, with similar exponents for gain and for loss (.38 and .39 respectively). Analogous trends were found at the individual level. Since estimations were performed on a single subject basis, a power exponent could also be derived for every participant. Out of 30 subjects, only 1 approached linearity ($0.9 \leq \alpha < 1$) in the gain domain and 2 in the loss domain. A paired *t*-test between the mean exponents for gain and for loss (.42 and .43) disclosed a nonsignificant result, $t\ (29) = .22$, $p = .65$. Overall, these results seemingly converge with standard assumptions in PT and disagree with rival assumptions of linearity of the value function (see Lopes, 1996; Lopes & Oden, 1999). On the other hand, the exponents found were considerably lower than the reference value of .88 suggested by Kahneman & Tversky (1992), implying a more pronounced curvature of the value functions.

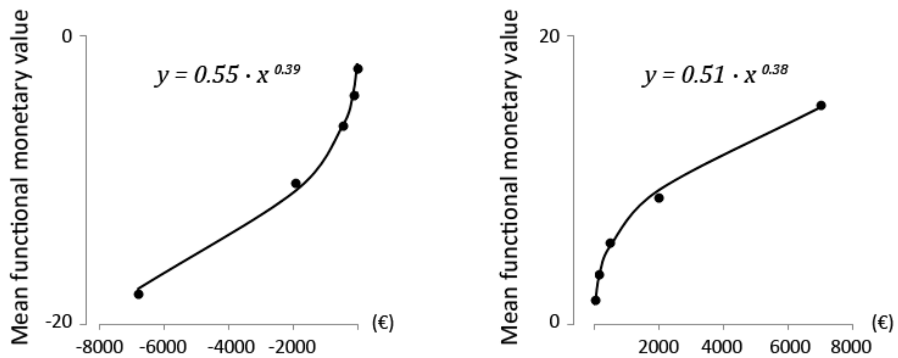

Figure 5. Psychophysical functions of Value. Mean functional estimates of monetary value are plotted against monetary outcomes. Dots represent empirical data, lines the best least-squares adjusted functions (power functions). In the equations, *y* represents the variable "functional monetary value" and *x* the variable "monetary outcome".

Figure 6 shows the psychophysical curves for probability, based on mean P_i functional estimates ("mean functional probability", in the ordinate). A convex overall shape is apparent in both domains, which was well adjusted by exponential functions with similar exponents. The noticed circumstance that probabilities in this experimental setting can only have a relative meaning (due to incertitude in the mapping of colored areas into the 0-1 range) prevents additional considerations from being made. In particular, nothing can be said concerning the psychophysical underweighting/overweighting of probabilities, and the found nonlinearity cannot be asserted to hold over the entire range of the probability function.

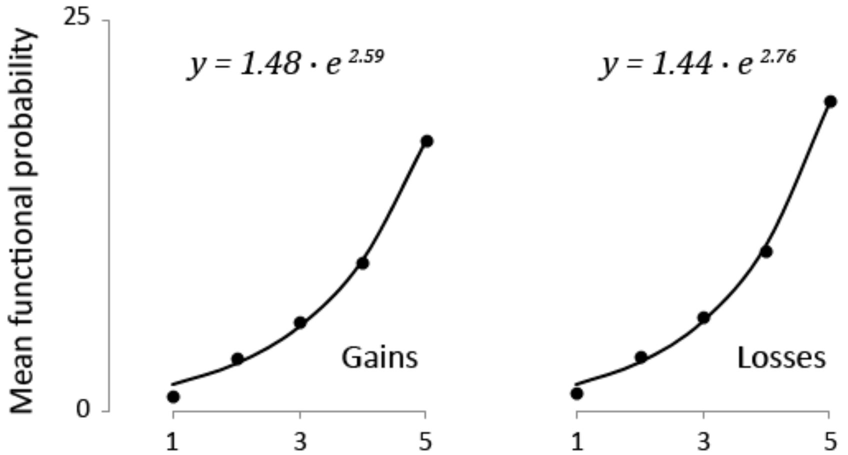

Figure 6. Psychophysical functions of Probability. Mean functional estimates of probability are plotted against the extent of colored surfaces in the gain and loss sectors of the roulette. Incertitude in how these surfaces, physically equally-spaced, map onto the 0-1 range, disallows the use of probability values in the abscissa. Dots represent empirical data, lines the best least-squares adjusted functions. In the equations, *y* represents the variable "functional probability" and *e* the base of the exponential function.

Loss aversion. A measure of LA was computed as in Study 1 (see above) for each participant in the Value Experiment. The mean value of loss aversion was 1.11, quite below the reference value of 2.25 adopted in PT (Kahneman & Tversky, 1992). A rough classification of participants as below or above LA = 1, the point of neutral attitude, resulted in the predominance of loss averse participants. However, 6 out of 30 participants (20%) still qualified as gain seeking subjects.

DISCUSSION

Study 2 was aimed at exploring the measurement capabilities of the compound relative ratio model, with a view to checking commonly made assumptions regarding the psychophysics of value and probability. One concern was with enlarging the number of functional estimates of value and probability, which was achieved by tailoring two separate experiments to include a larger range and number of monetary outcomes in one case, and a larger range and number of probability levels in the other. Both experiments supported the relative ratio model established in Study 1, which was thus successfully replicated under these changed conditions, and with a new sample of participants. Another concern was with stepping up the level of the functional measures of monetary value and probability. This was achieved by estimating and subtracting the C_0 parameter of the multiplicative model from the previous estimates, which endowed them with ratio properties. Taken as a function of monetary outcomes and probabilities, these ratio level measures could then be fitted by functions with interpretable curvature parameters, comparable moreover across gains and losses.

The ensuing characterization of the psychophysical functions may be summarized as follows. Characteristic nonlinearity is apparent in the value curves for both gains and losses, best adjusted in both domains by power functions with similar exponents. Nonlinearity is also featured in the probability curves, best adjusted in both domains by exponential functions with close exponents. Differently from curvature, height or steepness of curves could not be compared across domains (gain, loss) due to lack of a common unit

GENERAL DISCUSSION

The foregoing studies illustrate the usefulness of the FM approach for simultaneously measuring the psychological value of gains and losses under risk. The first required step in this approach is to empirically document the existence of a cognitive model deployed by subjects in integrating the several information dimensions included in the task. This was done mostly in Study 1, which established an overall relative ratio model for the integration of subjective expected gains and losses, encompassing in addition a multiplicative operation between subjective value and subjective probability. The two experiments in Study 2 added strength to this result, by fully replicating the graphical and statistical findings in Study 1.

Only then can functional measurement be used to derive the subjective metrics of the stimuli implicit in the algebraic model (Anderson, 1981; 1982). Functional measurement of expected gains and losses (i.e., of utility weighted by probability) was pursued in both studies. It rested on the overall relative ratio rule and provided functional scales with a common known zero and a common unit across gains and losses. Functional measurement of monetary value and probability was also addressed in both studies, but was more fully developed in Study 2. It rested on the embedded multiplying rule, and it provided functional scales with ratio properties (in Study 2), though lacking a common unit across gains and losses.

One key comparative advantage of the FM approach is that it dispenses with arbitrary assumptions invoked in alternative approaches. Empirically establishing an integration model evades the need for conjectured composition rules, like the one that weighted utilities should be added. Also, as functional metrics rely upon the validity of the integration model, nothing has to be assumed regarding the form of the utility or the probability weighting functions (unlike customary practice: see Abdellaoui et al., 2007; Abdellaoui et al., 2008). To be sure, FM can be met with limits. The lack of a common unit across gains and losses in the derived scales of value and probability can be taken as an example of that. However, as before, such limits are also not set by assumption and once for all. Rather, they are empirically settled in the process of exploring the measurement capabilities of the model, with a link to the substantive problems at hand (i.e., to their measurement conditions).

Another advantage of the approach is affording insights on how the task is construed by participants. This is a direct benefit of the emphasis on the integration model, not accessible to measurement frameworks whose foundations lie outside the psychological realm (see Anderson, 1981, chap. 5, and Anderson, 2001, pp. 716-721). The two algebraic rules subsumed under the found model thus have intrinsic interest as psychological structures, not just as a basis for measurement. The multiplying rule, which operates upon subjective probability and subjective value, illustrates a general expectancy × value model often documented in FM studies (see Anderson, 1991, pp. 108-109). The broad psychological significance of this model has been shown among others in developmental studies (Schlottmann & Anderson, 1994; Schlottmann, 2001), and in studies of the everyday usage of probability (Anderson & Schlottmann, 1991).

The relative ratio rule, which operates on subjective expected values, suggests on its turn that subjects conceive of the task as a competition between two opposite tendencies, expressing trough their judgments the compromise found between them (for a decision rule of this same form, applying to dichotomous competing responses, see Anderson, 1981, p. 77, and Anderson, 1996, p. 59). The degree to which this construal of the combination of expected gains and losses by participants depends on peculiarities of the task cannot be assessed at present. However, that it may have generality beyond the devised setting is supported by evidence that even strictly positive gambles with two outcomes might be viewed as entailing a compromise between competing tendencies (Schlottmann, 2001).

One worth noting result of the approach is the level at which expected values could be measured, on a ratio scale with common unit and common zero across losses and gains. This measurement level allows computing legitimate indices of loss aversion, except that these indices would now concern expected values, instead of proper gain and loss values in isolation from probabilities. Notwithstanding this departure from the standard definition (see Kahneman & Tversky, 1979; Tversky & Kahneman, 1991; Tversky & Kahneman, 1992), such indices would actually be consistent with alternative views of loss aversion, as the one propounded in Brooks & Zank (2005): «a potential loss is perceived more harmful than an equally likely gain of the same magnitude is perceived to give pleasure» (p. 303). To views of this latter sort, which underline the behavioral relevance of expected value in everyday life and do not require disentangling utility and probability, the relative ratio model does provide for a meaningful quantification of loss aversion.

The mean values of loss aversion obtained from the model in both studies were considerably below the reference values of 2.25 indicated in Kahneman & Tversky (1992). More importantly still, they disclosed a significant role of individual differences. While loss-averse subjects predominated (loss aversion > 1), a non negligible percentage of participants (14% and 20% in Studies 1 and 2, respectively) classified as gain seeking subjects (loss aversion < 1; see Abdellaoui et al., 2007). This latter finding is hardly compatible with modeling loss aversion as a structural component of the value function, and more in keeping with a dispositional or motivational account of it.

One additional result obtained with the FM approach concerns the characterization of psychophysical curves of value and probability. The measurement level achieved for subjective value and probability was enough in both studies for that, even if in Study 2 it was taken further to the ratio level, which allowed fitting meaningful curvature parameters to data. Both at the aggregate and the individual levels, all curves displayed marked nonlinearity, best fitted by power functions in the case of value, and exponential functions in the case of probability. This is in overall agreement with standard assumptions concerning, in particular, the value function (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992), and diverges from a view of probability weighting as the major, if not exclusive, locus of nonlinearity in decision under risk (Lopes & Oden, 1999).

It should be noted that psychophysical regularities, in this approach, are subordinate and derived from the psychological rules embodied in the integration model. They are not endowed with the status of general psychophysical laws. In particular, the finding that the value data were best adjusted by power functions entails no more than the flexibility of the power function as a fitting tool (see Anderson, 1981, pp. 341-342). However, the kind of nonlinearity observed in the curves can be thereby characterized, and the adjusted curvature parameters (power exponents) compared across the two domains of gain and loss. Close exponents were found for value in both domains, in the range of 0.3 to 0.4, expressing a similar rate of diminishing returns for gains and for losses.

In conclusion, it seems fair to say that the compound relative ratio model opens way for addressing key issues in the simultaneous measurement of subjective gains and losses under risk. There is no fundamental reason to believe that other integration models cannot be found by varying the nature of the task, which might prove equally profitable for the quantification of gains and losses under complex models of decision. Nor do the problems addressed in this paper (e.g., loss aversion and the psychophysics of value) exhaust the possibilities afforded by the current model. The work here reported may thus be more generally envisaged as one further illustration of the lasting potential of Functional Measurement to beneficially contribute to the fields of judgment and decision making.

REFERENCES

Abdellaoui, M. (2000). Parameter-free elicitation of utility and probability weighting functions. *Management Science, 46*(11)*,* 1497-1512.

Abdellaoui, M., Bleichrodt, H., & L'Haridon, O. (2008). A tractable method to measure utility and loss aversion in prospect theory. *Journal of Risk and Uncertainty, 36,* 245-266.

Abdellaoui, M., Bleichrodt, H., & Parashiv (2007). Measuring loss aversion under Prospect Theory: A parameter-free approach. *Management Science, 53*,1659-1674.

Anderson, N. H. (1981). *Foundations of information integration theory*. New York: Academic Press.

Anderson, N. H. (1982). *Methods of information integration theory*. New York: Academic Press.

Anderson, N. H. (1991). Cognitive theory of judgment and decision. In N. H. Anderson (Ed.). *Contributions to information integration theory. Vol. 1: Cognition* (pp. 105-142). Hillsdale, NJ: Lawrence Erlbaum Associates.

Anderson, N. H. (1992). Integration psychophysics and cognition. In D. Algom (Ed.). *Psychophysical approaches to cognition* (pp. 13-113). Amsterdam: Elsevier Science Publishers.

Anderson, N. H. (1996). *A functional theory of cognition*. New Jersey: Lawrence Erlbaum Associates.

Anderson, N. H. (2001). *Empirical Direction in Design and Analysis.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Anderson, N. H. (2008). *Unified Social Cognition*. Psychology Press.

Anderson, N. H., & Schlottmann, A. (1991). Developmental study of personal probability. *Contributions to information integration theory: Vol. III. Developmental* (pp. 111-134). Hillsdale, NJ: Erlbaum.

Anderson, N. H., & Shanteau, J. C. (1970). Information integration in risky decision making *Journal of Experimental Psychology, 84,* 441-451.

Bernoulli, D. (1738/1954). Expositions of a new theory of the measurement of risk. *Econometrica, 22*, 23–36.

Brooks, P. & Zank, H. (2005). Loss averse behavior. *Journal of Risk and Uncertainty*, *31*(3), 301-325.

Edwards, W. (1955). The predictions of decisions among bets. *Journal of Experimental Psychology, 50*, 201–214.

Fennema, H., & van Assen, M. A. (1998). Measuring the utility of losses by means of the tradeoff method. Journal of Risk and Uncertainty, *17,* 277-295.

Fox, C. R. & Poldrack, R. A. (2009). Prospect theory and the brain. In P. W. Glimcher, C. F. Camerer, E. Fehr, & R. A. Poldrack. *Neuroeconomics: Decision making and the brain* (pp. 145-174) Amsterdam: Academic Press.

Hershey, J., Kunreuther, H. & Schoemaker, P. (1982). Sources of bias in assessment procedures for utility function, *Management Science, 28*(8), 936-953.

Hershey, J. & Shoemaker, P. (1985). Probability versus certainty equivalence methods in utility measurement: Are they equivalent? *Management Science, 31*(10), 1213-1231.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 4,* 263-291.

Köberlling, V., & Wakker, P. P. (2005). An index of loss aversion. *Journal of Economic Theory, 122*, 119-131.

Lopes, L. L. (1996). When time is of the essence: Averaging, aspiration, and the short run. *Organizational Behavior and Human Decision Processes*, *65*(3), 179-186.

Lopes, L. L., & Oden, G. C. (1999). The role of aspiration level in risky choice: A comparison of Cumulative Prospect Theory and SP/A theory. *Journal of Mathematical Psychology, 43*, 286-313.

Lynch, J. G. & Cohen, J. L. (1978). The use of subjective expected utility theory as an aid to understand variables that influence helping behavior. *Journal of Personality and Social Psychology, 36*(10), 1138-1151.

Masin, S. C. (2004). Tests of functional measurement theory for multiplicative models. In A. M. Oliveira, M. Teixeira, G. F. Borges, & M. J. Ferro (Eds.). *Fechner Day 2004. Proceedings of the 20th Annual Meeting of the International Society for Psychophysics* (pp. 447-452). Coimbra, Portugal: The International Society for Psychophysics.

McCord, M. R., & de Neufville, R. (1986). Lottery equivalents. Reduction of the certainty effect problem in lottery assessment. *Management Science, 32*(1), 56-60.

Rabin, M. (2000). Risk Aversion and Expected-utility Theory: A Calibration Theorem, *Econometrica, 68,* 1281—1292.

Savage, L. J. (1954). *The Foundations of Statistics*. New York: Wiley.

Schlottmann, A. (2000). Children´s judgments of gambles: A disordinal violation of additive utility. *Journal of Behavioral Decision Making, 13,* 77-89.

Schlottmann, A. (2001). Children´s Probability Intuitions: Understanding the expected value of complex games. *Child Development, 72*(1)*,* 103-122.

Schlottmann, A., & Anderson, N. H. (1994). Children's judgments of expected value. *Developmental Psychology, 30*(1), 56–66.

Shanteau, J. (1974). Component processes in risky decision making. *Journal of Experimental Psychology, 103*(4)*,* 680-691.

Shanteau, J. (1975). An information integration analysis of risky decision making. In M. F. Kaplan and S. Schwartz (Eds.), *Human judgment and decision processes* (pp. 109-137). New York: Academic Press.

Teixeira, N. S., & Oliveira, A. M. (2008). A perceptual-cognitive dividing model for the integration of velocity and traveled distance of a moving target by localization responses. In B. A. Schneider and B. M. Ben-David (Eds.). *Fechner Day 2008: Proceedings of the 24th Annual Meeting of the International Society for Psychophysics* (pp. 99-105). Toronto, Canada: The International Society for Psychophysics.

Troutman, C. M., & Shanteau, J. (1976). Do consumers evaluate products by adding or averaging attribute information? *Journal of Consumer Research, 3*, 101-106.

Tversky, A. & Kahneman, D. (1991). Loss aversion in riskless choice: A reference dependent model, *Quarterly Journal of Economics, 107*(4), 1039-1061.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5,* 297-323.

Viegas, R. G., Oliveira, A. M., & Garriga-Trillo, A. (2009). A relative ratio model for the integration of gains and losses in a mixed regular roulette-type game. In M. Elliott, S. Antonijecic, S. Berthaud, P. Mulcahy, B. Bargary, C. Martyn, & M. Schmidt (Eds.). *Fechner Day 2009: Proceedings of the 25th Annual Meeting of the International Society for Psychophysics* (pp. 545-550). Galway, Ireland: The International Society for Psychophysics.

Viegas, R. G., Oliveira, A. M., & Garriga-Trillo, A. (2010). Loss aversion and the locus of nonlinearity in decision under risk: A test between Prospect Theory and SP/A theory with functional measurement. In A. Bastianelli & G. Vidotto (Eds.). *Fechner Day 2010: Proceedings of the 26th Annual Meeting of the International Society for Psychophysics* (pp. 345-350). Padua, Italy: The International Society for Psychophysics.

Wakker, P. P., & Deneffe, D. (1996). Eliciting von Neumann-Morgenstern utilities when probabilities are distorted or unknown. *Management Science, 42*(8), 1131-1150.

Weiss, D. (2006). *Analysis of variance and functional measurement: A practical guide*. Oxford: Oxford University Press.

Zalinski, J., & Anderson, N. H. (1987). *AVERAGE Program & Manual*. San Diego: University of California.

(Manuscript received: 29 November 2011; accepted: 26 March 2012)