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The intuitive physics of the equilibrium of the lever and of the hydraulic pressures: Implications for the teaching of elementary physics

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The research field of intuitive physics focuses on discrepancies between theoretical and intuitive physical knowledge. Consideration of these discrepancies can help in the teaching of elementary physics. However, evidence shows that theoretical and intuitive physical knowledge may also be congruent. Physics teaching could further benefit from understanding the reasons for this congruence. The present study explored these reasons by investigating the intuitive physics of the equilibrium of the lever and of the hydraulic pressures. It was found that the intuitive-physics law of the lever was multiplicative for all participants while the intuitive-physics law of the hydraulic pressures differed among participants. Since these laws are equally simple and the layman probably has had extensive experience with the lever and scarce or no experience with the hydraulic lift, these findings support the general hypothesis that physical laws and corresponding intuitive-physics laws are congruent when people have had experience with the respective phenomena. The results and theoretical considerations suggest two strategic principles for teaching elementary physics.

The teaching of physics can largely benefit from findings of research in intuitive physics. Typically, this field of research studies the discrepancies between the physicist's theoretical knowledge and the layman's intuitive knowledge of the everyday physical world (Clement, 1982; Lipmann & Bogen, 1923; McCloskey, 1983; Nersessian & Resnick, 1989; Renn, Damerow, & McLaughlin, 2003; Shanon, 1976; Sherin, 2006; Smith & Casati, 1994; Taber, 2004). Clearly, awareness about such discrepancies can help teachers anticipate those aspects of physical explanations that could be more difficult for the student to understand.

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Recent evidence increasingly shows, however, that theoretical and intuitive physical knowledge may also be congruent. That is, beginning with Anderson's (1983a, 1983b) seminal work, studies using information integration methodology have found that intuitive-physics laws—the mathematical relations between the cognitive variables defining a person's implicit knowledge of an everyday physical phenomenon—may be formally identical or may be formally dissimilar to the respective physical laws (Cocco & Masin, 2010; Corneli & Vicovaro, 2007; De Sá Teixeira, Oliveira, & Silva, 2013; De Sá Teixeira, Oliveira, & Viegas, 2008; Karpp & Anderson, 1997; Krist, Fieberg, & Wilkening, 1993; Léoni & Mullet, 1993; Léoni, Mullet, & Chasseigne, 2002; Masin & Rispoli, 2010; Mullet & Montcouquiol, 1988; Silverberg, 2003; Vicovaro, 2012; Wilkening, 1981; Wilkening & Huber, 2002; Wilkening & Martin, 2004). The following are one example of identity and one of dissimilarity of laws.

IDENTITY OF LAWS

For a linearly elastic spring with rest length L hanging vertically from a fixed support, Hooke's law is about the spring elongation E caused by a load with weight W hung on the lower end of the spring. It states that

$$E = k \cdot L \cdot W \tag{1}$$

with k constant. A physical test of this equation consists in measuring E for each factorial combination of fixed values of L and W and in plotting these measures against W separately for each L. The test supports the law if the resulting factorial curves are straight lines fanning out from the origin.

Let λ and ω denote the subjective counterparts of L and W, respectively. For each factorial combination of fixed values of these variables, Cocco and Masin (2010) had participants look at the spring (and thus perceive λ), heft the load (and thus perceive ω), and then estimate the imagined elongation ε in subjective centimeters that would have occurred in the event that the load was hung on the lower end of the spring. Functional measurement theory shows that the means of these estimates made separately for each W are linear measures of the respective ω (Anderson, 1982, p. 73). The curves obtained by plotting estimated ε against these linear measures of ω separately for each λ were straight lines fanning out from the origin.

This obtained pattern of straight lines intersecting at the origin—the same as that obtained by the physical test of Hooke's law mentioned above—supports the intuitive-physics law of elasticity

$$\varepsilon = k' \cdot \lambda \cdot \omega \tag{2}$$

with k' constant. This law is formally identical to Hooke's law.

DISSIMILARITY OF LAWS

For an object of weight W and volume V freely immersed in a fluid of density D, Archimedes' law of buoyancy states that the object floats when the ratio of the immersed volume to the total volume of the object is

$$R = \frac{W}{V \cdot D} \; . \tag{3}$$

Let ω, ν, and δ denote the subjective counterparts of W, V, and D, respectively. For each factorial combination of fixed values of these three variables, Masin and Rispoli (2010) had participants heft the object while looking at it (and thus perceive ω and ν), visually inspect the viscosity of a fluid (and thus perceive δ), and then estimate the ratio ϱ of the immersed volume to the total volume of the object that would occur in the event that the object was dropped in the fluid. The pattern of factorial curves resulting from plotting these estimates against ω separately for each ν was in agreement with a multiplicative relation between these variables, while the pattern resulting from plotting these estimates against ω separately for each δ was in agreement with an additive relation between these variables, supporting the intuitive-physics law of buoyancy

$$\rho = \frac{\omega}{\nu} - \delta \, . \tag{4}$$

In a study of the mastery of the relations between mass, volume, and density, Léoni & Mullet (1993) and Léoni et al. (2002) also found that participants integrated density additively. The law expressed by Equation 4 is dissimilar from Archimedes' laws of buoyancy.

HYPOTHESIS

The following study explored the reasons why theoretical and intuitive physical knowledge may be congruent, as in the case of Hooke's law and the respective intuitive-physics law. Knowing these reasons may further help teachers ameliorate their physical explanations.

Archimedes' law is more complex than Hooke's law. Since Archimedes' law and the corresponding intuitive-physics law are dissimilar, simplicity of laws could be one possible reason for the identity of a physical law with an intuitive-physics law.

People have most probably had extensive experience with buoyancy of objects in water. However, people may have had less or no experience with buoyancy of objects in fluids with density higher than that of water. The participants' use of information about fluid density as encoded in Expression 4 rather than as in Equation 3 agrees with this possibility.

One may then hypothesize that another condition for the identity of physical laws with intuitive-physics laws is the extent of experience that people have with the respective physical phenomena. In agreement with this hypothesis, Fischbein (1987) has argued in general that experience determines intuitive knowledge and Millar and King (1993) and Liégeois, Chasseigne, Papin, and Mullet (2003) have suggested in particular that lack of experience may be responsible for the often observed lack of understanding of the concept of potential difference in simple electric circuits.

Considering that the hydraulic lift involves machinery less likely for the layman to encounter in everyday life, it seems probable that the layman has had extensive experience with the equilibrium of moments in the lever and not with the equilibrium of pressures in the hydraulic lift. These equilibriums are governed by equally simple multiplication laws. Being governed by laws of equal simplicity, these phenomena should thus allow testing whether the formal identity of physical laws with the respective intuitive-physics laws depends on people's experience with the phenomena. Using information integration methodology, the following experiments provided the test of this hypothesis by determining the intuitive-physics laws of the lever and of the hydraulic lift.

THE EQUILIBRIUM OF THE LEVER

Consider a lever consisting of a horizontal beam with a fulcrum located at its center. For a load of weight W_1 placed on the beam on the left at a distance D_1 from the fulcrum, and a load of weight W_2 placed on the beam on the right at a distance D_2 from the fulcrum, Archimedes' law of the lever says that the beam remains horizontal when

$$D_1 \cdot W_1 = D_2 \cdot W_2 \tag{5}$$

(Heiberg, 1881). One may hypothesize that the relations between the relevant variables involved in the adults' implicit knowledge of the equilibrium of the lever mimic those in Archimedes' law. That is, one may hypothesize that

$$\delta_1 \cdot \omega_1 = \delta_2 \cdot \omega_2 \tag{6}$$

with δ_1, δ_2, ω_1, and ω_2 the subjective counterparts of D_1, D_2, W_1, and W_2, respectively.

The following evidence suggests that the relations between the variables involved in the intuitive-physics law of the equilibrium of the lever could be more complex than those expressed in Equation 6. Surber and Gzesh (1984) and Wilkening and Anderson (1982) empirically tested Equation 6 on adults using a balance scale with a relatively short beam equipped with pegs or hooks that allowed placing loads at different distances from the fulcrum. Their design may be described as follows. For each combination of fixed values of D_2 and of W_2 (right of fulcrum) the participants selected the physical distance D_1 at which a load of fixed weight W_1 (left of fulcrum) had to be placed so that the beam would remain horizontal.

Equation 6 predicts that participants may have selected D_1 such that

$$\delta_1 = \frac{\delta_2}{\omega_1} \cdot \omega_2 . \tag{7}$$

With ω_1 fixed, Equation 7 predicts that curves relating δ_1 to δ_2 separately for each value of ω_2 are straight lines fanning out from the origin. Since δ_1 and δ_2 tend to 0 when D_1 and D_2 tend to zero, respectively, this prediction is supported if the curves relating D_1 to D_2 separately for each value of ω_2 intersect at the origin. Contrary to this prediction, Surber and Gzesh (1984, p. 262) and Wilkening and Anderson (1982, p. 230) found that the intercept of the curves relating D_1 to D_2 varied with ω_2.

Wilkening & Anderson (1982) suggested that this disagreement of empirical data with Equation 7 could be due to the participants using a multiplication rule combined with an addition rule. Such a composite rule may be expressed by the equation

$$\delta_1 = \frac{c \cdot \delta_2 \cdot \omega_2 + (1-c) \cdot (\delta_2 + \omega_2)}{\omega_1} \tag{8}$$

with c a weight factor representing a compromise between the addition and multiplication rules or the relative proportion with which participants alternatively used these rules. Using the balance-scale task with children, Ferretti, Butterfield, Cahn, and Kerkman (1985) found data indicating that participants might have independently used both of these rules.

Wilkening and Anderson (1991, p. 56) argued that end effects may have biased Surber and Gzesh's (1984) and Wilkening and Anderson's (1982) results. Indeed, when one end of a fixed distance from the fulcrum was close to one end of the beam, participants could not select distances far beyond the other end of the beam with the consequence that participants' responses were probably biased. A similar argument applies to fixed distances close to the center of the beam.

The following experiment attempted to test Equations 7 and 8. The weights W_1 and W_2 were varied factorially and D_1 was used as the dependent variable with D_2 fixed. End effects were minimized using a scale beam that allowed a wide variation of D_1. Differently from previous studies, no pegs or hooks on the beam were used since these objects would have been extra features that the participants could have used to estimate a weighted mean torque with the weights of this mean varying with D_1 (Palmieri, 2008).

The test was based on the following assumptions.

(i) Judgments of subjective distance in subjective centimeters are related linearly to subjective distance and to physical distance. This assumption is supported by empirical data (Baird, 1970, pp. 42–43; Masin, 2008, 2012; Stevens & Galanter, 1957).

(ii) The study used equidistant values for W_1 and W_2. It is plausible that these values yielded practically equidistant values of subjective weight since the relation of subjective weight to physical weight has been found to be linear for objects with size as that of the objects used in this experiment (Stevens & Rubin, 1970) and since the range of weight values used in the present study was small.

For fixed values of δ_2 and ω_1, Equation 7 predicts that direct estimates of δ_1 plotted against ω_2 yield curves that intersect at the origin. On the other hand, Equation 8 predicts that the intercept of these curves varies with ω_1.

EXPERIMENT 1

METHOD

Participants and stimuli

Eight university students in non-physical sciences participated individually for pay. A white wooden beam measuring 241 cm (width) × 7 cm (height) × 0.5 cm (depth) was displayed throughout the experiment. The beam was held on the participant's frontal parallel plane by a vertical rod fixed on a stand. This rod was a wooden parallelepiped with a base 4 cm × 4 cm and height 60 cm. The beam had a 0.7 cm circular hole in its center. On the vertical axis of the rod facing the participant, a cylindrical pivot with a diameter as that of this hole was mounted centrally at 4 cm from the top of the rod. The pivot was passed through the hole in the beam allowing the beam to rotate. A device on the rod, invisible to the participant, could hold the beam in a horizontal position.

The participant's head was about 165 cm in front of the center of the beam while the participant sat at a table located between the participant and the beam. This table had a top surface made of Formica laminate. It was holding a 13-cm tall, 1.5-kg tripod carrying a laser pointer that constantly projected a 0.1-cm (width) × 7 cm (height) vertical red line on the beam. By both hands, the participant could smoothly move this vertical line along the beam by easily sliding the tripod on the table.

On the right of the center of the beam, a 0.03-cm (width) × 7 cm (height) vertical black line was drawn on the beam 10 cm from this center. A tape meter invisible to the participant was stuck along the top edge of the beam to measure distances of the red laser line from the center of the beam.

The beam was placed 80 cm from a parallel gray wall and was illuminated by a neon light on the ceiling located 130 cm right above the center of the beam. At the participant's location the illumination level was about 1 klx.

Four opaque gray bottles weighing 100, 200, 300, or 400 g were used as additional stimuli. All had the same shape and a capacity of 125 ml. Each had a circular plastic ring on top which allowed insertion of the index finger.

Procedure

At the beginning of the experiment the participant was shown that the beam rotated around the pivot of the rod. Subsequently the beam was set horizontally and kept in this position for the entire duration of the experiment.

On each trial, the participant hefted each of two stimulus bottles presented successively for about 0.5–1 sec. One experimenter presented these bottles to the participant with an interval between the presentations of about 5–10 sec. Stimulus bottles were hidden from view when they were not presented to the participant for hefting. The participant hefted each bottle by inserting the index finger in the ring on top of the bottle.

On each trial, the participant was asked to imagine one stimulus bottle hanging at the point of the lower edge of the beam coinciding with the lower end of the vertical black line (on the right) and then to position the vertical red line so that its lower end coincided with the point of the lower edge of the beam where the other stimulus bottle should be hung (on the left) to keep the beam horizontal.

When the participant had terminated positioning the red line on the beam, the participant was asked to leave the red line standing in that position for the rest of the trial and to estimate its distance from the center of the beam in centimeters and fractions thereof. For this estimation, the part with the first 10 cm cut from a meter tape was displayed horizontally on the table in front of the participant at a distance of about 50 cm. After the participant made this estimation, on the meter tape stuck on the top edge of the beam the experimenter read the physical distance of the red line from the center of the beam with precision to the nearest millimeter.

The weight of the bottle to be imagined hanging on the left was 100, 200, or 300 g and that of the bottle to be imagined hanging on the right was 100, 200, 300, or 400 g for a total of 12 combinations of weights. On each trial, the bottle to be imagined hanging on the right was presented either first or second and the initial position of the red line was either the left end or the center of the beam. The four combinations of these orders were used twice for each combination of weights. The resulting 96 combinations were presented randomly, one for each trial.

At the end of the experiment a questionnaire was submitted to each participant to assess their knowledge of Archimedes' law and whether they had made mental numerical calculations during the experiment.

RESULTS AND DISCUSSION

Figure 1 shows mean estimates of δ_1 in subjective centimeters plotted against W_2 separately for each W_1 of 100, 200, or 300 g, represented by crosses, open circles, and filled circles, respectively. The results for the physical measures D_1 of the distance of the red laser line from the center of the beam are omitted since they are qualitatively similar to those for δ_1.

Figure 1a shows the mean results for eight participants. All participants produced the same pattern of curves. Figure 1b exemplifies the results for one participant. Comparison of Figures 1a and 1b shows that the unit of subjective centimeter varied considerably among participants.

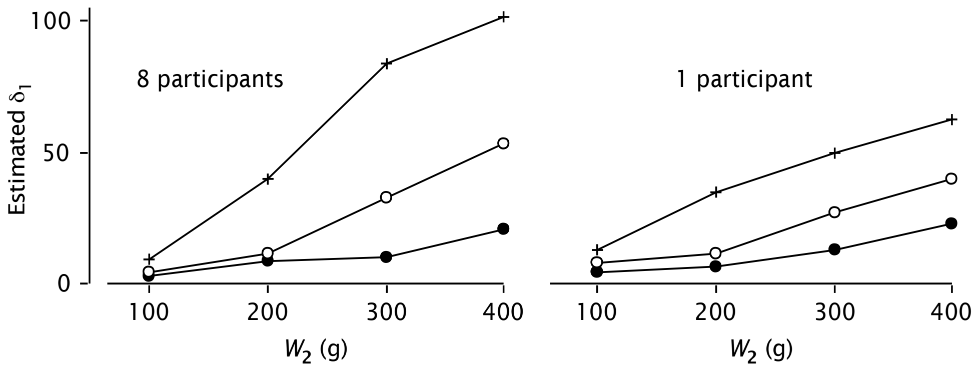

Figure 1. Mean estimate of perceived distance δ_1 in subjective centimeters of a weight W_1 of 100 g (+), 200 g (○), or 300 g (●) required to be placed on the left of the fulcrum to keep a beam horizontal when a weight W_2 would be placed at a fixed distance on the right of the fulcrum. Left: mean results for 8 participants. Right: example of mean results for 1 participant.

In Figure 2, the solid lines show the value of D_1 as a function of W_2 calculated by Equation 5. For each participant, this calculated value of D_1 was smaller than the physical distance corresponding to the respective mean estimate of δ_1. Given the equation $R = m \cdot \delta_1 + n$ with m and n constants, each R of each participant was transformed using this equation with the same values of m and n for each participant such that these values minimized the root mean square deviation of the mean transformed estimates of δ_1 from the respective values of D_1 calculated by Equation 5. Figure 2 shows these mean transformed estimates of δ_1 plotted against W_2 for m = 0.33 and n = 4.8 (rmsd = 1.84). The linear trend of W_2 was significant with the quadratic and cubic trends not significant, $F(1,7) = 89.5$, $p<.001$, $F(1,7) = 0.4$, $F(1,7) = 4.7$, respectively.

Since the mean transformed estimates of δ_1 were close to the respective value of D_1 calculated by Equation 5, the results in Figure 2 show that mean estimates of δ_1 closely agreed with Archimedes' law up to a linear transformation. The close convergence of the respective factorial curves toward the origin supports the hypothesis that Archimedes' law (Equation 5) and the intuitive-physics law of the equilibrium of the lever (Equation 6) are formally identical.

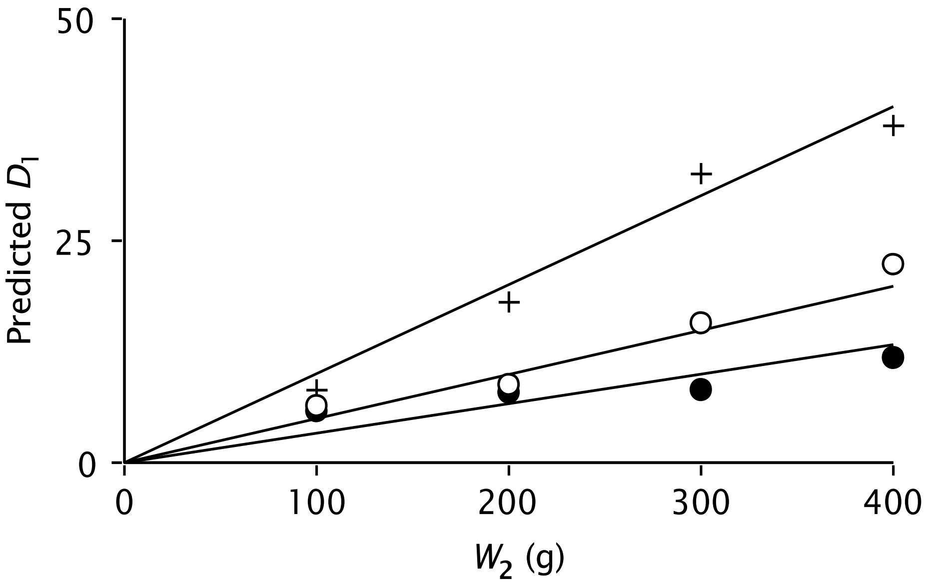

Figure 2. Mean linearly transformed estimates of perceived distance δ_1 that minimized deviation from corresponding physical distance D_1 predicted by Archimedes' law (solid lines).

Replies to the questionnaire submitted at the end of the experiment showed that participants reported knowing the inverse relation of distance to weight necessary for the beam to remain in horizontal equilibrium and that they reported making no mental numerical calculation in complying with the instructions.

EXPERIMENT 2

Young adults have had extensive experience with levers throughout their lives. In agreement with this experience, the above results indicate that Archimedes' law and the intuitive-physics law of the equilibrium of the lever are formally identical.

Archimedes' law of the lever and the law of the equilibrium of the hydraulic pressures are multiplicative and equally simple. The following experiment explored whether the physical and intuitive-physics laws of the equilibrium of the hydraulic pressures are formally identical. Since hydraulic pressures occur in mechanisms that may be rarely encountered in everyday life, finding formal identity of the physical with the intuitive-physics law for the equilibrium of the hydraulic pressures would indicate that people's experience of this phenomenon is not relevant for the identity of these laws while finding dissimilarity of the physical with the intuitive-physics law would indicate that this experience is relevant for the identity of these laws.

Figure 3 shows two of the stimuli used in Experiment 2. Each stimulus depicts two connected cylinders. These cylinders were simulations of glass cylinders containing water covered with a lid. The lids were positioned horizontally at the same level and were described to the participants as being of negligible weight and acting as pistons.

For a given stimulus, consider a load of weight W_1 resting on the left lid of area A_1 and a load of weight W_2 resting on the right lid of area A_2. The law of the equilibrium of the hydraulic pressures asserts that the two lids remain at the same level when

$$\frac{W_1}{A_1} = \frac{W_2}{A_2} \tag{9}$$

(Pascal, 1663).

The hypothesis that people implicitly know this law is expressed by the equation

$$\frac{\omega_1}{\alpha_1} = \frac{\omega_2}{\alpha_2} \tag{10}$$

with ω_1, ω_2 , α_1, and α_2, the subjective counterparts of W_1, W_2, A_1, and A_2, respectively. In the following experiment, α_2 was the dependent variable with α_1 fixed and with ω_1 and ω_2 varied factorially. Equation 10 implies that

$$\alpha_2 = \alpha_1 \cdot \frac{\omega_2}{\omega_1} \ . \tag{11}$$

With α_1 fixed, this equation predicts that curves relating α_2 to ω_2 separately for each value of ω_1 are straight lines fanning out from the origin. Since ω_2 tends to 0 when W_2 tends to zero, this prediction is supported if the curves relating α_2 to W_2 for each value of ω_1 intersect at the origin. Accordingly, the following experiment tested whether direct estimates of α_2 plotted against W_2 yielded curves intersecting at the origin.

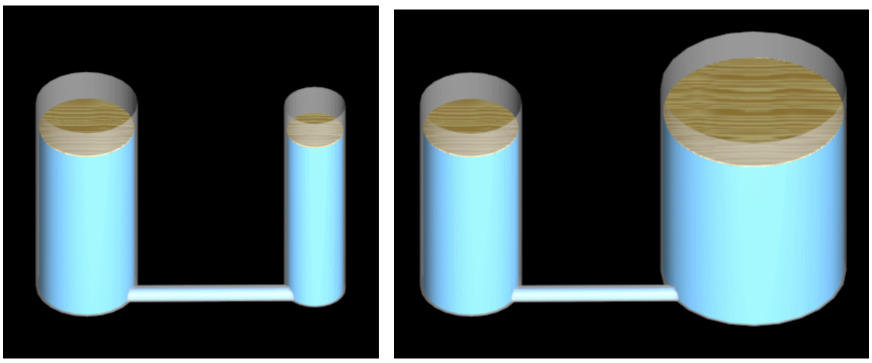

Figure 3. Examples of the stimuli used in Experiment 2.

METHOD

Participants and stimuli

Ten university students in non-physical sciences participated individually. None of them had participated in Experiment 1.

Stimuli were created by computer graphics software (Autodesk 3ds Max). As shown in Figure 3, they were the orthographic projection of cylinders viewed from above at an angle of 45 degrees. Stimuli looked like two gray glass cylinders each containing a column of blue water connected through a horizontal cylinder, with each column of water covered with a variegated light brown lid. Stimuli were displayed on a black 380-mm (width) × 210-mm (height) frontoparallel monitor screen (Philips Brilliance 190B). Viewing distance was about 1.2 m.

The height of the vertical cylinders was 95 mm and that of the respective column of water was 83 mm. The horizontal cylinder had a diameter of 9 mm and a length of 80 mm. The lids were 1 mm thick with a diameter equal to that of the respective column of water. The lids were placed horizontally on the water surface.

The internal diameter of the left cylinder was 45 mm and that of the right cylinder could be varied by the participant from 6.5 to 198 mm in discrete steps, with a difference between internal and external diameters of 3 mm. This variation was carried out using two keys, one to increase and one to decrease the size of the right cylinder. This size increased only horizontally with the height of lid from the base of the cylinder kept constant. The relation between the internal diameter of the right cylinder, y, and the number of these steps, x, counted from 1 to 50 from the minimum internal diameter of the right cylinder was defined by the polynomial $y = 6.205 + 0.0089 \cdot x + 0.23 \cdot x^2 - 0.00307 \cdot x^3$.

The stimuli appeared in the middle of the screen with a gap of 23 mm between the left cylinder and the left side of the screen and a gap of 50 mm between the largest possible right cylinder and the right side of the screen.

Four loads were used as additional stimuli. They were the bottles with a ring on top for lifting previously used for Experiment 1. Their weight was 50, 100, 150, or 200 g.

Procedure

Preliminarily, the participants were asked to imagine that the lids were made of rigid material of negligible weight, acting as pistons perfectly adhering to the internal surface of the glass cylinders. Using an animation, they were shown that an imaginary weight resting on one lid was causing this lid to descend and the other lid to ascend.

The instructions for each trial were the following. The participant was asked to lift each of two loads presented successively using the index finger of their preferred hand. This hand was kept hidden behind a screen so as to prevent the participant from seeing the load.

The participant was asked to imagine the first load lifted to be resting on top of the lid of the left cylinder and the second load lifted to be resting on top of the lid of the right cylinder. While doing this, they were also asked to select a right cylinder with its lid having the area that would make the left and right lids remain at the same equal level as that shown in the stimulus. Analysis of individual data subsequently showed that no participant selected the smallest or largest possible areas of the right lid except for two participants, each of whom only selected the smallest possible area only once.

After the area of the right lid had been selected, the participant was asked to estimate the area of the right lid by mentally counting how many left lids, including possible fractions, were necessary to completely cover the right lid.

The weight of the load to be imagined resting on the left lid was 50, 100, 150, or 200 g and the weight of the load to be imagined resting on the right lid was 50, 100, or 150 g for a total of 12 combinations of weights. These combinations were presented in random order twice consecutively.

At the end of the experiment a questionnaire was submitted to each participant to assess their knowledge of Pascal's law and whether they had made mental numerical calculations during the experiment.

RESULTS AND DISCUSSION

Figure 4 shows the mean estimates of the area of the right lid, α_2, plotted against W_2 separately for each W_1 of 50, 100, 150, or 200 g, represented by open circles, crosses, filled triangles, and filled squares, respectively. These group results show no clear structure in the pattern of factorial curves (hereafter called *factorial pattern*). On the other hand, analysis of individual data showed that there was a clear structure in the factorial patterns of seven participants and no clear structure in the factorial patterns of the remaining three.

Figure 5 shows four types of individual factorial patterns. Each diagram is for one participant. Participant 5 (upper left) exhibited a factorial pattern equal to that also exhibited by three other participants. As predicted by Equation 11, these four participants produced results supporting an information integration operation of the form $\alpha_2 = F(\omega_2/\omega_1)$ formally closely identical to that of the physical law for the equilibrium of the hydraulic pressures.

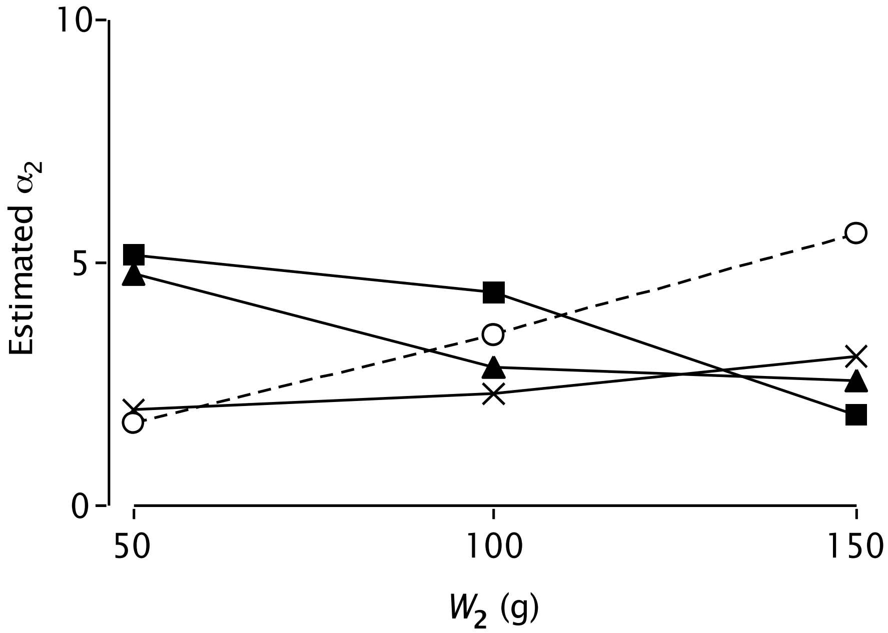

Figure 4. Group results of Experiment 2. Mean estimate of perceived area α_2 of the right lid required to balance the hydraulic pressures when a weight W_2 rests on the right lid and a weight of 50 g (○), 100 g (×), 150 g (▲), or 200 g (■) rests on the left lid.

The other participants exhibited very different factorial patterns. Participants 2 (lower left), 3 (upper right), and 7 (lower right) exhibited factorial patterns supporting information integration operations of the form $\alpha_2 = F[\omega_1 \cdot (k - \omega_2)]$, $\alpha_2 = F[\omega_1 + (k - \omega_2)]$, and somewhat approximately $\alpha_2 = F[(k - \omega_1) - |k' - \omega_2|]$, respectively, with k and k' constants.

The mean factorial pattern resulting from the seven participants with structured factorial patterns practically matched the irregular factorial pattern shown in Figure 4, indicating that the group data resulted from a mixing of differently structured individual factorial patterns.

Since seven participants produced structured factorial patterns probably generated by one single information integration operation, it seems plausible that the unstructured factorial patterns for the remaining three participants could have been due to them alternately using different information integration operations during the experiment.

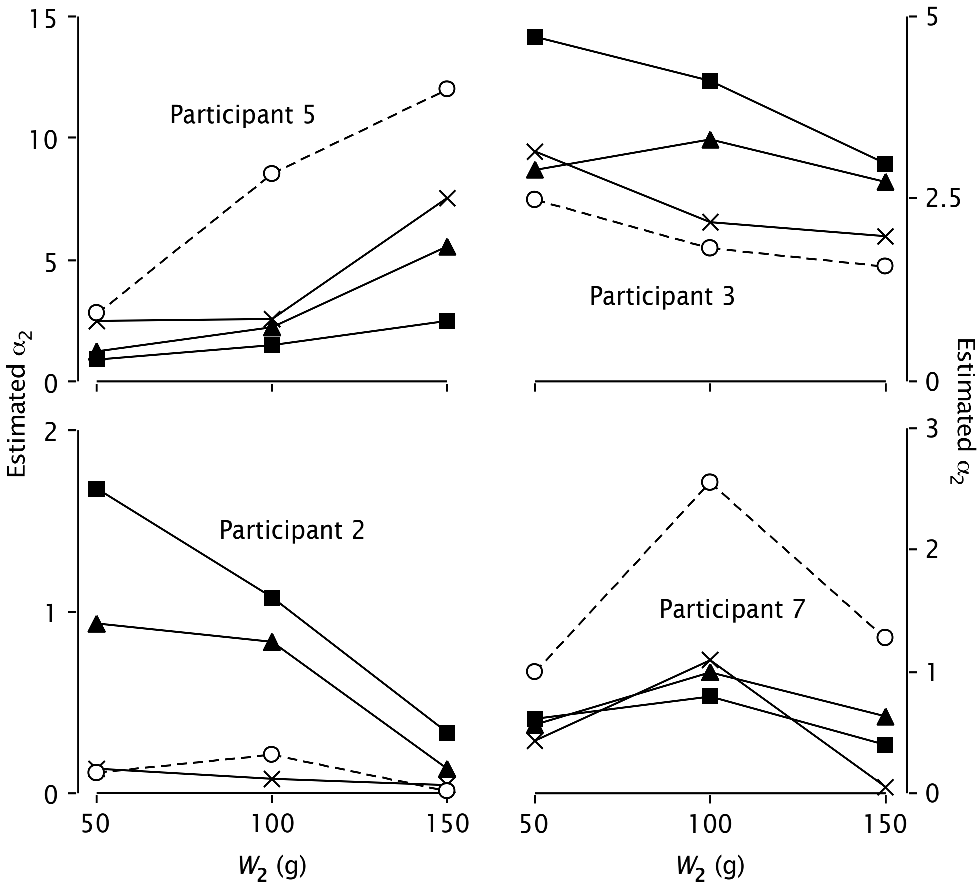

Figure 5. Results for four individual participants in Experiment 2. Mean estimate of perceived area α_2 of the right lid required to balance the hydraulic pressures when a weight W_2 rests on the right lid and a weight of 50 g (○), 100 g (×), 150 g (▲), or 200 g (■) rests on the left lid.

Replies to the questionnaire submitted at the end of the experiment showed that participants reported not remembering Pascal's law. All reported making no mental numerical calculation in complying with the instructions.

GENERAL DISCUSSION

The intuitive-physics law of the equilibrium of the lever was multiplicative for all participants while the intuitive-physics law of the equilibrium of the hydraulic pressures was multiplicative for some participants, was different from multiplicative for other participants, and did not come out in still other participants. Analogous results have been observed for the upward thrust exerted by fluids on objects (Mullet & Montcouquiol, 1988) and for the trajectory of spheres propelled horizontally off a cliff (Karpp & Anderson, 1997). Since the physical laws of the equilibrium of the lever and of the hydraulic pressures are both simple and multiplicative, the present results support the hypothesis that physical laws about everyday physical phenomena are formally identical to the respective intuitive-physics laws when these laws are simple and people have had experience with the phenomena.

How can these results be explained? In a wide variety of judgment tasks in all psychological fields—for example, in social cognition or person perception—a substantial body of evidence shows that people use four main cognitive operations for the integration of information: adding, multiplying, and weighted or unweighted averaging (Anderson, 1991, 1996). Since these information integration operations are used generally, either alone or in combination, it is plausible that people use these operations in judgment tasks about physical phenomena (Anderson, 1983a). The formal identity of a physical law with the corresponding intuitive-physics law could mean that an information integration operation formally identical to the physical law had been associated with the corresponding physical phenomenon through this person's experience with the phenomenon.

What is the learning process that leads to the formal identity of a physical law with an intuitive-physics law? When a person is experiencing a physical phenomenon, the person is probably making predictions about the quantitative relations among relevant variables of the phenomenon. For example, two persons playing on a seesaw need to predict their position on the seesaw while they consider distances from the fulcrum and the persons' weights. To make a prediction, these persons activate different information integration operations. One of these operations (multiplication/division) leads to a correct prediction. Reaching a desired physical state of equilibrium of the seesaw reinforces the correct operation and inhibits inadequate operations. The extent of this reinforcement and of this inhibition determines the stability of the association of the correct information integration operation with the physical phenomenon.

The present results for the equilibrium of the hydraulic pressures agree with this interpretation. The finding of four different types of factorial patterns for the phenomenon of the hydraulic lift indicates that participants were activating different information integration operations in dealing with this phenomenon. Since participants had most probably little or no prior experience with the hydraulic lift, the association of the activated operations with the hydraulic lift had not been previously reinforced nor inhibited. Also, the finding of participants who exhibited unstructured factorial patterns suggests that these participants had no prior experience with the hydraulic lift and were presumably switching from one integration rule to another during the experiment. Clearly, further research is needed to assess the validity and generality of these conclusions.

How extensive should people's experience be with a physical phenomenon so that the correct information integration operation is stably associated with the phenomenon? Although a definite answer to this question seems presently impossible, there are indications that the process of association may be relatively fast. For example, when younger children are asked to judge the area of rectangles whose width and height are varied factorially, judged area plotted against width separately for each height yields parallel straight lines (Anderson & Cuneo, 1978; Lautrey, Mullet, & Pâques, 1989; Wilkening, 1979; Wolf & Algom, 1987). These results indicate that younger children solve this problem of intuitive geometry by arbitrarily applying an additive information integration operation—presumably due to the simplicity of the operation. Wolf (1995) obtained results indicating that the process of associating the correct (multiplicative) integration operation to a rectangle occurs rapidly. He had younger children rate the size of rectangular chocolate bars with the width and height of bars varied factorially. Before they rated this size, children either never handled the chocolate bars or handled them for ten minutes. The resulting factorial curves were parallel straight lines when the children never handled the bars, but diverged in agreement with the correct multiplicative integration operation after the children had handled the bars. Similarly, Chasseigne, Lafon, & Mullet (2002) and Lafratta (2007) found that one session of feedback suffices to change the structure of factorial patterns from additive to multiplicative. However, it is not known how long the learning process should be to have a stable effect such as that occurring for the equilibrium of the lever.

The present results and theoretical considerations suggest two important strategic principles for teaching elementary physical science. Teaching would benefit if teachers were made aware of the variety of possible integration operations schoolchildren may use to predict functional relations between relevant variables of the everyday physical phenomena. Knowing the types of the operations most frequently used may help anticipate typical presuppositions about these functional relations.

Equally, if not more important, is that schoolchildren should have the opportunity to interact with those physical phenomena whose laws they will have to study. For example, regarding Pascal's law, they could preliminarily use real cylinders as those represented in Figure 3 with weights to be placed on the lids to obtain specific balance conditions. This experience would serve to associate the correct information integration operation to the physical phenomenon. This correct operation would subsequently become automatically activated—appearing as implicit intuitive knowledge—in schoolchildren's reasoning carried out while the teacher explains the phenomenon and its implications.

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