# Scatter Search – Wellsprings and Challenges

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## 1. Introduction

I came up with the idea of editing this volume in the summer of 2002 while working on the book "Scatter search - methodologies and implementations in C" with Manuel Laguna in the University of Colorado at Boulder. There, Fred Glover kindly let me use his office, where I found a copy of the "Tabu Search Methods for Optimization" special issue that he edited in 1988 for the European Journal of Operational Research. This encounter made me realize that Scatter Search has reached a level of maturity as an optimization method that has parallels with what Tabu Search was experiencing in the late eighties. So I thought that the moment was perfect to embark on this project, which has been supported enthusiastically by Professor Roman Slowinski.

The Scatter Search (SS) methodology was first introduced in 1977 by Fred Glover and extensive contributions have been made by Manuel Laguna. I am indebted to both of them for sharing their knowledge and experience with me. As will be shown in the next section, SS has evolved since it was first proposed, and this volume is intended to be one of the main future references on the application of this methodology. The book by Laguna and Martí (2003) covers standard implementations of both basic and advanced SS designs. These designs are based on the five methods identified in the next paper in this volume, titled "Principles of Scatter Search." The following papers describe applications in which these methods are clearly identified. Special attention is given to the Path Relinking (PR) methodology, which is an important extension of SS. PR has been shown to be very successful both by itself and in connection with other metaheuristics, including multi-start procedures such as GRASP. In the next paper we unify the terminology for SS and PR, defining the common central role of the Reference Set of solutions in both methods. Interesting PR applications also are found in subsequent papers of this volume.

Fred Glover prefaced the volume "Tabu Search Methods for Optimization" with the paper "Tabu Search - Wellsprings and Challenges". Since I conceived the current volume as an extension of that one, I took the liberty of using the same title to make my intentions apparent from the very beginning. Fred's paper starts with a discussion about the origins of the name Tabu Search. Clearly, the word 'tabu' evokes more interesting and colorful associations than 'scatter' but, on the other hand, in the context of optimization, the word 'scatter' provides a meaningful first approximation to one of the most important principles of the method.

SS was motivated by the idea of using a systematic process to enable solution combinations to meet the desired characteristics or restrictions. Nowadays it is a well established method within the metaheuristic community and, as we show in the following sections, it has been successfully applied to a wide range of optimization problems. However, as observed in Section 3, general awareness of the method still lags behind that of other population-based methods such as genetic algorithms and evolutionary strategies, thus another goal of this volume is to help foster a fuller understanding of SS and its potentials.

## 2. Origins and Evolution

The Scatter Search methodology was first introduced in 1977 (Glover, 1977) as a heuristic for integer programming, based on strategies presented at a management conference held in Austin, Texas in 1967. Given the popularity of the so called genetic algorithms (GAs), also introduced in the seventies and also based on maintaining and evolving a set of solutions, several papers (Glover 1994a, 1994b, 1995) have been devoted to clarify their different origins, perspectives and their common potentials. GAs were initially proposed as a mechanism to perform hyperplane sampling, rather than optimization and over the years they have morphed into a methodology whose primary concern is the solution of optimization problems. In contrast, scatter search was conceived as an extension of a heuristic in the area of mathematical relaxation, which was designed for the solution of integer programming problems: surrogate constraint relaxation. However, links between the approaches have increased in recent years and we hope that both methods evolve in closer harmony.

In the original proposal Glover described scatter search as a method that uses a succession of coordinated initializations to generate solutions. He introduced the *reference set* (RefSet) of solutions and several guidelines, including that the search takes place in a systematic way as oppose to the random designs of other methods (e.g. GAs). The approach was conceived to begin by identifying a convex combination of the reference points. This central point, together with subsets of the initial reference points, was then used to define new sub-regions. Thereupon, analogous central points of the sub-regions were examined in a logical sequence. Finally, these latter points were rounded (in a broad sense, depending on the solution representation) to obtain the desired solutions.

As stated in Laguna and Martí (2003), Scatter Search was never applied or discussed again until 1990, when it was re-introduced at the EPFL Seminar on Operations Research and Artificial Intelligence Search Methods (Lausanne, Switzerland). An article based on this presentation was published in 1994 (Glover 1994b) in which new implementation details are given and the range of application is expanded to nonlinear, binary and permutation problems. The procedure is coupled with Tabu Search (TS), using forms of adaptive memory and aspiration criteria to influence the selection of points in a reference set consisting of several subsets. The concept of weighted combinations is introduced as the main mechanism to generate new trial points on lines that join reference points. This version of Scatter Search emphasizes linear searches and the use of weights to sample points from the line. Moreover, it introduces the concepts of combining high quality solutions with diverse solutions and the structured weighted combinations to handle discrete optimization problems directly.

Glover, Kelly and Laguna (1995) describe SS as a link between early Tabu Search and Genetic Algorithm ideas. They explore the nature of connections between these methods and show a variety of opportunities for creating hybrid approaches. In this line, Glover (1995) showed that there are benefits to be gained by going beyond a perspective constrained too tightly by the connotation of the term “genetic”. The author proposes concepts and strategies (especially in the context of structured combinations) not yet exploited in the genetic tradition, and that exhibit special properties for exploiting combinatorial optimization problems.

In 1998 Glover published the *Scatter Search template* (Glover, 1998). This paper presents an algorithmic description of the method and can be considered a milestone in the SS literature, since many different applications were developed after it. In some ways this version is a simplification of the previous one, but it incorporates many implementation and algorithmic details that sparked the interest of researchers and practitioners.

This version of the method generates a starting set of solution vectors to guarantee a critical level of diversity and applies heuristic processes designed for the problem considered as an attempt to improve these solutions. Then, a subset of the best vectors (in terms of quality and diversity) is designated to be reference solutions. New solutions are created by means of structured combinations of subsets of the current reference solutions and the heuristic processes applied above are used again to improve the new solutions. Finally, a collection of the “best” improved solutions is added to the reference set. The notion of “best” is once again broad; making the objective value one among several criteria for evaluating the merit of newly created points. These steps are repeated until the reference set does not change.

This template incorporates three notable features:

- Its structured combinations are designed with the goal of creating weighted centers of selected subregions.
- Strategies for selecting particular subsets of solutions for combination are introduced. These strategies are typically designed to make use of a type of clustering to allow new solutions to be constructed "within clusters" and "across clusters".
- The method is organized to use improving mechanisms that are able to operate on infeasible solutions, removing the restriction that solutions must be feasible in order to be included in the RefSet.

The fact that the mechanisms within scatter search are not restricted to a single uniform design allows the exploration of strategic possibilities that may prove effective in a particular implementation. These observations and principles lead to the well known template that consists of the following five methods.

1. Diversification Generation Method
2. Improvement Method
3. Reference Set Update Method
4. Subset Generation Method
5. Solution Combination Method

The interplay among these methods constitutes the core of an SS algorithm according to this template. Details about these methods appear in the next paper of this volume titled "Principles of Scatter search".

In order to summarize the methodological contributions of SS and PR, we also refer to the paper by Reeves and Yamada (1999) that includes a study of the landscape in connection with several metaheuristics.

Laguna and Martí (2003) provide detailed descriptions of the five methods mentioned above to three classes of problems with different solution representation: permutation vectors, continuous and binary variables. The authors also address unconstrained and constrained problems to provide specific implementations of the methodology to these classes of problems. They introduce C code that implements both basic and advanced search mechanism, such as:

- Dynamic RefSet Updating
- RefSet Rebuilding
- RefSet Tiers
- Diversity Control
- Subset Generation Method
- Use of Memory
- Path Relinking

The "Principles of Scatter Search" paper summarizes these mechanisms and provides insights to efficient implementations of associated strategies. In line with our goal of establishing a standard, all the papers in this volume follow the descriptions and notation used in this initial paper. I would like to thank the authors for their effort to follow this cannon.

## 3. Applications

In this section we include all the scatter search and path relinking implementations developed up to now. To establish a standard of quality, we restricted our attention to published papers (and book chapters) and we do not include technical reports, unless they have been already accepted for publication. Therefore, our references, as far as we know, are a complete list of the published material in SS and PR.

We maintain the web site http://www.uv.es/~rmarti/scattersearch in order to keep this list of references updated. Table 1 classifies them according to specific problems or areas of application. We also have added the theoretical or methodological papers (most of them mentioned in the previous section) to make an exhaustive list.

| Description | References |
|---|---|
| Methodology | Glover (1998), Binato et al. (2001), Glover (1994a-1999), Glover, Laguna and Martí (2003a-b, 2004a-b) Greistorfer (2004), Greistorfer and Voss (2004) Laguna (2002), Laguna and Martí (2003), Martí et al. (2004), Reeves and Yamada (1999), Resende and Ribeiro (2002, 2003b) |
| Assignment | Alfandari et al. (2001), Alfandari et al. (2004), Cung et al. (1997), Martí et al. (2000), Oliveira et al. (2003), Yagiura et al. (2002), Yagiura et al. (2004) |
| Binary Problems | Amini et al. (1999) |
| Clustering / Selection | Cotta (2004), García-López et al.( 2004), Scheuerer and Wendolsky (2004), Ribeiro et al. (2003) |
| Coloring | Hamiez and Hao (2002) |
| Commercial Soft. | Laguna and Martí (2002) |
| Continuous | Fleurent et al. (1996), Herrera et al. (2004), Trafalis et al. (1996), Ugray et al. (2002) |
| Graph Drawing | Laguna and Martí (1999) |
| Graph Problems | Alvarez et al. (2001), Bastos et al. (2001), Cavique et al. (2001), Dell'Amico et al. (2004, Piñana et al. (2004), Souza et al. (2003), Xu et al. (2000), Festa et al. (2002), Ribeiro and Rosseti(2002), Ribeiro et al. (2002), Zhang and Lai (2004) |
| Knapsack | Da Silva et al. (2004), Díaz et al. (2004) |
| Linear Ordering | Campos et al. (1999), Campos et al. (2001) |
| Mixed Integer Prog. | Glover et al. (2000) |
| MultiObjective | Beausoleil (2001), Beausoleil (2004) |
| Neural Networks | Kelly et al. (1996), Laguna and Martí (2000), El-Fallahi and Martí (2003), El-Fallahi et al. (2004) |
| p-Median | Díaz and Fernández (2004), García-López et al. (2003), Pérez et al. (2004) |
| Permutation Problems | Campos et al. (2003), Martí, Laguna and Campos (2004) |
| Routing | Chu et al. (2004), Corberán et al. (2002), Greistorfer (1999), Greistorfer (2001), Rego (2000), Resende and Ribeiro (2003a) Chiang and Russell (2004) |
| Scheduling | Aiex et al. (2003), Debels et al. (2004), Nowicki and Smutnicki (2004a-b), Reeves and Yamada (1998), Yamashita et al. (2004) |
| Software Testing | Sagarna and Lozano (2004) |

Table 1. SS and PR published papers

If we classify the contributions in Table 1 according to the year of publication, we can study the impact of the SS methodology over the last ten years. Figure 1 shows the frequency of publications from 1994 until now (including the papers in this volume). It also depicts the cumulative frequency, which shows the dramatic increase in the number of publications dealing with SS.

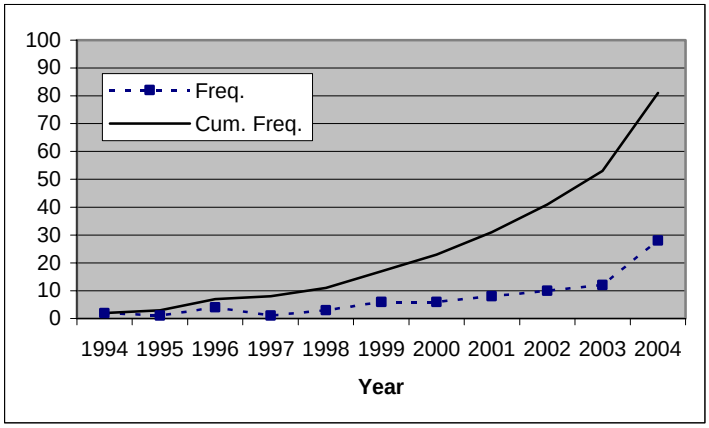


Figure 1. Number of SS publications

It should be noted that we are restricting our attention to published papers in journals or books, therefore, the quantities in Figure 1 are a lower bound on the real number of works in this area. If we want to measure the global impact of the SS methodology, we should consider not only the published papers, but also the technical reports, conference presentations and even projects under development.

A simple first approach to measuring the global impact of a methodology could be to perform a search over the Internet. We have used four well known search engines: *Alltheweb*, *Google*, *Msn* and *Yahoo*. We made three different queries: "tabu search" optimization, "genetic algorithms" optimization and "scatter search" optimization. Table 2 summarizes the number of entries found by each engine for each query. We can see that the impact of the Scatter Search methodology is still moderate compared with the other two solving procedures considered (TS and GAs present a number of links several orders of magnitude larger than those achieved by SS). On the other hand, GAs have about eight times more hits than Tabu Search.

| Search Engine | TS | GA | SS |
|---|---|---|---|
| *Alltheweb* | 7,180 | 48,962 | 737 |
| *Google* | 17,800 | 121,000 | 1,870 |
| *Msn* | 3,241 | 21,277 | 325 |
| *Yahoo* | 18,800 | 122,000 | 1,290 |

Table 2. Number of search results

## 4. The current special volume

The papers in this volume cover a wide range of SS and PR implementations. Regarding complexity, we can find basic designs as well as advanced implementations, while from a hybridization viewpoint we can find "pure" SS designs and hybrid algorithms, in which the SS elements are combined with other metaheuristics, such as tabu search or GAs.

When considering advanced strategies in a metaheuristic framework, the goal of improving performance is often in conflict with the goal of designing a procedure that is easy to implement and fine tune. Advanced designs generally translate into higher complexity and additional search parameters. This is certainly true for some of the designs included in this volume.

The hybrid applications included in this volume show that other metaheuristics can be improved when combined with SS. However, some SS advanced elements are excluded in these particular studies, and the effect that may be achieved by incorporating these elements remains an open question. The key idea

is to identify the method with the required strategy and to justify the selection of this method from another metaheuristic.

We have grouped the papers in this volume into the following categories:

- Foundations
- Nonlinear Optimization
- Optimization in Graphs
- Parallel Optimization
- Prediction and Clustering
- Routing
- Scheduling

We hope that they cover an area wide enough to attract the interest of the researchers in the metaheuristic field. Since we have experimented with SS for a number of years, we have learned some valuable lessons that are the result of implementing ideas and engaging on extensive experimental testing. As stated in Laguna and Martí (2003), the development and implementation of metaheuristic procedures usually entails a fair amount of experimentation and reliance on past experiences. Metaheuristic methodology is based on principles and not necessarily on theory that can be spelled out with theorems and proofs. In other words, as *Don Quijote* would say, for metaheuristics "the proof of the pudding is in the eating." Consequently, our primary goal in this volume has been to provide practical implementations to solve hard optimization problems. We know that a great deal remains to be learned about Scatter Search and we hope that the successes of the different contributions reported in this volume give us insights to develop and learn more about this amazing methodology.

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