Cooperative Games: the Unobservable Routing and the Communication Networks Queuing games

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We consider *cooperative games with transferrable utilities*. Games are defined by a respective set N of n players and a *characteristic function* V(·) that returns the cost for any coalition of players in N. *Sub-additivity* of the characteristic function guarantees that the total cost is minimized if all players join forces to form a single coalition that contains all players, called the *grand coalition.* Sub-additivity implies *economies of scope* meaning that gains due to cooperation are possible. We pose the question of how to fairly allocate the cost V(N) among the players of N. More specifically, we enquire the *core* of the game: a cost allocation is in the core if no coalition has an incentive to quit the grand coalition. A central first question when investigating the core is whether it is non-empty. The literature proposes two sets of conditions for answering this question. In this talk we present what we believe is a novel additional set of conditions that guarantees the non-emptiness of the core.

We consider games where each potential player of N is associated with its identity and a vector of quantitative properties. A coalition is defined by the identity of its members. The cost inflicted by a coalition, may depend only on the size of the coalition, and the vectors of properties of its members, but otherwise it is independent of the identities of the players within the coalition. This assumption allows us to consider also virtual coalitions, i.e., coalitions of any size which are not necessarily subsets of N. We call such games *regular games*. Many games considered in the literature are regular games. We propose a new property for regular games: for that sake, for any integer m≥1, let the *m-cloning* of player i of N, be m players identified as (i,1),(i,2),…,(i,m) - all having the same vector of properties as player i.

Definition: The characteristic function V of a regular game G=(N,V), is *homogeneous of degree one* if for any integer m ≥1, and any subset S of N, the cost of the coalition that consists of the m-cloning of each player i in S, is mV(S).

Homogeneity of degree one means that when m disjoint coalitions, associated with exactly the same set of vectors of properties, cooperate, they cannot do better than when acting individually. At the same time, they do not disturb each other. The resulting cost of cooperation is just m times the cost of an individual coalition. This in fact means lack of economies of scale. This property leads to our main result:

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Theorem: If the characteristic function V of a regular game G=(N,V) is sub-additive and homogeneous of degree one, then the core of the game is non-empty.

As examples we present two cooperative queuing games based on systems that consist of a set of n servers N={1,2,…,n}, where server i is associated with an exponential service rate μ_i, and a Poisson arrival process at a rate λ_i, with $\lambda_i<\mu_i$. The first game refers to a system called *the unobservable routing with outsourcing*, where the controller of N may re-route the customers to any of the existing servers in N, or alternatively to an external service provider. In this system all customers (servers) require (provide) the same type of service. The objective of the controller is to minimize the long run average cost, which consists of the steady state congestion cost plus the outsourcing cost. The second game refers to a system called *the capacity split with outsourcing*, and it relates to a communication network, where n channels, each is designated to a certain type of incoming data (as opposed to the first system), are responsible for transmitting the incoming data. The controller may reallocate all or part of the total capacity among the n channels, so that each channel is capable to transmit its incoming data. The remaining capacity, which is not allocated to the servers, is outsourced to other communication networks. The two systems without outsourcing, have been solved about 20 years ago by Bell and Stidham (1983) and Kelly (1978), respectively. We use these solutions to solve the optimal re-routing and capacity allocation when outsourcing is possible. Thereafter, we consider the two cooperative games generated by these systems, looking for core cost allocations. The two known sets of conditions for proving the non-emptiness of the core turn out to be non-applicable here. However, we show that these two games fall within the framework of regular games with a sub-additive and homogenous of degree 1 characteristic function, guaranteeing that their core is non-empty.

References

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