

Analysis of the Maximum Level Policy in a Production–Distribution System

Claudia Archetti⁽¹⁾ *Luca Bertazzi*⁽¹⁾ *Giuseppe Paletta*⁽²⁾
M. Grazia Speranza⁽¹⁾

(1) *Department of Quantitative Methods, University of Brescia, Italy,*

(2) *Department of Economics and Statistics, University of Calabria, Italy*

1 Introduction

We study an integrated system in which one commodity is produced at a production facility and shipped to several retailers over a finite time horizon. Shipments from the production facility to the retailers are performed by a fleet of vehicles. Each vehicle has a given transportation capacity. The commodity is consumed by the retailers in a deterministic and time-varying way. Each retailer determines a minimum and a maximum level of the inventory of the commodity and can be visited at most once for each time instant and several times during the time horizon. The production policy, the retailers replenishment policies and the transportation policy have to be determined so as to minimize the total system cost. The cost includes the fixed and variable production costs at the production facility, the inventory costs at the production facility and at the retailers, and the routing costs. We study two different types of replenishment policies: *Maximum Level* (ML) and *Order-Up to Level* (OU). In the former type of policy, if the retailer i is visited at time t , the quantity delivered to i is such that the level of the inventory at i is not greater than its maximum level. In the latter type of policy, if the retailer i is visited at time t , then the quantity shipped is such that the level of the inventory at i reaches exactly its maximum level. OU policies are inspired, in a deterministic setting, by the classical stochastic order-up-to level policy, widely studied in inventory theory. The problem we study is NP-hard, both when the ML and the OU policies are applied, since it reduces to the VRP in the class of instances in which the time horizon is composed of one time instant only, the fixed and variable production costs are zero, the inventory costs are zero, and all the retailers need to be served.

Our first aim is to investigate the computational complexity of the problem when the transportation is outsourced, both when the ML and the OU policies are applied. We prove that the problem is NP-hard in both cases. We also show the worst-case performance of the OU policy

with respect to the ML policy, proving that, in the worst case, the ratio between the optimal cost of the former policy and the optimal cost of the latter tends to infinity. Therefore, the solution obtained by solving the problem with the OU policy can be very bad if used as a heuristic solution of the problem with the ML policy.

The problem with the OU policy has been heuristically solved in Bertazzi, Paletta and Speranza (2005), while the problem with the ML policy has never been solved. Since it is very complex and the exact solution would be impractical in general, we design a heuristic algorithm. We decompose the problem into two subproblems, one concerning the production and the other concerning the distribution. We first solve the distribution subproblem and then the production subproblem. The subproblem concerning the production is optimally solved, while the subproblem concerning the distribution is solved by applying a constructive heuristic algorithm in which at each iteration one retailer is inserted in the solution. For each retailer, a problem, which is a generalization of the dynamic lot-size problem with time-varying capacity constraints, is optimally solved. The exact algorithm we propose is based on properties of the optimal solution and on feasibility and dominance relations among partial solutions. Finally, the heuristic solution obtained by hierarchically solving the two subproblems is improved by applying a procedure which coordinates production and distribution. To evaluate the performance of the heuristic, we design a branch-and-cut algorithm inspired by the one proposed in Archetti et al. (2007) for the solution of the problem in which one vehicle only can be used at each delivery time instant and compare the optimal solution of this problem with the solution generated by the heuristic on the basis of specific randomly generated problem instances with one vehicle. The results show that the problem is really difficult as the branch and cut approach is not able to solve systematically instances with up to 20 nodes within a time limit of two hours. On the other hand, the heuristic algorithm proved to be really efficient giving small errors in a very short computational time. Finally, we compare the performance of the ML policy with respect to the OU policy on instances with a higher dimension and a fleet of more than one vehicle. Computational results, obtained on a large set of randomly generated problem instances, show that the ML policy significantly reduces the total cost with respect to the OU policy.

References

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