The Traveling Salesman Problem with Draft Limits

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Abstract

We introduce a new variant of the Traveling Salesman Problem inspired from applications within maritime transportation. The problem is to find a minimum cost tour of a set of ports, but includes draft limitation of some ports. This problem is henceforth referred to as the Traveling Salesman Problem with Draft Limits (TSPDL) and is an important subproblem in complex routing problems in maritime transportation.

The draft limits dictate the maximum draft of a vessel when entering (leaving) a port. The draft of a vessel is determined by the draft of the empty ship, plus a function of the load onboard the ship. Thus, a vessel's ability to enter a port is dependent of the load onboard. We can find several practical applications where draft limits are restrictions in maritime routing and scheduling problems, see e.g. Song and Furman (2010) and Christiansen et al. (2011).

The TSPDL is defined on an directed graph $\mathcal{G}=(\mathcal{V},\mathcal{A})$, where $\mathcal{V}=\{1,...,n\}$ is the set of nodes(ports) and $\mathcal{A}=\{(i,j):i,j\in\mathcal{V},i\neq j\}$ is the arc set. A symmetric cost c_{ij} is defined for each arc, while the draft limit l_j and demand d_j are given at each node $j\in\mathcal{V}$. We assume that all draft limits and demands are non-negative. Further, assume that the vessel leaves the depot (node 1) fully loaded and the demand of the depot is zero. We wish to find a Hamiltonian Circuit $\mathcal{G}^*=(\mathcal{V},\mathcal{A}^*)$ of $\mathcal{G}$, abiding all draft limits and demands, with the least cost $\sum_{(i,j)\in\mathcal{A}} c_{ij}$. A basic model for the TSPDL can be formulated by modifying one of the formulations from Gavish and Graves (1978) and adding draft limit constraints. The binary flow variable x_{ij}, $(i,j)\in\mathcal{A}$ equals 1, if the vessel sails from node i directly to node j, and 0 otherwise, while the network flow variable, y_{ij}, $(i,j)\in\mathcal{A}$, describes the load onboard the vessel when sailing arc (i,j).

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$$\text{Minimize} \sum_{(i,j)\in\mathcal{A}} c_{ij} x_{ij} \;, \tag{1}$$

subject to

$$\sum_{i\in V} x_{ij} = 1, \qquad \forall j \in \mathcal{V}, \tag{2}$$

$$\sum_{j\in\mathcal{V}} x_{ij} = 1, \qquad \forall i \in \mathcal{V}, \tag{3}$$

$$\sum_{i\in\mathcal{V}} y_{ij} - \sum_{i\in\mathcal{V}} y_{ji} = d_j, \qquad \forall j \in \mathcal{V}\backslash\{1\}, \tag{4}$$

$$\sum_{j\in\mathcal{V}} y_{1j} = \sum_{j\in\mathcal{V}} d_j \;, \tag{5}$$

$$\sum_{i\in\mathcal{V}} y_{i1} = 0, \tag{6}$$

$$0 \le y_{ij} \le l_j x_{ij}, \qquad \forall (i,j) \in \mathcal{A}, \tag{7}$$

$$x_{ij} \in \{0,1\}, \qquad \forall (i,j) \in \mathcal{A}. \tag{8}$$

The objective function (1) minimizes the costs of the tour. Constraints (2) and (3) impose that the in-degrees and out-degrees, respectively, are equal to one. The load flow conservation constraints at each node are given in constraints (4). Constraints (5) and (6) ensure that the cycle starts and ends in node 1. The draft limits are stated in constraints (7). Finally, the formulation involves binary requirements (8) on the arc variables.

Variants of this model including valid inequalities will be discussed. Preliminary results together with future plans will be presented.

References

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