

A branch-price-and-cut method for the min-max k -windy rural postman problem

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Introduction

Given the edges of a graph, the *postman problem* consists of determining for a vehicle (postman) a shortest route that traverses all edges at least once. Each edge can be traversed in either direction and, when an edge is traversed more than once, the vehicle is said to be deadheading after its first traversal. The *rural postman problem* is the same as the postman problem except that the edges are divided into two sets: required edges that must be serviced, and optional edges that can be used for deadheading. The *windy rural postman problem* is a generalization of the rural postman problem where the distance along an edge depends on the direction used to traverse it. The *k -windy rural postman problem* is the same as the windy rural postman problem except that k vehicles are available to service all the required edges. Each vehicle must start and end at a depot and the objective consists of minimizing total distance. Finally, the *min-max k -windy rural postman problem (MM- k WRPP)* is the same as the k -windy rural postman problem except that the goal is to minimize the length of the longest route, aiming at balancing the workload between the vehicles.

The MM- k WRPP was introduced in [6] and proved to be *NP*-hard. Currently, the state-of-the-art methodology for solving the MM- k WRPP is branch-and-cut [2, 3]. Our main contribution is to develop an efficient branch-price-and-cut method that outperforms the best known branch-and-cut method [3] for certain classes of MM- k WRPP instances.

Model and solution method

We model the MM- k WRPP as an integer program involving two variable types. For each vehicle v , a variable z_v specifies the length of the route assigned to vehicle v . For each feasible route r and each vehicle v , a binary variable y_{rv} indicates whether or not route r is assigned

to vehicle v . These variables are generated as needed by column generation. Given an upper bound U on the optimal value (provided by a heuristic), a route is feasible if its total length does not exceed U . The model includes one set partitioning constraint per required edge to ensure its service, one constraint per vehicle to assign a route to it, one constraint per vehicle to compute z_v in function of the variables y_{rv} , and constraints ordering the vehicles according to their route length (the first vehicle has the longest route, the last the shortest one). The latter constraints help breaking symmetry in the branch-and-bound search tree.

To solve this model, we develop a branch-price-and-cut method (i.e., a column generation method embedded into a branch-and-cut method) similar to the one proposed in [5] for the vehicle routing problem with time windows. Column generation is used to solve the linear relaxations encountered in the search tree. There is one column generation subproblem per vehicle, which corresponds to an elementary shortest path problem with one resource constraint (ESPPRC). The resource limits the length of a route to be less than or equal to U . In the underlying network, every required edge is represented by two nodes, one for each traversal direction, and an arc between two such nodes represent the shortest path linking the corresponding two required edges/directions in the original graph. Because the ESPPRC is NP -hard, we rather solve a relaxation of it that yields weaker lower bounds. This relaxation, called *ng*-paths [1], allows cycles $i_0 - i_1 - \dots - i_q - i_0$ if node i_0 does not belong to the predefined neighborhood of at least one of the nodes i_1, \dots, i_q . This relaxation is solved by a labeling algorithm. However, because it is also not easy to solve, a fast tabu search algorithm is applied first to generate columns.

To tighten the lower bounds, we generate three types of cutting planes: subset row inequalities [7] limited to three set partitioning rows (required edges), aggregate R-odd cut inequalities [2, 4], and some kind of lifted odd-cycle inequalities based on three arcs [8]. At the root node of the branch-and-bound search tree, we may also impose a lower bound on the value of z_1 , the length of the route of the first vehicle. This lower bound is found by adjusting temporarily the upper bound U to a lower value through a dichotomic search. When the associated linear relaxation is infeasible, then the lower bound is set to $U + 1$ and the upper bound is reset to its initial value. When branching is required, three types of decisions can be imposed: assigning a required edge to a vehicle, fixing the sum of the flows on the arcs linking two required edges, and fixing the flow on a specific arc.

Computational results

We tested our branch-price-and-cut method on the MM- k WRPP instances of [2, 3]. There are 144 instances involving between 4 and 78 required edges. Each instance was tested for

five values of k , namely, $k = 2, 3, 4, 5, 6$. Within a one-hour time limit, our method solved to optimality 112, 115, 119, 123 and 119 instances with $k = 2, 3, 4, 5, 6$, respectively. In comparison, the best known branch-and-cut method of [3] solved 144, 130, 124, 113 and 101 instances. Hence, our method performs better than this branch-and-cut method for the 5- and 6-vehicle cases. For the cases with less vehicles, the branch-and-cut method remains the best method. Detailed computational results will be presented at the conference.

References

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