Improved Column Generation for Highly Degenerate Master Problems

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Branch-and-price, that is, column generation embedded into a branch-and-bound scheme is established as a leading solution methodology for many large-scale integer programming programs. The column generation approach is a method used for solving linear problems with a huge number of variables. This classical decomposition method (Dantzig and Wolfe 1960, Gilmore and Gomory 1961, 1963) well exploits modeling aspects. It solves a *restricted* master problem RMP comprising a subset of the variables and generates new ones as needed by solving a structured pricing subproblem SP. However, it often suffers from tailing-off effect, this being mainly due to the degeneracy of the master problem solutions. Fifty years later, we propose the *Improved Column Generation* method (ICG) which takes advantage of degeneracy (Desrosiers et al. 2011). It additionally decomposes the master problem constraints based on the values of the current basic solutions and is thus dynamic by design. The *row-reduced* restricted master problem, namely rMP, not only discards variables otherwise considered by the column generation process, but also reduces the number of constraints. The row-size of rMP is the number of non-zero basic variables. Therefore, ICG reduces the computational effort to evaluate the basis inverse.

Non-basic variables are characterized as compatible or not with respect to rMP, and compatible ones allow for a strict decrease of the objective function (non-degenerate pivots). Therefore, two subproblems are used: cSP for the compatible ones, and rSP (itself solved by column generation) to price out the incompatible ones. Pricing subproblem cSP is the original SP augmented with a set of linear constraints imposing compatibility requirements. It selects compatible columns as long as they are useful for non-degenerate pivots in rMP. When the minimum reduced cost of the compatible columns is greater than zero (or a specified threshold), rSP is rather solved. It looks for a convex combination of incompatible columns such that, again, the objective function strictly decreases when they all enter the basis. In that case, the row-size of rMP is updated.

Combining column generation and dynamic updates of the row-size of the master problem takes advantage of both its formulation and the degenerate values of its solutions. A special case of the proposed methodology is the *Improved Primal Simplex* method (IPS) used for the solution of highly degenerate linear programs (Elhallaoui et al. 2010a, Raymond et al. 2010). The *Dynamic Constraints Aggregation* method (DCA) used for the solution by branch-and-price of set partitioning problems (Elhallaoui et al. 2005, 2010b) is a specialized implementation of ICG for vehicle routing and crew scheduling applications. On highly degenerate instances, recent computational experiments with these algorithms show that the row-reduction of a problem might have a larger impact on the cpu time than the classical column-reduction. As for any Simplex or column generation algorithms, more research is needed on ICG, mainly on implementation strategies. Amongst these are the moment for an update of the row-partition of rMP, efficient solvers for cSP and rSP, which pricing subproblem to call, and much more on relaxed compatibility and restricted incompatibility.

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