Analysis and Branch-and-Cut Algorithm for the Time-Dependent Travelling Salesman Problem

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Given a graph whose arc traversal times vary over time, the Time-Dependent Travelling Salesman Problem (TDTSP) consists in finding a Hamiltonian tour of least total duration covering the vertices of the graph. The contribution of this paper is twofold. First, we describe a lower and upper bounding procedure that requires the solution of a simpler (yet NP-hard) Asymmetric Travelling Salesman Problem (ATSP). In addition, we prove that this ATSP solution is optimal for the TDTSP if all the arcs share a common congestion pattern. Second, we formulate the TDTSP as an integer linear programming model for which valid inequalities are devised. These inequalities are then embedded into a branch-and-cut algorithm that is able to solve instances with up to 40 vertices.

Key words: travelling salesman problem; time dependence; lower and upper bounds; branch-and-cut.