Natural and Extended formulations for the Time-Dependent Travelling Salesman Problem

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Abstract

Consider a graph G=(V,A), where V ={1,2,..,n} and A={(i,j): i,j=1,..,n, $i \neq j$}. The Time-Dependent Travelling Salesman problem (TDTSP) is to find a minimum cost Hamiltonian circuit, starting and ending on node 1, where arc costs depend on its position in the tour. Thus, to each arc (i,j) in A and each possible position h of the arc in the tour we associate a cost c_{ij}^h. Clearly, an arc (1,j) leaving the depot can be only in position 1 and an arc (i,1) entering the depot can be only in the last position. Every other arc (i,j), i,j≠1, can be located in positions h=2,…,n-1.

Several formulations for the TDTSP described in the literature can be obtained by using the binary variables z_{ij}^h for all $(i,j) \in A$ and $h = 1,..,n$, indicating whether or not arc $(i,j) \in A$ is in the h^{th} position of the circuit. A formulation that uses only the z_{ij}^h variables is called a *natural* formulation. We strart by reviewing the well known formulation by Picard and Queyranne (1978), whose main feature is that it uses, as a subproblem, an exact description of the n-circuit problem. An n-circuit is a circuit with n arcs which may repeat nodes and even arcs.

Then we introduce new models for the problem. These models are built on two features: i) use a stronger subproblem - a n-circuit subproblem with the additional constraint that a given node is not repeated in the circuit and ii) combine these subproblems for all nodes. The new formulation will use extra variables (besides the z_{ij}^h variables) and thus, it will

fall in the class of so-called *extended* formulations. Although the model has more variables and constraints than the original PQ model, the results given from our computational experiments show that the linear programming relaxation of the new model gives, for many of the instances tested, gaps that are close to zero. Thus, the new model is worth investigating for solving TDTSP instances, either by using it within available ILP packages or as the subject of determining what inequalities are implied by the linear programming relaxation of the new model and are not redundant in the linear programming relaxation of the Picard and Queyranne model. In fact, this is also a topic of our work and we will relate a set of such inequalities with the inequalities described in Abeledo at al. (2010). We should emphasize that our goal is not to obtain a formulation that provides fast lower bounds. The main aim is to propose a formulation that produces very tight lower bounds permitting us to get more insight on the structure of the problem (eg., projected inequalities, which subproblems are strong for a given commodity). However, in the conclusion, we will suggest some alternative ways for handling the proposed formulation.