Solving a Vehicle Routing Problem with a non-linear load dependant cost function *

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Abstract

In this paper we consider an exact solution approach for a vehicle routing problem with a non-linear cost function. In particular, we examine the vehicle routing problem with time windows (VRPTW) with objective to minimize the overall travel cost minus a bonus gained for each route based on the quality and quantity of the load of that route. This new objective function turns out to a non-continuous stepwise linear function.

The problem at hand comes from a real-life application in the waste collection industry. Specifically, a set of vehicles have to collect paper of various quality from a number of customers in the area of Copenhagen, Denmark. The vehicles start and end at a depot and are allowed to perform multiple trips during the time horizon. Each load of paper collected is priced depending on its quality, e.g., white paper with no print is more valuable than old newspapers, and a bonus is obtained that is subtracted from the overall travel costs. The paper quality is defined to be equal to the least quality paper load collected. Since the paper quality influence the objective the vehicles are allowed to only collect some of the total load at a customer at a time thereby avoiding to decrease the current paper quality of the load on vehicle. However, all paper must be collected at some point and we enforce that a vehicle must take all available paper of a given quality. All customers must be visited within a time window and waiting is allowed. For simplicity we assume that the fleet is unlimited, hence we disregard the possibilities of multiple trips. Also we simply split customers with paper in several qualities into distinct pickups.

Column generation algorithms are suitable for vehicle routing problems with non-linear objectives since the non-linearity often can be handled when solving the sub-problem with a dynamic programming algorithm. In many cases (including the one considered in this paper) this is be done by using a strict dominance criterion, i.e., using equality signs when comparing labels. However, this is not a very strict dominance criterion and it may be beneficial to try an mimic the behaviour of the less than or equal signs that are used for comparing labels in the classical linear case.

The contribution of this paper is the development of an improved dominance criterion (instead of an equality sign on all resources during comparison of two labels) and fathoming rules to be used in the dynamic programming algorithm when solving the sub-problem of the column generation algorithm. The improvements are based on the definition of envelope functions for the non-linear term in the objective function. The envelope functions are used to calculated upper and lower bounds for the cost of the partial paths (labels) when comparing labels in the dynamic programming algorithm. The idea of envelope functions is similar to previous presented dominance enhancements such as Chabrier [1]

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where customer resources were relaxed, and by Jepsen et al. [2] when handling subset-row inequalities in the dynamic programming algorithm.

In our implementation we have a set partitioning master problem that is solved in a column generation fashion to obtain a valid lower bound. Our main goal is to examine the improvements in the dynamic programming algorithms, hence we do not consider cuts or branching in the master problem. Our experimental results are carried out on an extension of the Solomon benchmarks. Preliminary experiments show that the envelope functions have a significant positive impact on the solution time.

References

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