The Minimum Labeling Hamiltonian Cycle Problem

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The Minimum Labeling Hamiltonian Cycle Problem (MLHCP) is defined on an undirected and unweighted graph $G = (V; E)$, where V is the vertex set, $n = |V|$, and E is the edge set. Let $\delta(e)$ be a label (or colour) associated with edge e, let C be the set of all labels, $\zeta(k) = \{e \in E \mid \delta(e) = k\}$, and $p = |C|$. A label $k \in C$ is said to be used if an edge $e \in \zeta(k)$ belongs to the cycle. The aim is to determine a Hamiltonian cycle with the least number of labels. Labels can represent transporters, types of telecommunication fibers or production technologies.

In this presentation, we also consider two variants of the MLHCP by associating a cost c_e to each edge $e \in$ E. In the first problem, the tour length cannot exceed a given limit. This problem will be referred to as the Minimum Labeling Hamiltonian Cycle Problem with Length Constraint (MLHCPLC). The second variant, the Label Constrained Traveling Salesman Problem (LCTSP), minimizes the tour length while imposing an upper bound m on the number of labels. To our knowledge, these two problems have not previously been studied.

We will present first mathematical models followed by valid inequalities and polyhedral results for the three problems just described. More precisely, we will give some relations between the solutions of the TSP and those of the MLHCP. Then we will show how any valid constraint for the TSP can be generalized to the MLHCP and demonstrate that some of the valid inequalities proposed are facet-defining for the MLHCP. Next, we will describe a heuristic and a branch-and-cut algorithm in which the valid inequalities are embedded. Finally, we report computational results for the MLHCP and the two variants. Results show that the branch-and-cut algorithm is capable of solving a majority of instances involving up to n = 200 nodes and p = 200 labels. Moreover the initial heuristic yields good results within a reasonable computing time when compared to the overall computational time.