The Split Delivery Capacitated Team Orienteering Problem

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Abstract

The Capacitated Team Orienteering Problem (CTOP) is the problem where a set of potential customers, each of them associated with a demand and a profit, is given. The subset of customers to be served by a fleet of capacitated vehicles has to be identified in such a way that the profit collected is maximized, while satisfying constraints on the maximum time duration of each route and the vehicle capacity constraints. In the CTOP each customer can be served by one vehicle at most. In this paper we study the Split Delivery Capacitated Team Orienteering Problem (SDCTOP), where a customer may be served by more than one vehicle.

The CTOP is the capacitated version of the Team Orienteering Problem (TOP), a well known problem that belongs to the class of routing problems with profits. The CTOP has been introduced in Archetti et al. [1] where heuristics and a branch-and-price algorithm were proposed. The CTOP is motivated by several applications where a carrier can select actual customers from a set of potential customers on the basis of their profit. Transactions between shippers and carriers more and more take place through the web. Databases of transportation demands from shippers are posted on the web in different forms and the carriers may access those databases and evaluate these demands. Typically carriers have to bid on the most profitable customers. It is crucial for the carriers to identify the customers that are most profitable, given their capacitated fleet of vehicles.

When split deliveries are allowed each customer may be served by more than one vehicle. In the Split Delivery Vehicle Routing Problem (SDVRP), that is the traditional Vehicle Routing Problem where split deliveries are allowed, it is well known that split deliveries may reduce the cost of routes and the number of vehicles used (see Archetti, Savelsbergh, Speranza [2], where it is shown that the cost may be halved). In this paper we show that the profit collected in the SDCTOP is, in the best case, as large as twice the profit collected in the CTOP.

We then present an exact branch-and-price algorithm and two heuristics for the solution of the SDCTOP. A first heuristic makes use of the columns generated during the execution of the branch-and-price to obtain heuristic solutions. A mixed integer linear programming (MILP) problem is solved on subsets of promising columns. The other heuristic is an hybrid algorithm that combines a tabu search scheme with an improvement phase where an ad hoc MILP problem is solved to intensify the search. We show the effectiveness of the proposed approaches on benchmark instances and on a new set of instances that also allow us to evaluate the impact of split deliveries.

References

[1] Archetti, C., Feillet, D., Hertz, A., Speranza, M.G. (2009), The capacitated team orienteering and profitable tour problem, *Journal of the Operational Research Society* 60, 831-842.

[2] Archetti, C., Savelsbergh, M., Speranza, M.G.(2006), Worst-case analysis for split delivery vehicle routing problems, *Transportation Science* 40, 226-234.