An ILP Local Search Algorithm for the Capacitated m-Ring-Star Problem

Paolo Toth
DEIS, University of Bologna, Italy.
paolo.toth@unibo.it

Zahra Naji-Azimi
Department of Management, Imam Reza University, Mashhad, Iran.
z.naji-azimi@imamreza.ac.ir

Majid Salari
Department of Industrial Engineering, Ferdowsi University of Mashhad, Mashhad, Iran.
msalari@ferdowsi.um.ac.ir

April 12, 2011

The Capacitated m-Ring-Star Problem (CmRSP) can be described as follows: a mixed graph $G = (V, E \cup A)$ is given, where V is the set of nodes, $E = \{\{i, j\} : i, j \in V, i \neq j\}$ is the set of edges (undirected arcs) and A is the set of arcs. The node set V is defined as $V = \{0\} \cup U \cup W$, where node 0, U and W represent, respectively, the central depot, the set of customers and the set of Steiner nodes. For each customer $i \in U$, $C_i \subset U \cup W$ denotes the subset of nodes to which customer i can be connected. The arc set A is defined as $A = \{(i, j) : i \in U, j \in C_i\}$. Each edge $e \in E$ has a non negative *visiting* cost c_e and each arc $(i, j) \in A$ has a non negative *allocation* cost d_{ij}.

We refer to a simple cycle consisting of a subset of nodes and the depot as a *ring*. If a customer is visited by a ring or allocated to a node of a ring, we consider the customer as *assigned* to that ring. Two input parameters m and Q are given, representing, respectively, the number of rings and the capacity of each ring. We assume $mQ \geq |U|$, so that a feasible solution for the considered CmRSP instance always exists.

A solution of the CmRSP is feasible if each customer is assigned to exactly one ring, no Steiner node is used more than once, and the number of customers assigned to a ring does not exceed the capacity Q. We impose as well that a Steiner node can belong to a ring only if one or more customers are allocated to it. The goal of the CmRSP is to find m rings with the minimum global cost, given by the sum of the *visiting* and *allocation* costs. CmRSP is known to be $\mathcal{NP}$-hard, since it is a generalization of the Symmetric Traveling Salesman Problem (TSP), arising when $m = 1$, $Q = |U|$, $W = \phi$, $A = \phi$.

In this talk we propose a new heuristic approach which combines both heuristic and exact ideas to solve the problem. Considering the general scheme of the Variable Neighborhood Search approach, the algorithm incorporates an Integer Linear Programming based improvement method which is applied whenever the heuristic procedure is not able to enhance the quality of the current solution. Extensive computational experiments, on benchmark instances of the literature and on a new set of instances, have been performed to compare the proposed approach with the most effective methods from the literature. The results show that the proposed algorithm outperforms the other approaches. Indeed, the proposed algorithm is able to obtain, within reasonable computing times, most of the optimal solutions and to improve many of the best known results proposed in the literature for CmRSP.