DETERMINACION DEL SIGNO DE UNA FORMA CUADRATICA.

Sea la forma cuadrática:

$$F(x_1, x_2, x_3, ..., x_n) = a_{11}\, x_1^2 + a_{22}\, x_2^2 + ... + a_{nn}\, x_n^2 +$$
$$2\, a_{12}\, x_1 x_2 + 2\, a_{13}\, x_1 x_3 + ... + 2\, a_{(n-1)\,n}\, x_{n-1} x_n$$

La matriz es:

$$H = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ .. & .. & ... & .. \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}$$

Los menores principales:

$H_1 = a_{11}$

$$H_2 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$H_3 = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$H_{n-1} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n-1} \\ a_{21} & a_{22} & \cdots & a_{2n-1} \\ .. & .. & \cdots & .. \\ a_{n-1\,1} & a_{n-1\,2} & \cdots & a_{n-1\,n-1} \end{pmatrix}$$

$$H_n = H = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ .. & .. & \cdots & .. \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

$H_1 > 0$, $H_2 > 0$, , ... $H_{n-1} > 0$, $H_n > 0$ DEFINIDA POSITIVA

$H_1 < 0$, $H_2 > 0$, , ... $H_{n-1(par)} > 0$, $H_{n(impar)} < 0$ DEF. NEGATIVA

$H_1 > 0$, $H_2 > 0$, , ... $H_{n-1} = 0$, $H_n = 0$ SEMIDEFINIDA POSITIVA

$H_1 < 0$, $H_2 > 0$, , ... $H_{n-1} = 0$, $H_n = 0$ SEMIDEF. NEGATIVA

$$HL(x,\lambda)=\begin{pmatrix} H_xL(x^*,\lambda^*) & -J^t h(x^*) \\ -Jh(x^*) & 0 \end{pmatrix}=\begin{pmatrix} \frac{\partial^2 L}{\partial x_1^2} & \cdots & \frac{\partial^2 L}{\partial x_1 \partial x_n} & -\frac{\partial h_1}{\partial x_1} & \cdots & -\frac{\partial h_m}{\partial x_1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2 L}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 L}{\partial x_n^2} & -\frac{\partial h_1}{\partial x_n} & \cdots & -\frac{\partial h_m}{\partial x_n} \\ -\frac{\partial h_1}{\partial x_1} & \cdots & -\frac{\partial h_1}{\partial x_n} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -\frac{\partial h_m}{\partial x_1} & \cdots & -\frac{\partial h_m}{\partial x_n} & 0 & \cdots & 0 \end{pmatrix}..$$

Recordemos que la matriz orlada de orden r (Grados de libertad: m+1,.., n) se construye tomando el menor principal conducente de orden r de H_rL (r primeras filas y columnas) orlado con las r primeras filas de $J^th(x)$ y las r primeras columnas de Jh(x).

La forma cuadratica restringida será **definida positiva** si:

$$(-1)^{\mathbf{m}} \left|H_r^* L\right| > 0 \qquad \forall r,\ r=m+1,...,n$$

definida negativa

$$(-1)^{\mathbf{r}} \cdot \left|H_r^* L\right| > 0 \qquad \forall r,\ r=m+1,...,n$$