

Optimizar $F(x,y,z) = x y z$

s.a.: $x + y + z = 9$

Lagrangiana:

$$L(x,y,z,\lambda) = x y z + \lambda (9 - x - y - z)$$

$$\nabla L = \begin{pmatrix} y z - \lambda \\ x z - \lambda \\ x y - \lambda \\ 9 - x - y - z \end{pmatrix} = 0$$

Operando: $s_1 = (3, 3, 3, 9)$

$$H L = \begin{pmatrix} 0 & z & y & -1 \\ z & 0 & x & -1 \\ y & x & 0 & -1 \\ -1 & -1 & 1 & 0 \end{pmatrix}$$

$$H L(3,3,3,9) = \begin{pmatrix} 0 & 3 & 3 & -1 \\ 3 & 0 & 3 & -1 \\ 3 & 3 & 0 & -1 \\ -1 & -1 & 1 & 0 \end{pmatrix}$$

$$H_2 L (3,3,3,9) = \begin{pmatrix} 0 & 3 & -1 \\ 3 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix} \quad H_2 L > 0$$

$$H_3 L (3,3,3,9) = H L (3,3,3,9) = \begin{pmatrix} 0 & 3 & 3 & -1 \\ 3 & 0 & 3 & -1 \\ 3 & 3 & 0 & -1 \\ -1 & -1 & 1 & 0 \end{pmatrix} \quad H_3 L < 0$$

$(-1)^r H_r > 0$: DEFINIDA NEGATIVA

$(-1)^2 H_2 > 0$ $(-1)^3 H_3 > 0$ SI

$(-1)^m H_r > 0$: DEFINIDA POSITIVA

$(-1)^1 H_2 > 0$ $(-1)^1 H_3 > 0$ NO

$s_1 = (3, 3, 3, 9)$ MÁXIMO

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VARIABLES
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```
X, Y, Z, F;
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```
X.L=1;
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```
Y.L=1;
```

```
Z.L=1;
```

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EQUATIONS
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```
OBJ, R1;
```

```
OBJ..      F =E= X*Y*Z;
```

```
R1..      X + Y + Z =E= 9;
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MODEL LAG01 /ALL/;
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SOLVE LAG01 USING NLP MAXIMIZING F;
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MODEL   LAG01                OBJECTIVE  F
TYPE    NLP                  DIRECTION  MAXIMIZE
SOLVER  CONOPT              FROM LINE 12

**** SOLVER STATUS          1 NORMAL COMPLETION
**** MODEL STATUS          2 LOCALLY OPTIMAL
**** OBJECTIVE VALUE                27.0000

                LOWER      LEVEL      UPPER      MARGINAL
---- EQU OBJ                .          .          .          1.000
---- EQU R1                 9.000    9.000    9.000    9.000

                LOWER      LEVEL      UPPER      MARGINAL
---- VAR X                 -INF     3.000    +INF     .
---- VAR Y                 -INF     3.000    +INF    2.2370E-6
---- VAR Z                 -INF     3.000    +INF    2.2370E-6
---- VAR F                 -INF    27.000    +INF     .

**** REPORT SUMMARY :          0      NONOPT
                                0      INFEASIBLE
                                0      UNBOUNDED
                                0      ERRORS

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Interpretación de los multiplicadores de Lagrange.

$$\begin{aligned} & \text{Max } f(x,y) \\ \text{s.a.: } & h(x,y)=b \end{aligned}$$

$$\frac{dL}{db} = \lambda$$

o lo que en términos de incrementos, sería:

$$\Delta F \approx \lambda \Delta b$$

$$\begin{aligned} \textit{Maximizar} \quad & F(x,y,z) = x y z \\ \text{s.a.: } & x + y + z = 9.1 \end{aligned}$$

Solución:

$$F' = F + \Delta F = 27 + 9 (0,1) \approx \mathbf{27.9}$$

VARIABLES

X,Y,Z,F;

X.L=1;Y.L=1;Z.L=1;

EQUATIONS

OBJ, R1;

OBJ.. F =E= X*Y*Z;R1.. X + Y + Z =E= 9.1;

MODEL LAG01B /ALL/;

SOLVE LAG01B USING NLP MAXIMIZING F;

MODEL	LAG01B	OBJECTIVE	F		
TYPE	NLP	DIRECTION	MAXIMIZE		
SOLVER	CONOPT	FROM LINE	12		
****	SOLVER STATUS	1	NORMAL COMPLETION		
****	MODEL STATUS	2	LOCALLY OPTIMAL		
****	OBJECTIVE VALUE		27.9100		
		LOWER	LEVEL	UPPER	MARGINAL
----	EQU OBJ	.	.	.	1.000
----	EQU R1	9.100	9.100	9.100	9.201
		LOWER	LEVEL	UPPER	MARGINAL
----	VAR X	-INF	3.033	+INF	.
----	VAR Y	-INF	3.033	+INF	9.7901E-7
----	VAR Z	-INF	3.033	+INF	9.7901E-7
----	VAR F	-INF	27.910	+INF	.