Waiting for a kidney in the País Valencià (Spain)

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Abstract

Bulk service queues and simulation tools have been used to analyze the congestion of the waiting list for renal transplants in the País Valencià, Spain, from January 1997 to December 1999. The main characteristics of the queue model are the arrivals of patients onto the waiting list, the process of cadaveric donations, and the number of kidneys, either one or two, provided by each donor. Bayesian inference is considered in order to estimate the acceptance, donation and transplantation rates. Predictions are provided for the number of new people entering the waiting list and for the number of donors and transplants during a period of time . A first measure of the whole congestion of the system, by comparing the acceptance rate with the transplantation rate, is also discussed.

KEY WORDS: Bayesian statistics; Health services; Queueing models; Renal transplants; Waiting lists.

1 Introduction

In this paper we present a first analysis of the congestion of the waiting list for renal transplants in the País Valencià, one of the seventeen autonomous regions into which Spain is divided. This study makes use of queueing models, Bayesian statistics and simulation tools.

Queueing systems occur any time *customers* demand a *service* from some facility with *servers*. They are the obvious probabilistic models when dealing with scenarios of congestion and blockages. Therefore, it seems very natural to think of a waiting list for a medical transplant as a queueing system: patients needing a kidney transplant wait in a queue to be served and leave the system when they receive a transplant.

Queueing models have a long tradition (Bailey, 1952 [1]) as a very useful tool for evaluating, at least approximately, the performance of health care systems in which waiting lists occur: appointment systems and waiting list management in outpatient clinics (Jackson et al., 1964 [2],

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Worthington, 1991, [3] etc.), managed markets for hospital treatment (Shwartz and Lenard, 1994, [4], Iversen, 2000, [5]), planning the capacity of emergency services (Ridge et al., 1998 [6]), designing, planning and staffing service units (Gorunescu et al., 2002 [7]), etc.. Queueing systems are also a valuable tool when analyzing waiting lists for transplants. Zenios (1999) [8] has developed a multiclass queueing model with reneging to represent a transplant waiting list and has also participated in many papers dedicated to kidney allocation problems in waiting lists in the United States. Su and Zenios (2002) [9] and Zenios et al. (2002) [10] are just some of his more relevant papers on the subject.

Waiting lists for medical transplants are such a very serious social and health care problem that they have attracted a great variety of quantitative studies with different objectives and technical procedures: David and Yechiali (1985) [11] analyze the decision problem associated with transplanting a kidney to a potential recipient within the general setting of the theory of optimal stopping; Davies and Roderick (1998) [12] use discrete event simulation to analyze the evolution of the number of patients needing a renal transplant in the United Kingdom (UK) over a period of 15 years; Smits et al. (1998) [13] use the competing risk method and survival tools in order to estimate the chance of a transplant within a period of time after registration; Gibbons et al. (2003) [14] analyse geographical differences in the total amount of time a patient waits for an organ in the framework of a general study of the National Academy of Sciences at the request of the US Congress.

This paper presents an analysis of the waiting list for renal transplants in the País Valencià within the framework of a single-server bulk service queueing model. This particular model allows us to jointly analyze the process by describing the number of patients with end-stage renal-disease who were accepted for a renal transplant and the main characteristics of the donor's source. This is a real-world problem and our essential information about it are data gathered within a given period of time: the number of patients entering the waiting list, the number of donors and the number of transplants. When using stochastic models to analyze real problems with data we need to use statistical methods in order to connect the real world of the data with the probabilistic one.

Compared to the consolidated tradition of Queueing Theory, the statistical analysis of queueing systems is a recent and unexplored field of research. Most of the papers in this area are in the framework of the frequentist approach but in recent years there has been increasing interest in Bayesian methods. See Bhat et al. (1997) [15] for a very good review on this topic, Armero and Bayarri (1999) [16] for a discussion of the advantages and difficulties of Bayesian analyses of queues and the recent paper by Ausín et al. (2003) [17] for a practical Bayesian modeling of the distribution of the length of the stay in hospital in order to optimize the number of beds in service.

Bayesian statistical methods are used to estimate the parameters of the proposed model: the expected number of new patients accepted onto the waiting list for a kidney transplant per year (usually known as the *acceptance rate*), the *effective donation rate* and the proportion of effective donors that provide their two kidneys. Also, we will take advantage of this knowledge in order to provide a first measure of the congestion of the system by comparing the *acceptance rate*with the *transplantation rate* through the *traffic intensity* in the system.

This paper is organized into seven sections, this Introduction being the first one. The rest of the paper is organized as follows. Section 2 is devoted to the available data. We explain the difficulties involved in working with our data and we present a brief summary of them. Section

3 deals with queueing models. We identify the main components of the waiting list in terms of queueing elements and propose a simple, easy and preliminary model that captures the main characteristics of the problem. In Section 4 we analyze the uncertainty involved in the admission of new patients onto the waiting list. The process of donations and transplants to recipients is discussed in Section 5. In these last two sections we pay special attention to predicting the number of daily entrances, effective donations and transplants. Section 6 combines the information provided by the arrivals and donations through the *traffic intensity*, the most basic measure of performance of the congestion in the system. The paper ends with a short Section devoted to some concluding remarks and future research.

2 The data

The National Organization of Transplants (Matesanz and Miranda, 1996 [18]) is the Spanish institution which coordinates the donation and transplantation of all types of organs, tissues and bone marrow. This network has one coordinator in each autonomous region, who is usually a member of the Health Authority of the Regional Government. The País Valencià (see Figure 1) is one of the Spanish autonomous regions that has taken on the management of all health services. In 1992, the Regional Government created the Registry of Transplants with the aim of computerizing, managing and keeping all the information relative to transplants in the País Valencià. This registry was structured into three sub-registries: people waiting on the list for a transplant, transplanted patients and donors. Each sub-registry is also divided according to the type of organ or tissue.

The data in this paper have been supplied by this registry. In particular, we have collected the daily number of patients entering the waiting list, the daily number of donors and the number of kidneys, one or two, provided by each donor. They were registered from January 1997 to December 1999. We ruled out data before January 1997 because in 1996 a new transplant hospital joined the network (making a total of four in the País Valencià) and that modified the management of the waiting lists.

During the period under analysis (1095 days in all), 531 new patients were accepted onto the renal transplant waiting list. 564 kidney transplants coming from 323 donations were carried out. This apparent discrepancy between donations and kidneys is due to the fact that a donor can provide one or both kidneys. Specifically, we have noticed 241 double donations and only 82 single. As for transplantations, 293 out of the 564 kidneys were transplanted to incident patients whereas the remaining where grafted onto prevalent ones.

Table 1 shows the daily frequency distribution (in terms of percentages) of the number of patients admitted onto the waiting list, the number of donations and of transplants. It is worth mentioning that on approximately 70% of the days there are no arrivals, but that there are a few days (19 days in all) with more than three inputs. The sample mean of the daily number of patients accepted for a transplant is 0.4849 patients/day, nearly 1 patient every two days, and the variance is computed as 0.8643 patients^2/day. There are almost 75 per cent of the days without donations (and, consequently, without transplants). The sample mean and variance of the daily donations is 0.2949, the same value for both quantities. Notice also the special behaviour of the sample distribution of the transplants with regard to the higher percentages in the even values. This peculiar property is due to the fact that double donations are more frequent than singles ones. The sample average of the daily number of transplants is 0.5151 transplants. Its variance is 0.9447 transplants^2, nearly twice as big as its mean.

Figure 2 displays the cumulative daily number of arrivals, donations and transplants recorded between January 1997 and December 1999. The growth rate corresponding to donations seems fairly constant over time. In contrast, the cumulative polygonal for arrivals and for transplants indicates a more irregular behaviour. In the case of the arrivals, there are some clustered data because they are just registered on working days and also as a consequence of the heterogeneity of the different supplier hospitals.

As we shall explain later, the daily number of arrivals and donations are the stochastic processes that provide the queueing structure for our analysis. We have used the Kolmogorov-Smirnov test in order to easily check the assumption of the Poisson distribution for modelling them. As we suspected, there was no evidence against that hypothesis for the donation process (*observed ks* = 0.0055, *p-value* = 1.0000, almost a textbook example), but there was some for the process of arrivals (*observed ks* = 0.0822, *p-value* = 0.0011). The long right tail in the frequency distribution of the daily arrivals seems to be principally responsible for this.

3 The queueing model

Simple waiting models are usually defined by specifying the arrival pattern, the service mechanism and the queueing discipline. Following the specific vocabulary of queueing systems, people entering the waiting list for a kidney are the *customers*. An arrival occurs when a patient-*customer* is admitted onto the waiting list for a graft. When a patient is transplanted he/she immediately leaves the system.

The identification of the service is not so evident. Remember that in queueing theory, the total time that a customer spens in the system has to be expressed as the sum of the queueing time and the service time. In our problem, we consider the total time spent in the system by a patient as his/her waiting time in the list for a kidney. Consequently, we need to distinguish the service time and the queueing time. Since the creation of the Registry of Transplants in 1992 there has always been a waiting list in the País Valencià. So there is always a patient waiting for a kidney and for this reason, we can identify the queueing time of a patient as the time elapsed from his arrival to the time the kidney of the preceding patient arrives. His(her) service time would be the time elapsed from the arrival of the kidney assigned to the previous patient to the time *his (her)* kidney arrives. Figure 3 shows a diagram illustrating the service mechanism with regard to the donation process. This representation is also appropriate when the same donor provides two kidneys. See Figure 4 for an extensive diagram of this bulk service. In a queueing system, the discipline is the way in which the customers are selected for service. There are four transplant team centers in the País Valencià. Each available kidney is allocated to a recipient depending on clinical and geographical criteria that follow a system from "the inside to the outside" according to the hierarchy in Figure 5.

The activity and relationships between the patients (as possible recipients) of these centers and the donations could be represented in terms of a managed internal market for service transplant as presented in Iversen (2000)[5]. The queueing discipline is a decisive element when analyzing the length of individual waiting times. In contrast, it is irrelevant when considering other types of variables characterizing the congestion of the queue. The traffic intensity, in which we are interested in this paper, is a clear example of this last statement. Thus, from this point on, we will not pursue it.

In accordance with the foregoing comments and the description of the data in Section 2, we start this study by selecting the structure of the Markovian queueing model $M/M^X/1$ as appropriate for the analysis of the waiting list for renal transplants: new patients enter the waiting list daily following, approximately, a Poisson process with unknown *entrance rate* λ; donors donate their kidneys according to a Poisson process with unknown *donor rate* μ; each donor provides $X = 2$ kidneys with unknown probability θ, or, $X = 1$ kidney with the remaining probability $1 - \theta$.

4 Joining the waiting list

Let us consider the Poisson process $\{N_A(t), t \geq 0\}$ with parameter λ that describes the daily number of new patients accepted for a renal transplant in the País Valencià. The parameter λ represents the *daily entrance rate* to the waiting list. It is unknown and we are going to estimate it by using Bayesian statistical tools. We assume a prior situation of prior ignorance and express this state of lack of knowledge by means of a flat prior distribution, $p(\lambda) = \lambda^{-1}$. Given λ, $N_A(1)$ is distributed as a Poisson random variable with parameter λ. Therefore, the likelihood function of λ corresponding to arrival data will be:

$$l(\lambda) \propto \prod_{i=1}^{1095} \frac{\lambda^{a_i}\, \mathrm{e}^{-\lambda}}{a_i!} \propto \lambda^{531} \mathrm{e}^{-1095\lambda},$$

where a_i denotes the number of new patients registered on day i and $t_a = \sum_i^{1095} a_i = 531$ is the total number of new arrivals onto the waiting list registered from January 1997 to December 1999.

Bayes theorem allows us to compute the posterior $p(\lambda \mid \text{data})$ distribution of λ easily , a Gamma distribution $\text{Ga}(\lambda \mid 531,\, 1095)$, which contains all the current information on the *daily entrance rate*. In particular, the posterior expectation is $\text{E}(\lambda \mid \text{data}) = 0.48493$ new arrivals every day with a posterior variance $\text{D}^2(\lambda \mid \text{data}) = 0.00044$. The central 95% posterior interval (0.44456, 0.52704) is very accurate due to the small variability obtained in the estimation of λ. During the years 1997, 1998 and 1999, the population in the País Valencià was around 4.0372 millions. Consequently, the expected *acceptance rate* (i.e. expected new patients taken on per year) reached 43.84 per million population (pmp from now on) with (40.20, 47.65) pmp as the corresponding central 95% posterior interval.

As important as this inference, we can deal with the prediction of the number of new patients $N_A(t)$ admitted in a period of t days, or the time T_A between two consecutive arrivals onto the waiting list. They are observable quantities that can give complementary information about the process of arrivals onto the waiting list. Let us start by first analyzing $N_A(t)$. Given λ, $N_A(t)$ is a Poisson random variable with parameter $\lambda\, t$; its posterior predictive distribution will be:

$$p(N_A(t) = k \mid \text{data}) = \int p(N_A(t) = k \mid \lambda)\, p(\lambda \mid \text{data})\, \mathrm{d}\lambda = \text{Gp}(k \mid 531, 1095, t), \qquad (4.1)$$

where $\text{Gp}(k \mid \alpha, \beta, t)$ stands for the probability at k of a GammaPoisson distribution defined as:

$$\text{Gp}(k \mid \alpha, \beta, t) = \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha + k)}{k!} \frac{t^k}{(\beta + t)^{k+\alpha}},\ \ k = 0, 1, 2, \ldots \qquad (4.2)$$

Figure 6 displays the posterior predictive distribution of the daily number $N_A(1)$ of new arrivals onto the waiting list. Its expectation is $\mathrm{E}(N_A(1) \mid \text{data}) = 0.4849$ patients, nearly one person every two days, with a posterior variance $\mathrm{D}^2(N_A(1) \mid \text{data}) = 0.4854$. It is worth pointing out the great variability involved in this predictive stage in comparison with that in the estimation process.

Now we analyze T_A, the elapsed time between two consecutive admissions onto the waiting list. Given λ, T_A is exponentially distributed with parameter λ. Thus, its posterior predictive density is easily computed as:

$$p_{T_A}(t \mid \text{data}) = \int p_{T_A}(t \mid \lambda)\, p(\lambda \mid \text{data})\, \mathrm{d}\lambda = \mathrm{Gg}(t \mid 531, 1095, 1), \tag{4.3}$$

where $\mathrm{Gg}(t \mid \alpha, \beta, n)$ stands for the density at t of a GammaGamma distribution as defined as:

$$\mathrm{Gg}(t \mid \alpha, \beta, n) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \frac{\Gamma(\alpha + n)}{\Gamma(n)} \frac{t^{(n-1)}}{(\beta + t)^{\alpha + n}}, \; t > 0. \tag{4.4}$$

For this predictive, we have computed $\mathrm{E}(T_A \mid \text{data})$=2.066 days and $\mathrm{D}^2(T_A \mid \text{data})$=4.2846. In other words, we expect approximately one new patient on the waiting list every two days, but with great irregularities.

5 Donors and transplants

Let us first discuss the process corresponding to donors. As in the previous Section, we consider that daily donations occur according to a Poisson process $\{N_D(t), t \geq 0\}$ with unknown parameter μ, the *daily effective donation rate*. With the same type of inferential process as carried out to estimate λ and the current data of donations (a total of 323 from January 1997 to December 1999) we have obtained the Gamma distribution Ga(323, 1095) as the posterior of μ. Therefore, we can compute $\mathrm{E}(\mu \mid \text{data}) = 0.2950$ donations/day with a small variability, $\mathrm{D}^2(\mu \mid \text{data}) = 0.0003$. If we think in terms of the *effective donation rate*, defined as the annual number of donations pmp, we will expect a total number of 26.6707 donations. As an indicator of its performance, we have computed (23.8393, 29.6544) as the central interval with probability 0.95.

The same procedure for predicting the number of new arrivals onto the waiting list, and the time between two consecutive arrivals, applies here to compute the posterior predictive distribution of the number $N_D(t)$ of effective donations managed in a period of t days, and the time T_D elapsed between two sequential effective donations. Thus, $N_D(t) \mid \text{data} \sim$ Gp(323, 1095, t) and $T_A \mid \text{data} \sim$ Gg(325, 1095, 1).

Figure 7 shows the posterior predictive distribution of $N_D(t)$ in the case of $t = 1$ day. As a summary of its behaviour we have computed its expectation as 0.295 donors and variance equal to 0.2953 donors^2. In the case of the time between two consecutive donations we would have, a mean period of about 3.4 days between two consecutive donations with 11.63 days^2 as the variance, again, a large quantity as is usual when dealing with predictions.

Now, let us discuss the transplants. Since each donor can provide one or two of his (her) two kidneys, the process of donors and transplants are not coincident. What is more, the number of transplants $N_T(t)$ in a period of t days depends on the number of effective donations $N_D(t)$ in this period and the probability θ of a double donation. Specifically, during a given period of time, the number of transplants is the sum of the number of donors plus the number of double donations.

This last quantity is a binomial random variable with parameters, the number of donations and the probability of having a double donation, that is,

$$N_T(t) \mid N_D(t), \theta \sim N_D(t) + \text{Binomial}(N_D(t), \theta). \tag{5.5}$$

Therefore, we first estimate θ and then we address the prediction of the number $N_T(t)$ of transplants in a period of t days. As θ is a probability, we can carry out a basic conjugate analysis in order to make inferences about it. We choose a non-informative prior distribution in the family of the Beta distributions. Specifically, we select the prior $p(\theta) = Be(1, 1)$. The likelihood function of θ corresponding to the data of effective donations (323 in all, 241 with two kidneys and 82 with only one) is proportional to $\theta^{241}\,(1-\theta)^{82}$. Consequently, the posterior distribution of θ will be a Beta distribution with parameters 242 and 83. From this posterior distribution we compute the expected value of θ as $\mathrm{E}(\theta \mid \text{data})$=0.7450, the variance as $\mathrm{D}^2(\theta \mid \text{data})$=0.0006, and (0.6959, 0.7905) as a 95% credible interval. In short, we have found out that approximately 3 out of every 4 effective donations are double.

Now we can return to the prediction of $N_T(t)$. As this variable depends on $N_D(t)$ and θ, we could express its posterior predictive distribution as follows:

$$p(N_T(t) = k \mid \text{data}) = \sum_{j=[\frac{k+1}{2}]}^{k} \int p(N_T(t) = k \mid \theta,\, N_D(t) = j,)\, p(\theta,\, N_D = j, \mid \text{data})\, \mathrm{d}\theta, \tag{5.6}$$

where $[x]$ denotes the largest integer $\leq x$. But since $N_D(t)$ and θ are posterior independent and $N_D(t)$ depends, in turn, on the *service rate* μ, we will have:

$$\begin{aligned} &p(N_T(t) = k \mid \text{data}) = \\ &\quad \sum_{j=[\frac{k+1}{2}]}^{k} \int p(N_T(t) = k \mid N_D(t) = j, \theta)\, p(N_D = j \mid \mu) p(\mu \mid \text{data}) p(\theta \mid \text{data})\, \mathrm{d}\theta\, \mathrm{d}\mu. \end{aligned} \tag{5.7}$$

This predictive does not have a closed expression and so we can use Monte Carlo integration in order to approximate it from a random sample $\{(\mu^{(i)},\, \theta^{(i)}), i = 1, \ldots, N\}$ of the posterior distribution of μ and θ ($\mathrm{Ga}(\mu \mid 323, 1095)$ and $\mathrm{Be}(\theta \mid 242, 83)$, respectively) as:

$$p(N_T(t) = k \mid \text{data}) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{j=[\frac{k+1}{2}]}^{k} p(N_T(t) = k \mid N_D(t) = j,\, \theta^{(i)}) p(N_D(t) = j \mid \mu^{(i)}). \tag{5.8}$$

This approximate distribution is presented graphically in Figure 8. It is interesting to point out the peaks in the even values of the distribution, which are a direct consequence of the great quantity of double donations. We have also calculated the mean and variance of the daily number of transplants as 0.5132 and 0.9454, respectively.

Finally, we can consider the daily *transplant rate* as defined as $\mu(1 + \theta)$. Because its posterior distribution does not have an analytic expression, we have approximated it by means of a simulated sample. Specifically, we have constructed a sample $\{\mu^{(i)}(1 + \theta^{(i)}), i = 1, \ldots, N\}$ from the posterior distribution of μ and θ. Figure 9 displays a kernel approximation of the posterior distribution of the daily *transplant rate*. As a naïve visual comparison between the rate of arrivals onto and departures from the waiting list, we have also included the draw of the posterior distribution

$\text{Ga}(\lambda \mid 531, 1095)$ of the *arrival rate*. Note the symmetrical shape of both distributions. We have approximated the posterior mean and variance of the daily *transplant rate* as 0.5146 and 0.00089, respectively. And also, the expected annual number of transplants ppm is 46.5146, with the interval (41.3636, 52.0041) as a 95% credible region of this rate.

6 The congestion

So far we have considered the arrivals and donations separately. From now on, we will combine both elements in order to provide the *traffic intensity*, the most basic measure of the congestion of the waiting list. This is a parameter that compares the *arrival rate* with the *transplant rate* in the form:

$$\rho = \frac{\lambda}{\mu(1+\theta)}. \tag{6.9}$$

The traffic intensity plays a remarkable role in Queueing Theory. Specifically, the queue will be stable (that is, the queue length will not go to infinity) if and only if $\rho < 1$. Figure 10 presents a graphical approximation to the posterior distribution of the traffic intensity with a vertical line indicating the frontier value $\rho = 1$. It has also been constructed from a random sample of the posterior $\text{p}(\rho \mid \text{data})$ which we have generated from the corresponding sample from the joint posterior distribution of λ, μ and θ. Its posterior expectation is $\text{E}(\rho \mid \text{data}) \approx 0.9452$ with variance $\text{D}^2(\rho \mid \text{data}) \approx 0.00457$. A credible interval with probability 0.95 is (0.8226, 1.0836). Also, we have computed, approximately, the posterior probability of the waiting list will eventually be stable as 0.796. This knowledge about ρ indicates a high level of congestion in the waiting list (McQuarrie (1983)[19] suggested the upper bound 0.9 as the limit to maintain quality in Health Services). It seems that, in the long run, the queue could reach the steady-state, but the evidence in favour of this assumption is not too strong. This information also suggests that the possible convergence to the equilibrium is very slow. In any case, from a managerial point of view, a very interesting feature would lie in predicting the temporal evolution of the number of patients on the waiting list in order to understand better the dynamic behaviour of the congestion. This information will be very valuable in any quantitative evaluation (resources, needs, costs, etc.) of the system. In Abellán et al. (2003)[20] there is a detailed analysis of this topic within the framework of several simulated scenarios of congestion.

7 Conclusions and future research

This paper is the result of the joint effort of a multidisciplinary team made up of epidemiologists and statisticians. It presents a simple analysis of a real problem which is tricky, complex and on which contains a lot of human suffering and happiness depends. It is, along with the paper by Abellán et al. (2003)[20], the first two results of our research devoted to statistically analyzing the waiting list for a renal transplant in the País Valencià.

At the beginning of the project, we dedicated most of our efforts to managing an appropriate data bank, to learn about several social and medical aspects of the problem, and to represent the dynamic of the waiting list through a probabilistic model. Our intention was to build, test and validate a simple and robust model that imitates the most basic features of the flow and interactions of the stochastic processes involved in the problem (the entrance of patients onto the waiting list and the arrival of donations).

Our first results are on the right track. In particular, the predictions in Abellán et al. (2003)[20] about the evolution of the number of patients waiting for a kidney seem to indicate a good performance of the model. This fact is also corroborated by the opinion of a group of nephrologists from the four Transplantation Units in the País Valencià who have decided to participate in the project.

We know that our work is in its early stages. Our main objective is to gain more knowledge about the problem. But we believe that the complexity of the subject suggests it should be tackled by making several complementary analyses. In particular, our more immediate efforts are dedicated to better understanding the arrival, donation and transplant processes in relation to the age, geographical area of residence and date of entrance of the recipients and donors.

From a technical point of view we intend to improve the queueing model by refining the Poisson assumption for the arrival process and also by incorporating the possibility that a patient needing a transplant will abandon the waiting list. We also know that we need to learn more about the simulation of stochastic systems, particularly discrete event simulation, in order to perform a more realistic and accurate simulation procedure.

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Figure 1: Spain and its autonomous regions. The País Valencià is the shaded area.

Daily arrivals	0	1	2	3	4	5	6	8
% of days	70.14	18.26	07.58	02.28	00.91	00.18	00.55	00.09
Daily donations	0	1	2	3				
% of days	74.34	22.28	02.92	00.46				
Daily transplants	0	1	2	3	4	5	6	
% of days	74.34	05.39	17.35	00.73	01.92	00.09	00.18	

Table 1: Frequency distribution of the daily number of arrivals, donors and transplants from January 1997 to December 1999

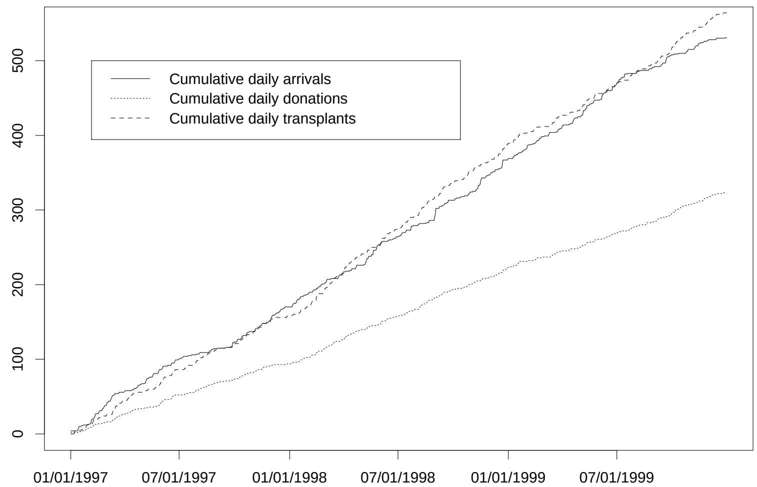

Figure 2: Cumulative number of daily arrivals, effective donations and transplants registered from January 1997 to December 1999.

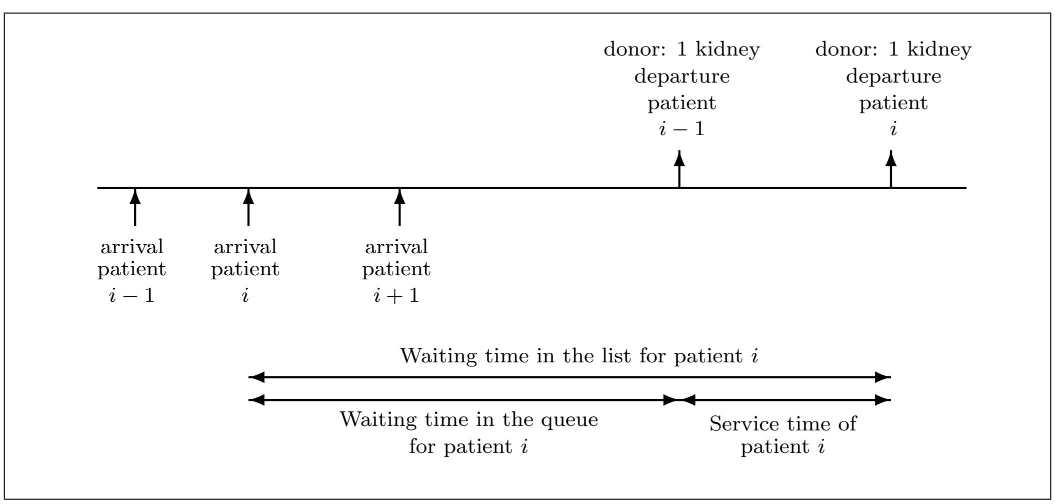

Figure 3: Graphical representation of the queueing time, service time and waiting time on the list by a patient when only one kidney is obtained from the donor.

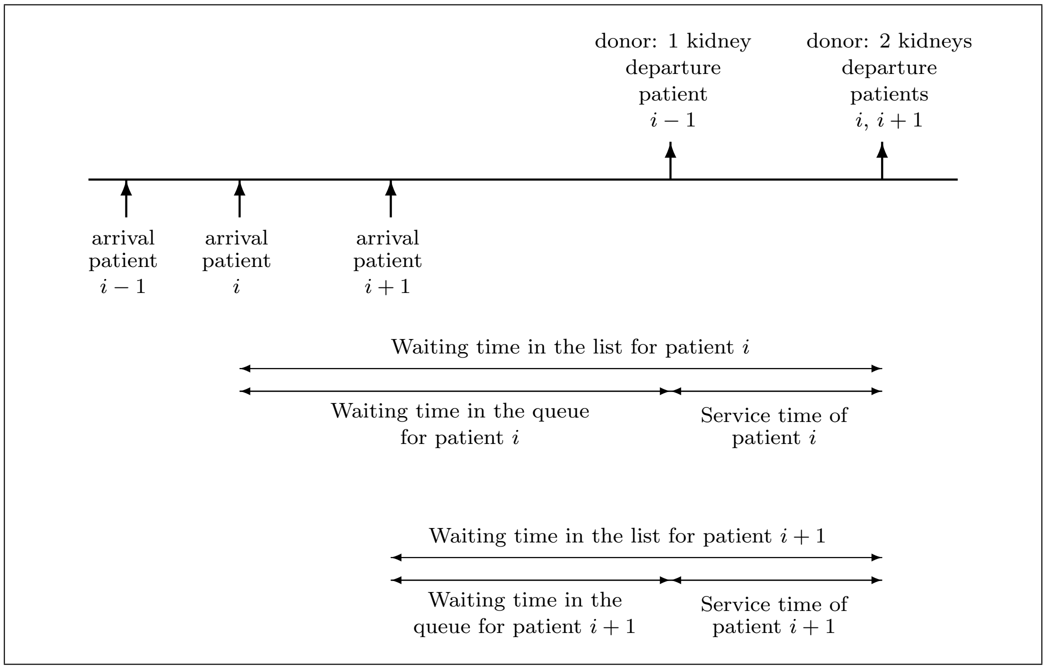

Figure 4: Graphical representation of the queueing time, service time and waiting time on the list by a patient when a donor provides two kidneys.

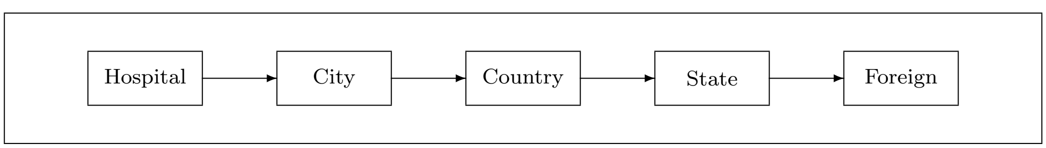

Figure 5: Geographical criteria of allocation of kidneys to recipients.

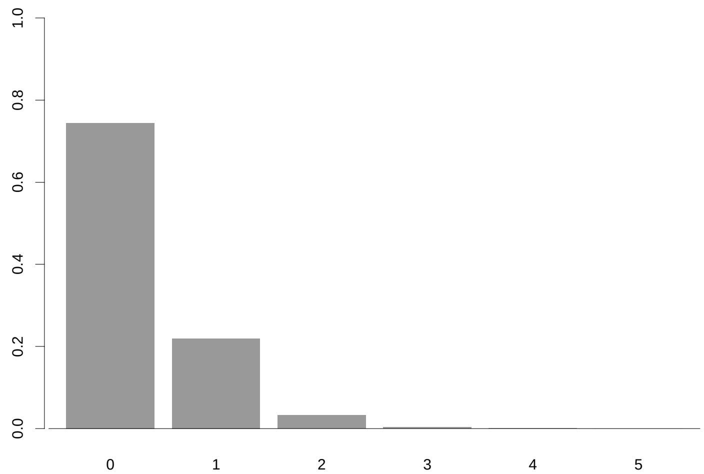

Figure 6: Posterior predictive distribution of the daily number of new arrivals onto the waiting list.

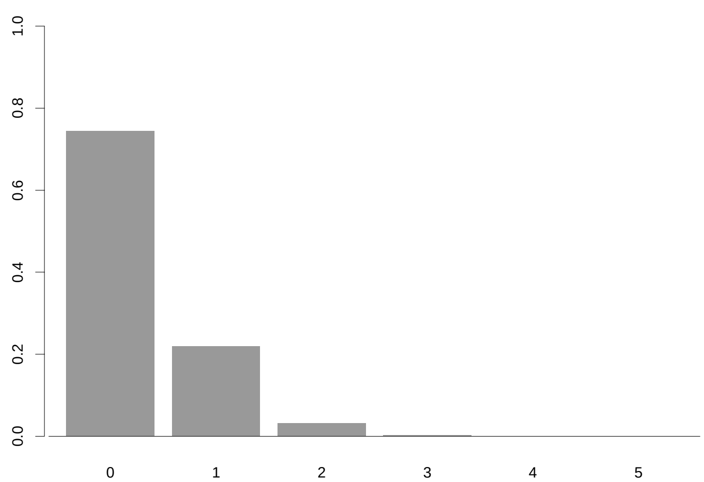

Figure 7: Posterior predictive distribution of the daily number of effective donations.

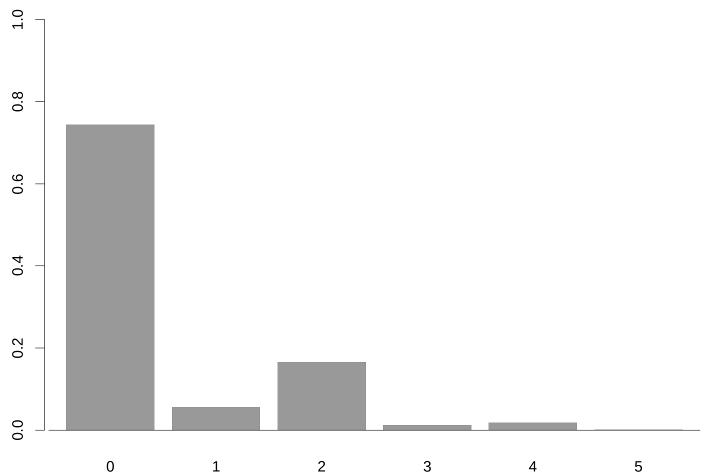

Figure 8: Approximate posterior predictive distribution of the daily number of transplants.

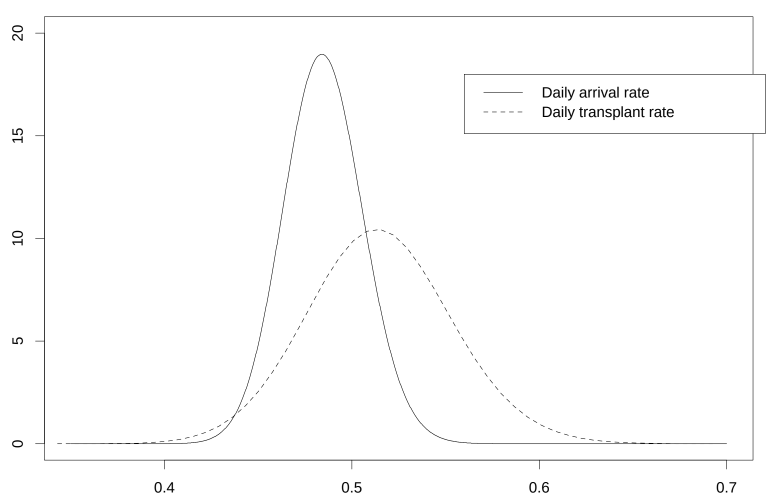

Figure 9: Posterior predictive distribution of the daily number of new arrivals onto the waiting list.

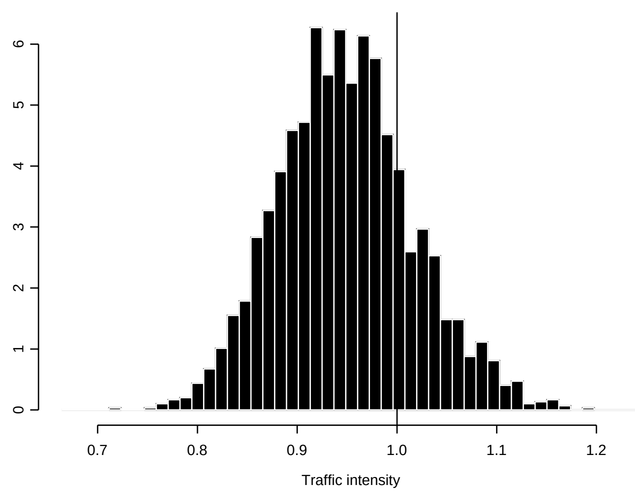

Figure 10: Approximate posterior distribution of the traffic intensity.