Multifluid hydrodynamics

(how to model relativistic “conduction” and “transfusion”)

Marco Antonelli

antonelli@lpccaen.in2p3.fr

CNRS & Laboratoire de Physique Corpusculaire, Caen

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Multifluid hydrodynamics

(how to model relativistic “conduction” and “transfusion”)

Work done with: L. Gavassino (Vanderbilt University), G. Camelio, B. Haskell (CAMK, Warsaw)

General theory:

- **Equilibrium thermodynamics** of a multifluid → arXiv:1906.03140
- Stability and causality of Carter’s multifluid → arXiv:2202.06760
- Multicomponent fluid with **bulk viscosity** → arXiv:2003.04609
- Dissipation in superfluids → arXiv:2012.10288 (vortices), arXiv:2110.05546 (heat & bulk viscosity)
- **Radiation hydrodynamics** (M1) as a Carter’s multifluid → arXiv:2007.09481

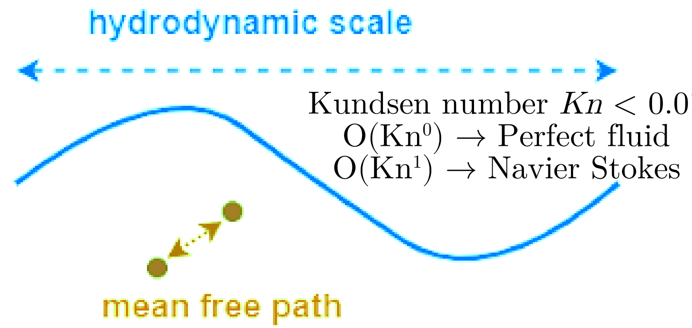
Some applications:

- **Glitches** in pulsars → arXiv:1710.05879 (glitch amplitude), arXiv:2001.08951 (glitch timescale)
- Effect of **bulk viscosity due to chemical reactions** in neutron star oscillations
  - → arXiv:2204.11809 (formalism)
  - → arXiv:2204.11810 (simualtions)

Reviews:

Gavassino & Antonelli *“Unified Extended Irreversible Thermodynamics and the stability of theories for dissipation”*
Andersson & Comer “Relativistic Fluid Dynamics” arXiv:gr-qc/0605010
Antonelli, Montoli, Pizzochero *“Insights into the physics of Neutron Star Interiors from Pulsar Glitches”*

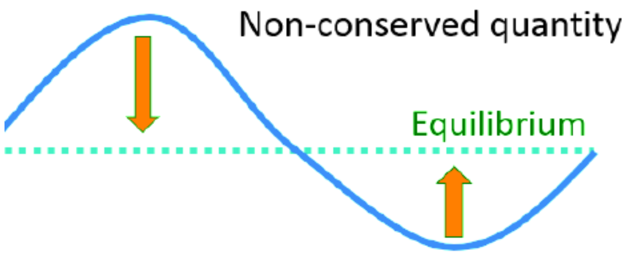
What is hydrodynamics?



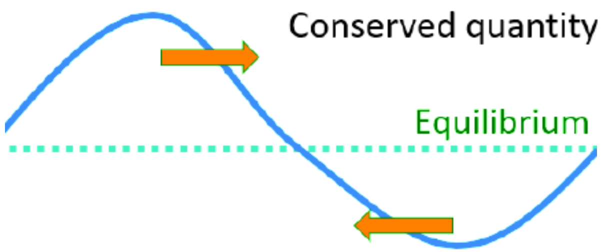
Emergent (large-scale) effective theory for **slow** processes

$\rightarrow$ "large-scale" wrt some relevant microscopic length
$\rightarrow$ "slow" wrt the microscopic timescales

Slow evolution characterised by those **few** DOF (**conserved** and **quasi-conserved** quantities) that equilibrate over macroscopic time-scales



Thermodynamic relaxation
Most DOF relax over the "micro" timescale
Local process (no need to "communicate")
$\rightarrow$ **fast** process $\sim$ collision time



Conserved quantity out of equilibrium
A conserved charge can only be moved around
The only way to equilibrate is transfer across regions
$\rightarrow$ **slow** process for large systems and small gradients

The "slow" DOF play the role of effective fields $\rightarrow$ hydrodynamics is the low-frequency field theory for such DOF

Relativistic hydrodynamics: relativistic **thermodynamics** + relativistic **classical field theory**

Irreversible dynamics
Final equilibrium state must be **stable**

Causality & **well-posedness**
of the initial value problem

Three "steps"

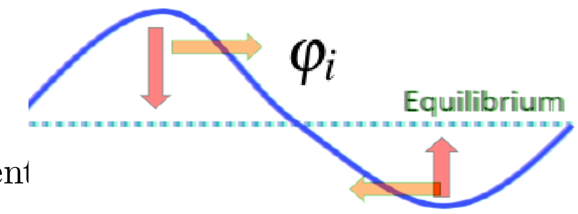
Review: Gavassino & MA, (2021)
Unified Extended Irreversible Thermodynamics and the stability of theories for dissipation

Defining a hydrodynamic model is a 3-step procedure:

1 – **Identify/choose the "slow" fields** φ (one for each **conservation** law)

Conservation of *energy-momentum* → e.g. velocity, temperature (4 quantities)

Conservation of *baryon number* → e.g. baryon chemical potential...



Additional field φ for each **quasi-conservation** law (relaxation due to "rare" event

Chemical fractions in the presence of slow chemical reactions → reaction affinity

Stresses in the presence of friction/viscosity → strains...

2 – The fields φ locally characterize the state of the system → we have to provide the **constitutive relations**

$$T^{\nu\rho} = T^{\nu\rho}(\varphi_i, \nabla_\sigma \varphi_i, \nabla_\sigma \nabla_\lambda \varphi_i, \ldots)$$
$$n^{\nu} = n^{\nu}(\varphi_i, \nabla_\sigma \varphi_i, \nabla_\sigma \nabla_\lambda \varphi_i, \ldots)$$
$$s^{\nu} = s^{\nu}(\varphi_i, \nabla_\sigma \varphi_i, \nabla_\sigma \nabla_\lambda \varphi_i, \ldots)$$

Ideally provided by some microscopic theory
They define the physical meaning of the model
If only 1 conserved curent → "simple fluid"

3 – Prescribe some **equations of motion (EOM)**

$$\mathfrak{F}_h(\varphi_i, \nabla_\sigma \varphi_i, \nabla_\sigma \nabla_\lambda \varphi_i, \ldots) = 0$$

Ideally consistent with:
Causality & well-posedness
Stability of the equilibrium state

Steps 2 & 3: How? You decide... but they must be at least consistent with:

$$\nabla_\nu T^{\nu\rho} = 0 \qquad \nabla_\nu n^{\nu} = 0 \qquad \nabla_\nu s^{\nu} \geq 0$$

For all the conserved quantities in (1)

II Law of Thermodynamics:
"=" non-dissipative
">" dissipative

Review: Gavassino & MA, (2021)
Unified Extended Irreversible Thermodynamics and the stability of theories for dissipation

Three "steps"

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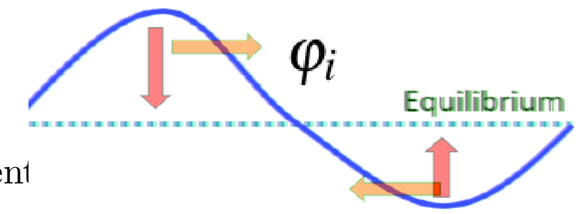

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$$T^{\nu\rho} = T^{\nu\rho}(\varphi_i, \nabla_\sigma \varphi_i, \nabla_\sigma \nabla_\lambda \varphi_i, ...)$$

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$$s^\nu = s^\nu(\varphi_i, \nabla_\sigma \varphi_i, \nabla_\sigma \nabla_\lambda \varphi_i, ...)$$

3 – Prescribe some **equations of motion (EOM)**

$$\mathfrak{F}_h(\varphi_i, \nabla_\sigma \varphi_i, \nabla_\sigma \nabla_\lambda \varphi_i, ...) = 0$$

Each conservation law can be used as EOM...

... but if there are **quasi-conserved** currents then you **need to supply a "model"** for how the current is dissipated.

Example: for a current affected by chemical reactions: $\nabla_\mu J^\mu - \Xi \mathbb{A} = 0$

computed via chemical kinetics

Reaction "affinity"

The simplest example/motivation

Review: Gavassino & MA, (2021)
Unified Extended Irreversible Thermodynamics and the stability of theories for dissipation

Zero-order in the deviation from equilibrium $\rightarrow$ perfect fluid

$$\nabla_\alpha \mathcal{T}^\alpha_\beta = 0 \qquad \nabla_\alpha J^\alpha = 0 \qquad \text{(4+1 conservation equations)}$$

Energy-momentum tensor and baryon current:

$$J_\alpha = n u_\alpha \qquad \mathcal{T}_{\alpha\beta} = (p+\varrho) u_\alpha u_\beta + p g_{\alpha\beta} \qquad p = p(\varrho, n)$$

The "3 steps" are trivial: (1) choose your fields, (2) constitutive relations, (3) EOM

1) 4+1 conservation equations $\rightarrow$ need 5 DOF $\quad (\varphi_i) = (u^\sigma, \varrho, n)$

   0 quasi-conservation equations $\rightarrow$ need 0 "extra" DOF

2) Constitutive relations: $J_\alpha = n u_\alpha \qquad \mathcal{T}_{\alpha\beta} = (p+\varrho) u_\alpha u_\beta + p g_{\alpha\beta} \qquad p = p(\varrho, n)$

3) EOM: the 4+1 conservation laws are enough

Neutron stars are "conductive" $\rightarrow$ many flows happen at the same time!

Electric current in MHD, superfluidity, heat conduction, neutrino and photon radiation...

...we typically need **more than 5 DOF**. Where do we get enough equations of motion?

Carter multifluid approach addresses thee three steps (1,2,3) for an arbitrary number of fluid "species".

Three "steps": the multifluid approach

Review: Gavassino & MA, (2021)
Unified Extended Irreversible Thermodynamics and the stability of theories for dissipation

Carter's multifluid provides a simple solution when the #DOF > 5

1 – **Identify/choose the "slow" fields** φ

Assume that there is a set of currents which completely specify the macrostate of the system

$$(\varphi_i) = (n_i^\sigma)$$

Two can be taken to be: n^σ s^σ

...the remaining ones depend on the non-equilibrium thermodynamic properties of the system.
Some currents may be "locked" together, others can flow independently.

2 – **Constitutive relations:** the only non-trivial one is the energy-momentum

$$T^{\nu\rho} = T^{\nu\rho}(n_i^\sigma)$$

...Carter assumes that it can be derived from a "master function" with constitutive relation:

$$\Lambda = \Lambda(n_i^\sigma, g_{\sigma\lambda})$$

3 – Prescribe some **equations of motion (EOM)**

...again, derived from the "master function" $\Lambda = \Lambda(n_i^\sigma, g_{\sigma\lambda})$

What is needed:

- Physically motivated identification of the set of currents (1)
- Constitutive relation for a single scalar function (2)

$$\Lambda = \Lambda(n_i^\sigma, g_{\sigma\lambda})$$

This has great practical value:

Consitutive relations of the energy-momentum are "derived"

The EOM are "derived"

Carter's multifluid

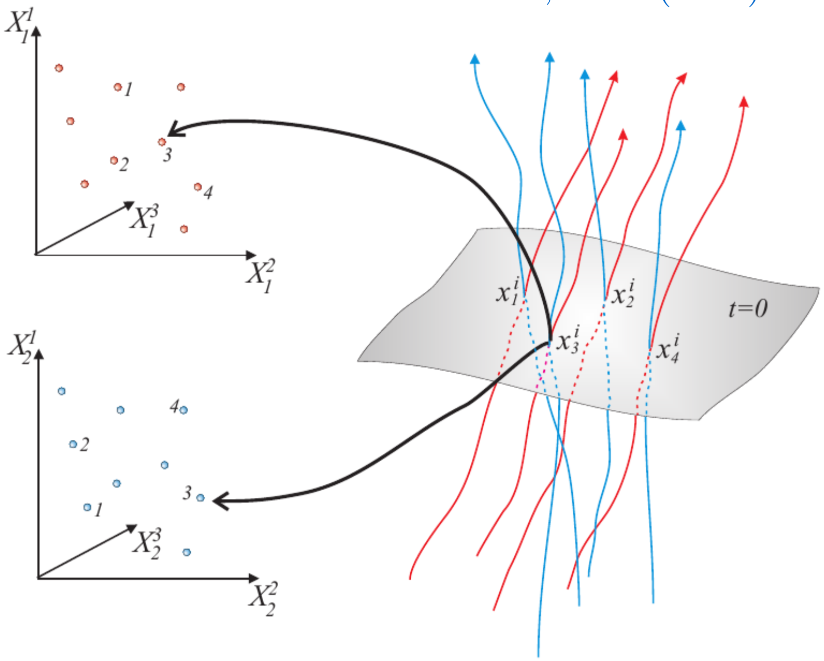
Carter, *Covariant Theory of Conductivity in Ideal Fluids* (1987) → Relativistic
Prix, *Variational description of multi-fluid hydrodynamics* (2002) → Newtonian

Variational approach based on Einstein-Hilbert+Matter action

$$I_{EH} = \int_{\mathcal{M}} \frac{R}{16\pi} \sqrt{-g}\, d_4 x \qquad I_m = \int_{\mathcal{M}} \mathcal{L} \sqrt{-g}\, d_4 x$$

$$T_{\nu\rho} = -\frac{2}{\sqrt{-g}} \frac{\delta I_m}{\delta g^{\nu\rho}}$$

Figure from
Andersson & Comer, LRR (2007)



→ simpler to prescribe a Lagrangian than the equations of motion
or the energy-momoentum tensor
→ easy to add extra macroscopic fields (e.g. MHD)
→ straightforward to incorporate additional fluid components
→ suitable for **conduction**

Basic requirements:

→ Should reduce to the usual **perfect fluid** if 1 current
→ Simple extensions of the perfect fluid (many constituents: "perfect multifluid") → $\nabla_\nu n^\nu_x = 0$
→ Connection with thermodynamics! Gavassino & MA, CQG (2020)
The "hydro" of the homogeneous state is just "thermo"
Equilibrium state with relativistic **persistent currents** (which frame?)

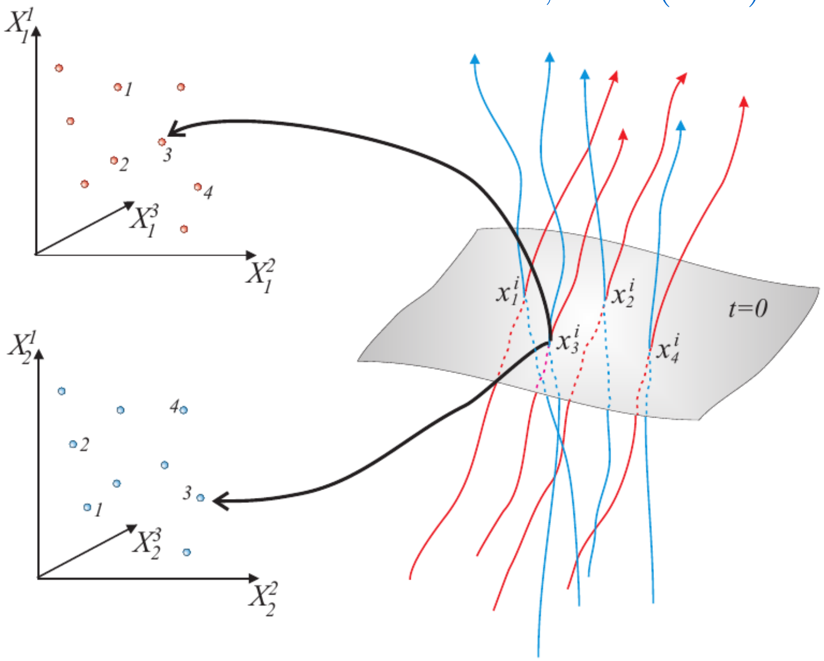
Seems reasonable to try with a tentative
"Lagrangian" of the kind

$$\Lambda(n^\nu_x) := \Lambda(-g_{\rho\nu} n^\rho_x n^\nu_y)$$

Carter's multifluid (unconstrained)

Tentative: proceed as in usual field theory (**unconstrained** variation)

Figure from
Andersson & Comer, LRR (2007)



→ Lagrangian: $\Lambda(n_x^\nu) := \Lambda(-g_{\rho\nu} n_x^\rho n_y^\nu)$

→ Canonical momenta: $\mu_\nu^x := \dfrac{\partial \Lambda}{\partial n_x^\nu} = \mathcal{B}^x n_{x\nu} + \sum_{y \neq x} \mathcal{A}^{xy} n_{y\nu}$

→ Entrainment: $\mathcal{B}^x := -2 \dfrac{\partial \Lambda}{\partial n_{xx}^2} \qquad \mathcal{A}^{xy} := -\dfrac{\partial \Lambda}{\partial n_{xy}^2}$

→ Energy-momentum: $T^\nu{}_\rho = \Psi \delta^\nu{}_\rho + \sum_x n_x^\nu \mu_\rho^x$

$$\Psi = \Lambda - \sum_x n_x^\rho \mu_\rho^x$$

Problem #1! Equations of motion (ignore surface terms in the action):

This goes with the "gravity" part

$$\delta\left(\sqrt{-g}\Lambda\right) = \sqrt{-g}\left[\sum_{\mathrm{x}} \mu_a^{\mathrm{x}} \delta n_{\mathrm{x}}^a + \frac{1}{2}\left(\Lambda g^{ab} + \sum_{\mathrm{x}} n_{\mathrm{x}}^a \mu_{\mathrm{x}}^b\right)\delta g_{ab}\right]$$

Trivial & useless dynamics!

Problem #2!
No conservation laws!

$$\nabla_\nu n_x^\nu = 0$$

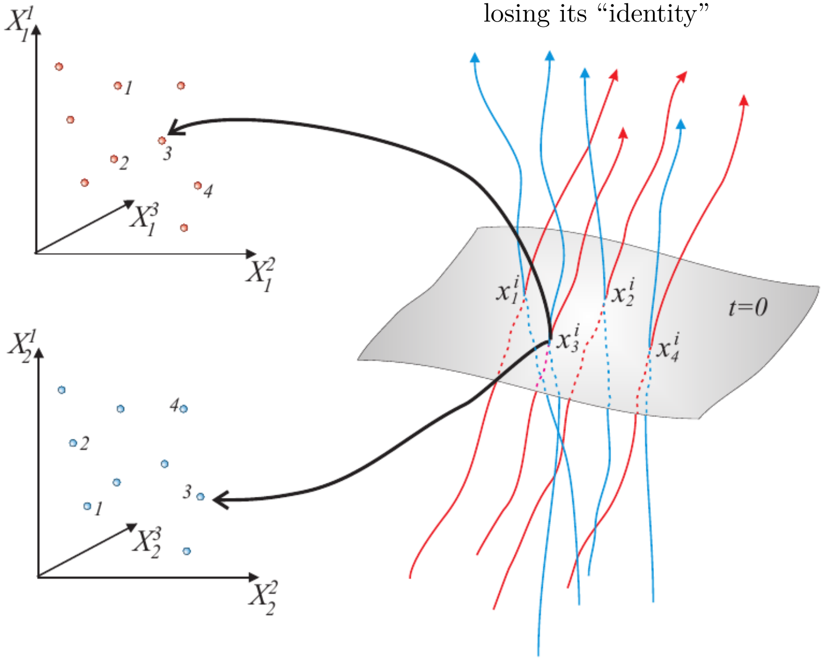
Where is this? Nowhere!
Not surprising since we used
unconstrained variations of the currents

Carter's multifluid (constrained)

Solution: we have to guarantee the identity of each fluid element's worldline!

...keep the **definitions**:

Each component has its own set of worldlines that it follows without losing its "identity"



→ **Not a Lagrangian** but "master function":

$$\Lambda(n_x^\nu) := \Lambda(-g_{\rho\nu} n_x^\rho n_y^\nu)$$

→ Canonical momenta: $\mu_\nu^x := \dfrac{\partial \Lambda}{\partial n_x^\nu} = \mathcal{B}^x n_{x\nu} + \sum_{y \neq x} \mathcal{A}^{xy} n_{y\nu}$

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...but modify the variation procedure (variations of the currents constrained to keep identity of worldlines)

The "real" Lagrangian is in terms of the "trajectories" (like for the point particle)

$$\mathcal{L}[g, X_s^\alpha, X_i^\alpha] = \Lambda(-g_{\rho\nu}\, n_x^\rho[g, X_x^\alpha]\, n_y^\nu[g, X_y^\alpha])$$

$$\nabla_\nu n_x^\nu = 0$$

Where is this?

The **domain of the action is restricted** so that conservation is ensured both on-shell and off-shell

Carter's multifluid: non-dissipative dynamics

Carter, *Covariant Theory of Conductivity in Ideal Fluids* (1987)

In a nutshell: $\Lambda = \Lambda(n^2_{xy})$ $\quad n^2_{xy} := -n^\nu_x n_{y\nu}$ $\quad I = \int \left(\frac{R}{16\pi} + \Lambda \right) \sqrt{-g}\, d^4x$

1) Variational procedure to ensure the conservation of the number density currents

Taub, PhysRev 94 (1954), Comer & Langlois CQG 10 (1993), Andersson & Comer, LRR (2007)

→ domain of the action restricted by imposing that $\nabla_\nu n^\nu_x = 0$ both on-shell and off-shell: variations of the currents are taken in the "Taub form"

$$\delta n^\nu_x = \xi^\rho_x \nabla_\rho n^\nu_x - n^\rho_x \nabla_\rho \xi^\nu_x + n^\nu_x \left(\nabla_\rho \xi^\rho_x - \frac{1}{2} g^{\rho\sigma} \delta g_{\rho\sigma} \right)$$

ξ_x = trajectory displacements

→ Variation of the action produced by the ξ_x (ignoring the boundary terms)

$$\delta I = \int \left(\sum_x f^x_\nu \xi^\nu_x \right) \sqrt{-g}\, d^4x \qquad f^x_\nu := 2 n^\rho_x \nabla_{[\rho} \mu^x_{\nu]}$$

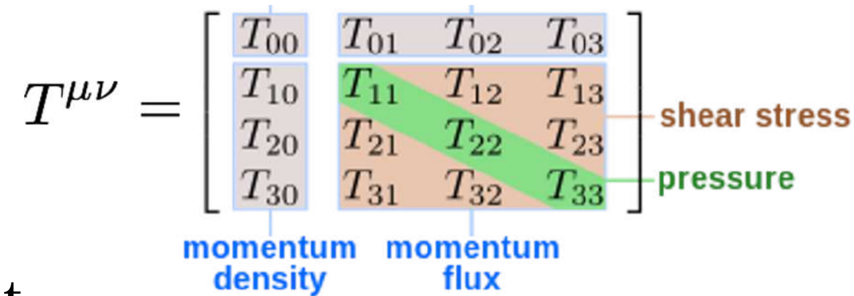
Equations of motion: $n^\rho_x \nabla_{[\rho} \mu^x_{\nu]} = 0$ for each constituent (all coupled by entrainment!)

$$\mu^x_\nu := \frac{\partial \Lambda}{\partial n^\nu_x} = \mathcal{B}^x n_{x\nu} + \sum_{y \neq x} \mathcal{A}^{xy} n_{y\nu}$$

If all EOM satisfied → $\nabla_\nu T^{\nu\rho} = 0$

(so this may replace 1 EOM)

Cold neutron star core



Single-component T=0 perfect fluid

Baryon current:

$$n^\mu = n u^\mu$$

Momentum per baryon:

$$\mu_\nu := \mu u_\nu$$

Euler relation:

$$e = -P + \mu n$$

Energy-momentum tensor:

$$T^\rho{}_\nu = P \delta^\rho{}_\nu + n^\rho \mu_\nu$$

Fluid rest-frame: isotropy

$$T_{ab} = \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{bmatrix}$$

Two-component T=0 perfect fluid

Neutron & proton currents:

$$n_p^\mu = n_p u_p^\mu \qquad n_n^\mu = n_n u_n^\mu$$

"Entrained" momenta per particle:

$$\mu_{p\nu} := \mathcal{B}_p n_{p\nu} + \mathcal{A} n_{n\nu}$$
$$\mu_{n\nu} := \mathcal{B}_n n_{n\nu} + \mathcal{A} n_{p\nu}$$

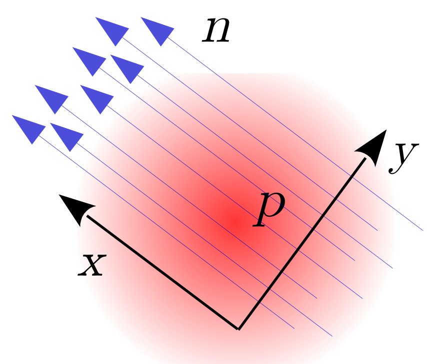
Euler-like relation: $e = -\Psi + n_p \mu_p + n_n \mu_n$

Energy-momentum tensor:

$$\Lambda = T^\nu{}_\nu - 3\Psi$$

$$T^\rho{}_\nu = \Psi \delta^\rho{}_\nu + n_p^\rho \mu_{p\nu} + n_n^\rho \mu_{n\nu}$$

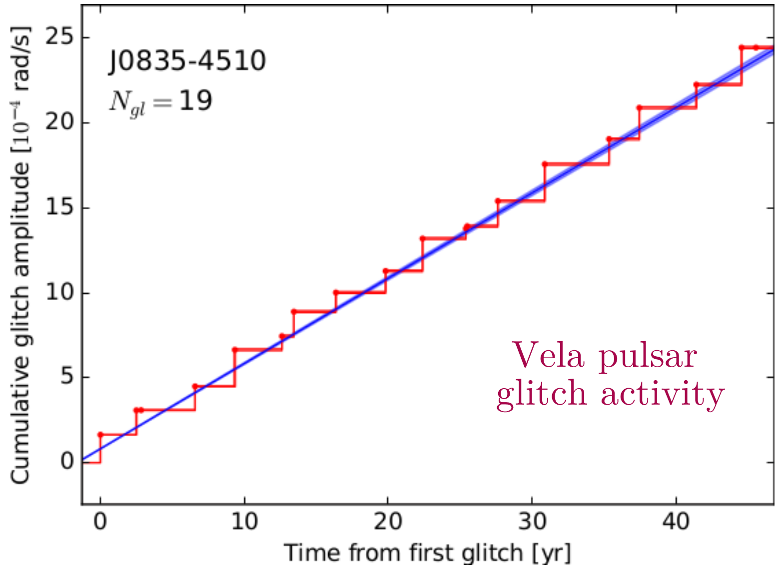
Collinear-frame: no shear but "cylindrical" pressure



$$T_{ab} = \begin{bmatrix} T_{00} & T_{01} & 0 & 0 \\ T_{10} & T_{11} & 0 & 0 \\ 0 & 0 & \Psi & 0 \\ 0 & 0 & 0 & \Psi \end{bmatrix}$$

Entrainment coupling

$$\mathcal{A}_a = \frac{\sum_i \Delta\Omega_i}{\sum_i \Delta t_i} = \frac{\sum_i \Delta\Omega_i}{t_{N_{\rm gl}-1} - t_0}$$



Link+1999 arXiv:9909146: glitch activity informs us about the effective moment of inertia of the region where pinning is possible

Chamel 2013 arXiv:1210.8177: the ***superfluid in the crust is not enough*** to explain Vela's activity (strong entrainment)

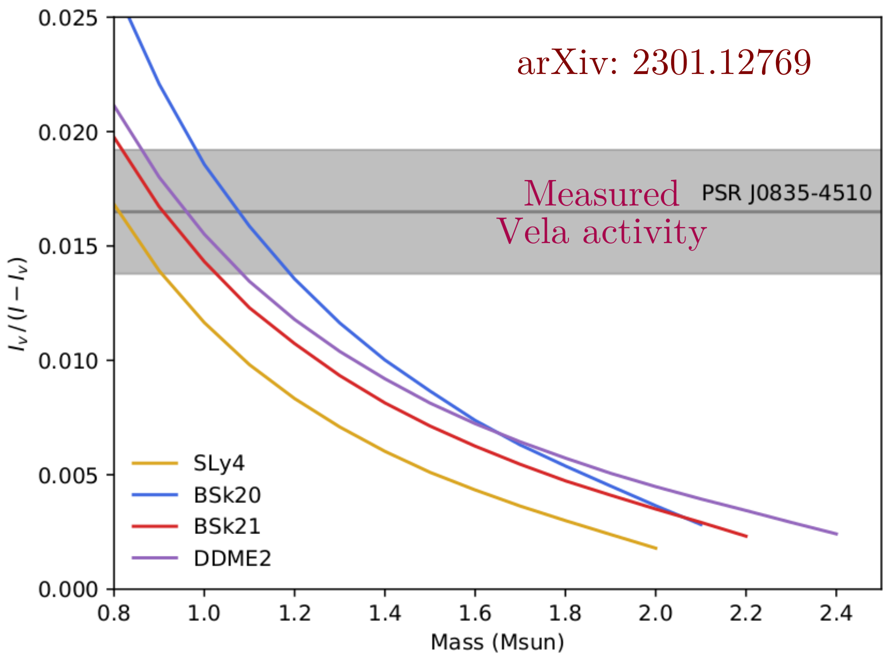
Revised argument: Montoli+ 2020 arXiv: 2012.01539

$$I_v = \frac{8\pi}{3}\int_0^{R_d} dr\, r^4 \, \frac{\rho_n(r)}{1-\epsilon_n(r)} \qquad \frac{\mathcal{A}}{|\dot\Omega|} < \frac{I_v}{I - I_v}$$

This is Newtonian, GR expression is similar (redshift & frame drag)

Review: MA, Montoli, Pizzochero arXiv: 2301.12769

Insights into the physics of neutron star interiors from pulsar glitches



Compare with bounds on the minimum mass of a NS:

Observed: M=1.174 M⊙ Martinez+2015 arXiv:1509.08805

CCS simulations: M≈1.15 M⊙ Suwa+2017 arXiv:1808.02328

Kelvin's theorem

Vortex motion and **Iordanksii force controversy** solution:
arXiv: 2012.10288 (2020)

Perfect multifluid → EOM are: $n^a_{\mathrm{x}} \nabla_{[a} \mu^{\mathrm{x}}_{b]} = 0$
...what's their meaning?

Consider the usual 1-component perfect fluid (at T=0 or "barotropic")

Take $\nabla_\alpha T^{\alpha\beta} = 0$ and project orthogonally to the 4-velocity with $\perp_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$

$$2 n^\mu \nabla_{[\mu} \mu_{\nu]} + \left(\nabla_\mu n^\mu \right) \mu_\nu = 0 \qquad \mu_\mu = \mu u_\mu$$

4-momentum per baryon

relativistic Kelvin theorem: vorticity is transported by the 4-velocity

... ofc this is useful to model cold neutron stars interiors (you'd like to know how vortices move)

→ **dissipation in superfluids:** when vortices do **NOT** flow with the current!

→ This also tells us how to extend Carter's perfect multifluid to include dissipation...

...the Lagrangian becomes a "generation function"

Carter's multifluid: "generating function"

Dissipation $\rightarrow$ entropy is not conserved... we have to break currents's conservation without falling into the useless "unconstrained" model

Keep the central postulate: there is a function $\Lambda = \Lambda(n_i^\sigma, g_{\sigma\lambda})$

Energy-momentum obtained as
$$T^{\nu\rho} = \frac{2}{\sqrt{|g|}} \frac{\partial(\sqrt{|g|}\Lambda)}{\partial g_{\nu\rho}}\bigg|_{\sqrt{|g|}\, n_i^\sigma}$$

The Lagrangian is DOWNGRADED to be just a "generating function" for the energy-momentum and the canonical momenta

$$\boxed{\begin{aligned} T^\mu{}_\nu &= \Psi \delta^\mu{}_\nu + \sum n^\mu_{\mathrm{x}} \mu^{\mathrm{x}}_\nu \\ \Psi &= \Lambda - \sum \mu^{\mathrm{x}}_\mu n^\mu_{\mathrm{x}} \end{aligned}}$$

$$2n^\mu \nabla_{[\mu}\mu_{\nu]} + (\nabla_\mu n^\mu)\, \mu_\nu = 0$$

$$\mu_\nu := \frac{\partial \Lambda}{\partial n^\nu_h}\bigg|_{n_i^\sigma, g_{\sigma\lambda}}$$

Equations of motion: just take the divergence of the energy-momentum and see...

$\Lambda = \Lambda(n_i^\sigma, g_{\sigma\lambda})$ does not give the EOM anymore!

$$\nabla_\nu T^\nu{}_\rho = \sum_h \left(\mu^h_\rho \nabla_\nu n^\nu_h + 2 n^\nu_h \nabla_{[\nu} \mu^h_{\rho]} \right)$$

$$\mu^h_\rho \nabla_\nu n^\nu_h + 2 n^\nu_h \nabla_{[\nu} \mu^h_{\rho]} = \mathfrak{R}^h_\rho$$

$$\sum_h \mathfrak{R}^h_\rho = 0$$

$$\mathfrak{R}^n_\rho n^\rho = 0$$

Baryon conservation

$$\frac{\mathfrak{R}^s_\rho s^\rho}{\mu^s_\lambda s^\lambda} \geq 0$$

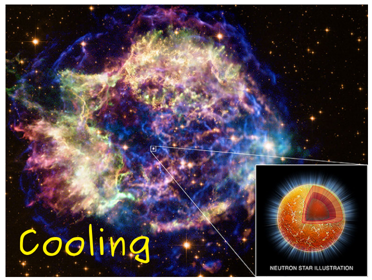
II Law

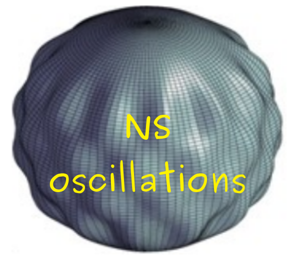
Is this a viable scheme for dissipation in relativity?

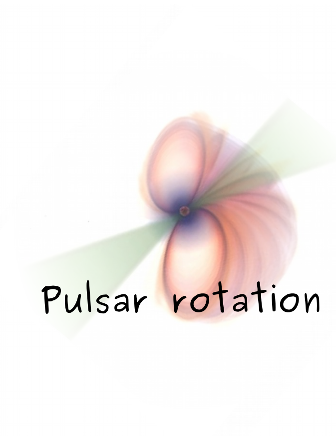
Is it more or less universal than "Israel-Stewart"?

Causality, stability? $\rightarrow$ difficult question but linearly stable & causal for "simple" forces

Dissipation in Neutron Stars







Define the R-modes
Instability window

Shear viscosity:
(out of equilibrium distribution)
(electron VS nuclei, protons, impurities)
(binary collisions of phonons)

For both cold NS
and proto-NS

According
to recent
estimates

Bulk viscosity:
(out of equilibrium distribution)
(nuclear reactions)
(phonon-phonon collisions)

Late-stage
cooling

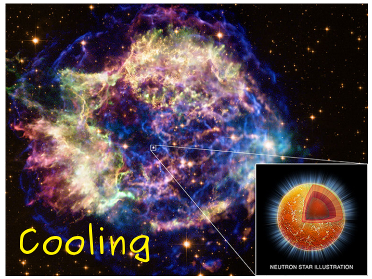
Vortex mediated friction:
(vortex motion in the superfluid)

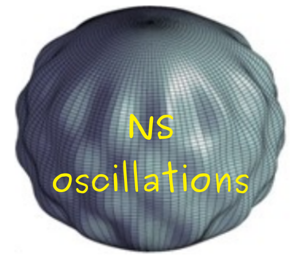
Fundamental
for pulsar glitch
recoveries

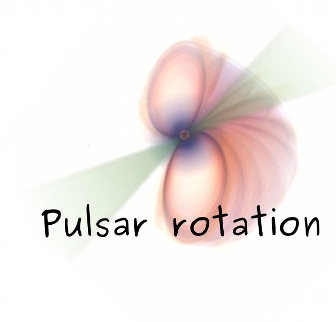
Luminosity/radiation
(photon/neutrino emission)

Dissipation in Neutron Stars

“Generating function” approach good for… ?







Shear viscosity:
(out of equilibrium distribution)
(electron VS nuclei, protons, impurities)
(binary collisions of phonons)

Maybe yes but NO
(need to introduce a “flux of a flux” together with the other currents)

Bulk viscosity:
(out of equilibrium distribution)
(nuclear reactions)
(phonon-phonon collisions)

Can be “better” than Israel-Stewart
→ can evolve multiple independent chemical fractions **(arXiv:2003.04609)**
→ upgrade Israel-Stewart to superfluid matter **(arXiv:2110.05546)**

Vortex mediated friction:
(vortex motion in the superfluid)

This is what Carter’s multifluid does and others can not **(arXiv:2012.10288)**

Luminosity/radiation
(photon/neutrino emission)

Simple fluid + non-conserved ultra-relativistic fluid
→ **M1 radiation hydrodynamics** (**arXiv:2007.09481**)

(Hiscock & Lindblom 1983)

Dissipative hydrodynamics

$$\Pi_{\alpha\beta} := g_{\alpha\beta} + u_\alpha u_\beta$$

$$\mathcal{T}_{\alpha\beta} := (\varrho + \mathcal{R}) u_\alpha u_\beta + (p + \mathcal{P}) \Pi_{\alpha\beta} + \pi_{\alpha\beta} + \mathcal{Q}_\alpha u_\beta + \mathcal{Q}_\beta u_\alpha$$

Quantities as in the perfect fluid but there are additional "dissipative fluxes"

"First order" theories → the "dissipative fluxes" functions of the "perfect fluid variables" & derivatives

$$\pi = \pi(\varrho, u, \partial\varrho, \partial u, \dots) \text{ etc.}$$

Navier
Stokes
Fourier

→ Philosophy is that of the **gradient expansion**

→ 5 algebraic DOF (the same as the perfect fluid)

EoM: $\nabla_\alpha \mathcal{T}^\alpha_\beta = 0$

Israel-Stewart
hydrodynamics

"Second order" theories → the "dissipative fluxes" are new DOF

→ DOF: 5 (perfect fluid) + 1 (bulk) + 3 (heat flux) + 5 (traceless shear) = **14**

→ Philosophy is that of **moments method**

→ if only bulk & heat → 9 algebraic DOF

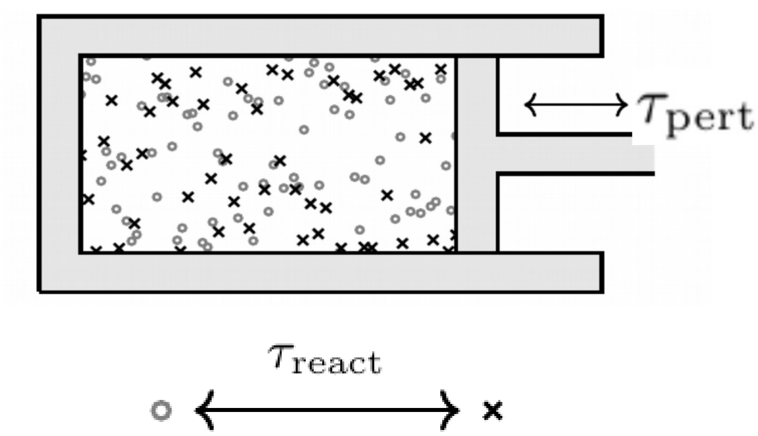
EoM: $\nabla_\alpha \mathcal{T}^\alpha_\beta = 0$ $\quad u^\mu \nabla_\mu \pi + \dots = 0$ etc.

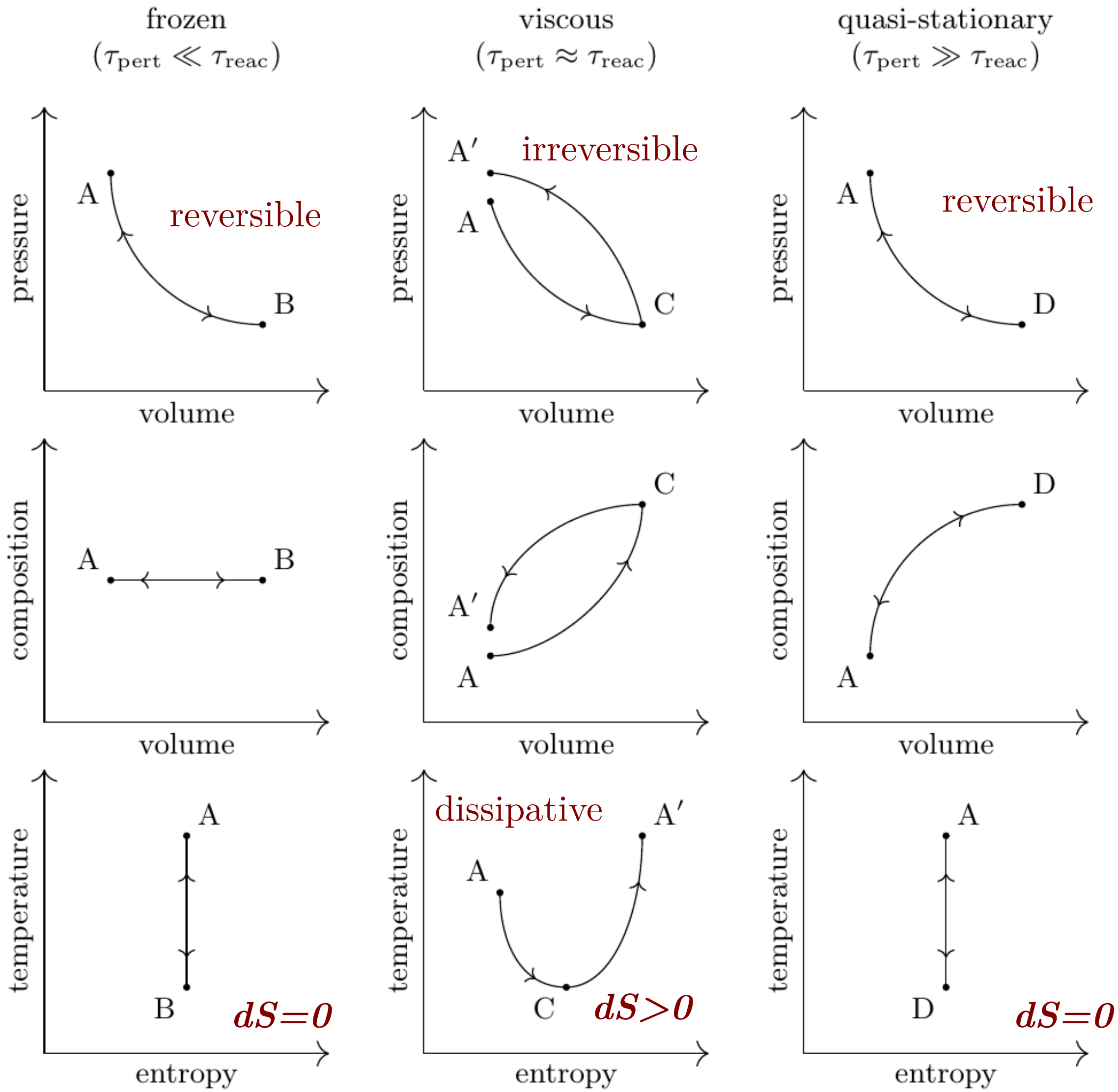
Bulk viscosity

Camelio+ arXiv:2204.11809
(to appear in PRD, 2023)

Bulk viscosity = dissipative response to compression and expansion (thermal/chemical re-equilibration)

Maximum dissipation when:
reaction time ~ perturbation period





Bulk viscosity (Navier-Stokes)

1 – Fields: 4-velocity (3 DOF)

s entropy density, n number density → equilibrium reference state

5 DOF of the Non-barotropic perfect fluid

2 - Constitutive relations: boil down to the perfect fluid for zero stress

$$T^{\mu\nu} = (e + p + \Pi)u^\mu u^\nu + (p + \Pi)g^{\mu\nu}$$

EOS ref. state: $p(n,s)$ $e(n,s)$

$$n^\mu = n u^\mu \qquad s^\mu = \left(s - \frac{\beta_0 \Pi^2}{2T} \right) u^\mu$$

Definition: $\Pi = -\zeta \nabla_\lambda u^\lambda$

3 - Hydrodynamic equations:

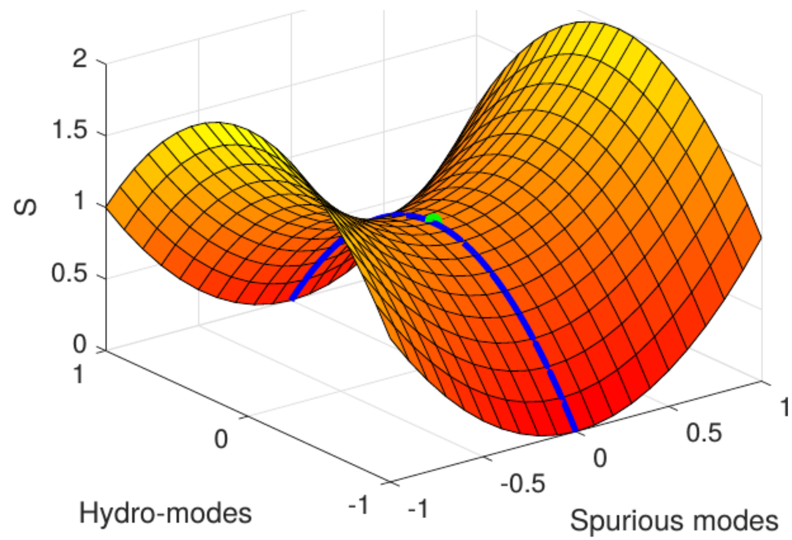
$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu n^\mu = 0$$

$$\nabla_\mu s^\mu = \frac{\Pi^2}{T\zeta} \qquad \textbf{(B)}$$

This theory is known to be **acausal** and **unstable** (Hiscock & Lindblom 1985)

→ homogeneous equilibrium state "tends to expolde"



Gavassino & MA
Frontiers 8:92 (2021)

Bulk viscosity (Israel-Stewart)

(Hiscock & Lindblom 1983)

1 – Fields: 4-velocity

s entropy density, n number density → equilibrium reference state

Additional field: bulk stress → genuine additional DOF (deviation from equilibrium)

5 + 1 DOF
Non-barotropic perfect fluid
+ bulk stress

2 - Constitutive relations: boil down to the perfect fluid for zero stress

$$T^{\mu\nu} = (e + p + \Pi)u^\mu u^\nu + (p + \Pi)g^{\mu\nu}$$

$$n^\mu = n u^\mu \qquad s^\mu = \left(s - \frac{\beta_0 \Pi^2}{2T}\right)u^\mu$$

(A)

EOS ref. state:

$p(n,s)$

$e(n,s)$

3 - Hydrodynamic equations:

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu n^\mu = 0$$

$$\nabla_\mu s^\mu = \frac{\Pi^2}{T\zeta} \qquad \textbf{(B)}$$

(A) & (B) → "Telegraph" equation for the bulk stress

Equation for the evolution of the bulk stress

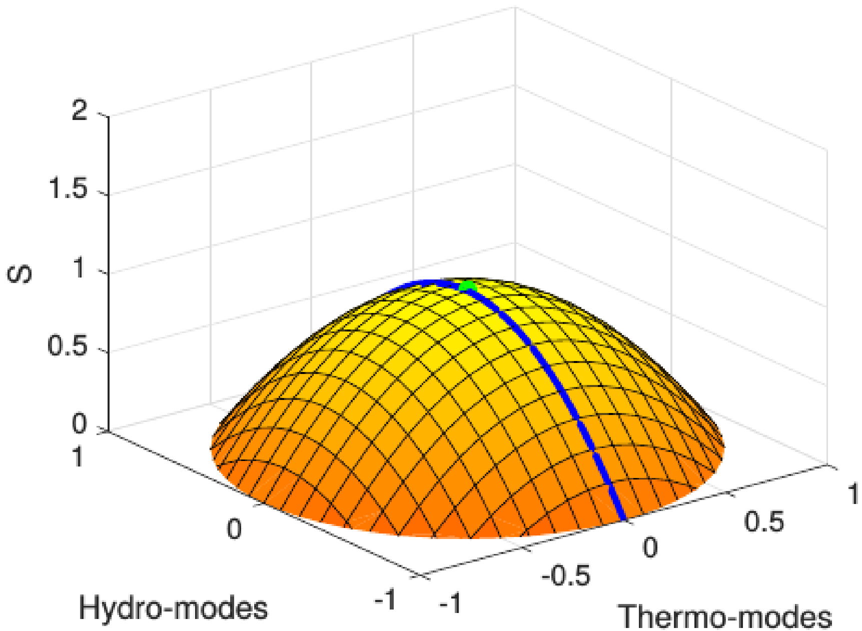
$$\boxed{\Pi = -\zeta\Big[\underbrace{\nabla_\mu u^\mu}_{\text{Navier-Stokes}} + \beta_0\, u^\mu \nabla_\mu \Pi} + \frac{1}{2}\Pi\, T\, \nabla_\mu\left(\frac{\beta_0 u^\mu}{T}\right)\Big]$$

Navier-Stokes

Maxwell-Cattaneo

Stability (entropy interpretation)

Review: Gavassino & MA, (2021)
Unified Extended Irreversible Thermodynamics and the stability of theories for dissipation



Physical entropy (generic **stable** fluid)

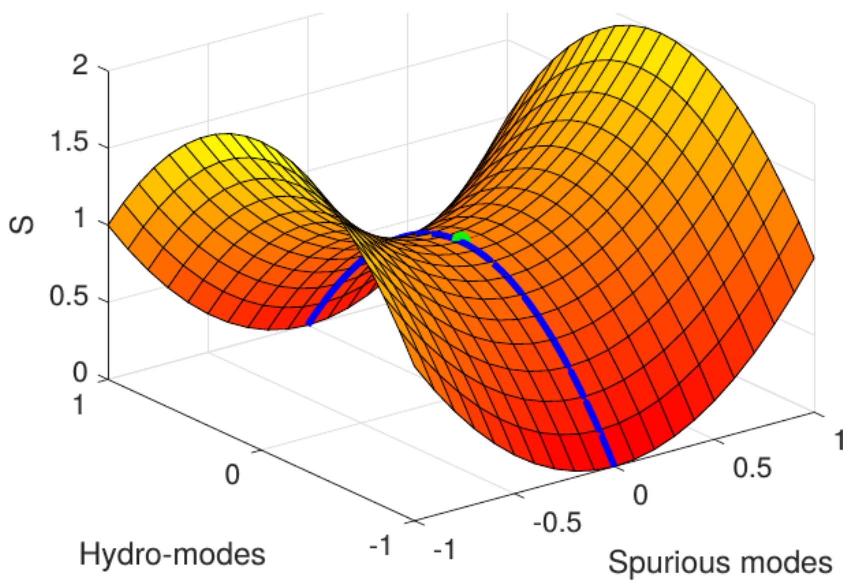
Maximum = equilibrium state (homogeneous perfect fluid state)

Blue line = states accessible by non-rel limit (Navier-Stokes)

Any deviation from equilibrium reduces the entropy and therefore must decay when the second law is imposed.

Thermo-modes (gapped) $\rightarrow$ Non-equilibrium thermodynamics

Hydro-modes (gapless) $\rightarrow$ Navier-Stokes-Fourier approach



Entropy of relativistic Navier-Stokes

Saddle point = equilibrium state (homogeneous perfect fluid state)

Blue line = states accessible by non-rel limit

Hydro-modes $\rightarrow$ damped if we impose the validity of the II Law

Spurious (gapped) modes $\rightarrow$ the II Law forces them to grow indefinitely, originating the instability.

Superfluid + bulk viscosity + heat

How to combine **superfluidity** with **dissipation**? Two different languages are used:

Relativistic theory for **superfluidity** → **Carter's perfect multifluid**

Relativistic theory for **dissipation** → "Israel-Stewart" / "second-order"

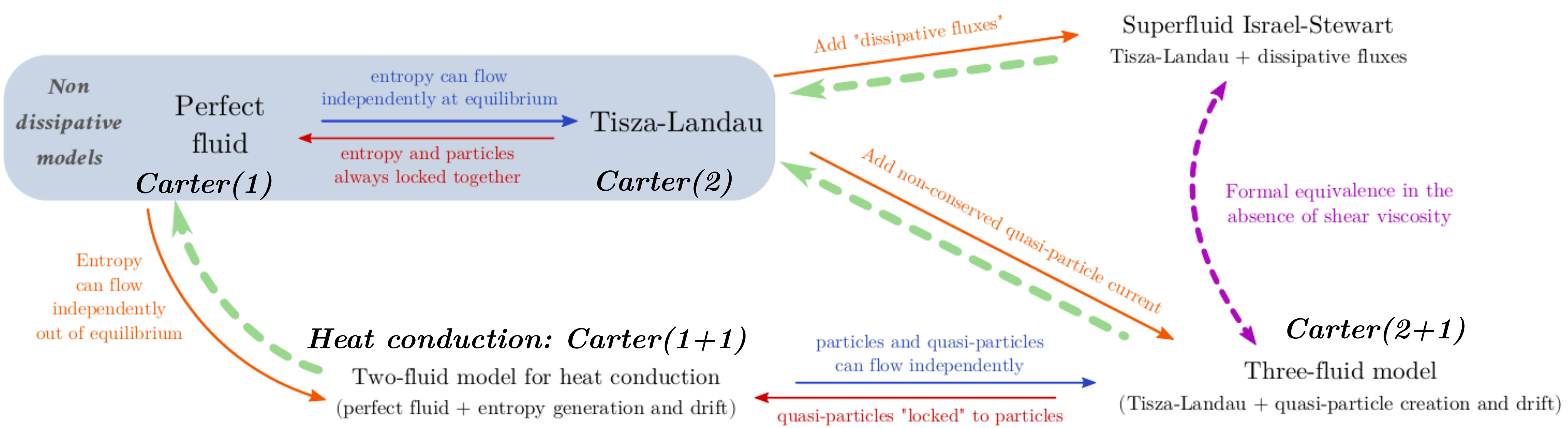
They do not seem to have anything in common → how to merge the two consistently?

Close to equilibrium: ***Carter(N+1)*** = ***Carter(N)*** + **Israel-Stewart dissipation (bulk viscosity & heat)**

→ If ***Carter(N)*** is non-dissipative, in ***Carter(N+1)*** we have to unlock the II Law

→ Advantage: easy to derive causality & stability conditions for *Carter(N+1)*

→ II Law valid both on and off-shell: the equations can be solved!



Message: How to include **bulk & heat dissipation** in a **superfluid?** → **Add 1 non-conserved current!**

Which current depends on microphysics (phonons, rotons, photons, non-superfluid baryons in NS crust)

ArXiv: 2110.05546

Superfluid + bulk viscosity + heat

Carter's dissipative multifluid with 3 currents: $\Lambda = \Lambda(n^2, s^2, z^2, n^2_{ns}, n^2_{nz}, n^2_{sz})$

$$\nabla_\nu n^\nu = 0 \qquad \nabla_\nu s^\nu \geq 0 \qquad \nabla_\nu z^\nu \neq 0$$

Non-conserved quasiparticle number

$z + z \rightleftharpoons z + z + z$

Equilibrium: non-dissipative 2-fluid of Tisza-Landau (persistent current of entropy wrt particles)

We only need density and entropy to define the state $\rightarrow$ $z^\nu = z^\nu_{\text{eq}}(n^\rho, s^\rho)$

Out-of-equilibrium: $z^\nu = z^\nu(n^\rho, s^\rho, \Pi, Q^\rho)$

Israel-Stewart is "perturbative": dissipative fluxes *(Π, Q)* are defined as deviations from equilibrium.

Expand around $z^\nu(n^\rho, s^\rho, 0, 0) = z^\nu_{\text{eq}}(n^\rho, s^\rho)$ and find: $Q^\nu = Q^\nu(n^\rho, s^\rho, z^\rho)$ $\quad \Pi = \Pi(n^\rho, s^\rho, z^\rho)$

12 algebraic DOF = 9 (I&S heat+bulk) + 3 (superflow) $\quad (n^\rho, s^\rho, \Pi, Q^\rho) \longleftrightarrow (n^\rho, s^\rho, z^\rho)$

Generating function formalism:

$$T^\nu{}_\rho = \Psi \delta^\nu{}_\rho + n^\nu \mu_\rho + s^\nu \Theta_\rho - z^\nu \mathbb{A}_\rho$$

$$\Psi = \Lambda - n^\nu \mu_\nu - s^\nu \Theta_\nu + z^\nu \mathbb{A}_\nu$$

Equations of motion

$$\mathcal{R}^n_\rho = 2 n^\nu \nabla_{[\nu} \mu_{\rho]} = 0$$

$$\mathcal{R}^s_\rho = 2 s^\nu \nabla_{[\nu} \Theta_{\rho]} + \Theta_\rho \nabla_\nu s^\nu$$

$$\mathcal{R}^z_\rho = -2 z^\nu \nabla_{[\nu} \mathbb{A}_{\rho]} - \mathbb{A}_\rho \nabla_\nu z^\nu$$

Need to find $\mathcal{R}^z_\rho = -\mathcal{R}^s_\rho$ from quasiparticle kinetics $\rightarrow$ $\nabla_\nu T^\nu{}_\rho = \mathcal{R}^n_\rho + \mathcal{R}^s_\rho + \mathcal{R}^z_\rho = 0$

ArXiv: 2110.05546

Superfluid + bulk viscosity + heat

Carter's dissipative multifluid with 3 currents: $\Lambda = \Lambda(n^2, s^2, z^2, n^2_{ns}, n^2_{nz}, n^2_{sz})$

$$\nabla_\nu n^\nu = 0 \qquad \nabla_\nu s^\nu \geq 0 \qquad \nabla_\nu z^\nu \neq 0$$

Non-conserved quasiparticle number

$z + z \rightleftharpoons z + z + z$

12 algebraic DOF $\quad (n^\rho, s^\rho, \Pi, Q^\rho) \longleftrightarrow (n^\rho, s^\rho, z^\rho)$

We have "Carter" from the "generating functional" $\rightarrow$ **expand to find "superfluid Israel-Stewart"**

Eckart frame of the "excitation gas" $\quad u^\nu := z^\nu / \sqrt{-z^\rho z_\rho}$

Dissipation is mediated by collisions between quasiparticles: local thermodynamic equilibrium = collinearity between $\boldsymbol{s}$ and $\boldsymbol{z}$

Heat: $s^\nu = s^E u^\nu + \dfrac{Q^\nu}{\Theta_E} \qquad Q^\nu u_\nu = 0$

Bulk: $\Psi = \Psi_{\text{eq}} + \Pi \quad \rightarrow \quad \Pi = \mathbb{A}_E \dfrac{\partial \Psi}{\partial \mathbb{A}_E} \quad$ where $\quad \mathbb{A}_E = -\mathbb{A}_\nu u^\nu$

Telegraph-type evolution for heat & bulk $\rightarrow$ consistent with non relativistic hydrodynamics of Khalatnikov

Entropy and dissipative force ($\boldsymbol{k}$ = heat conduction coefficient):

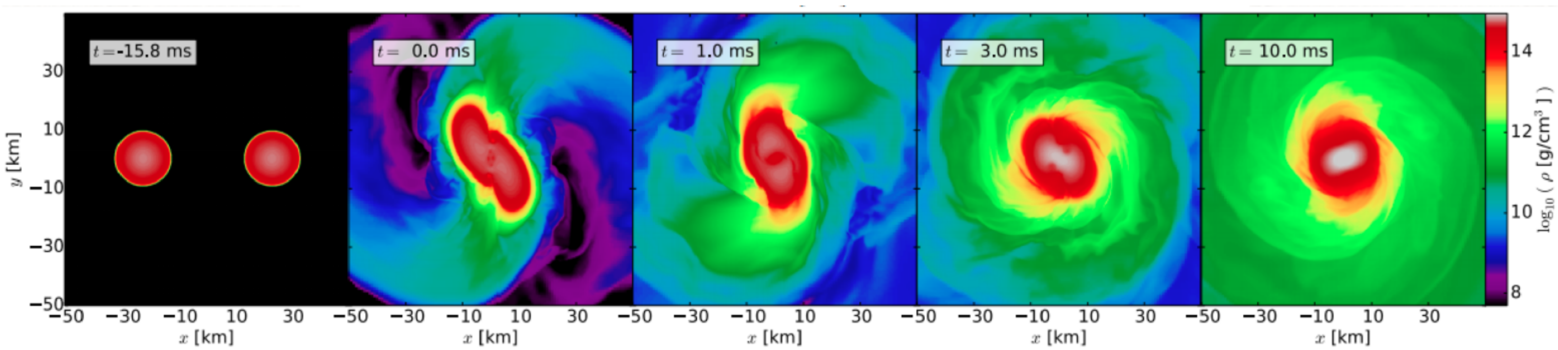
$$[\nabla_\nu s^\nu]_{\text{bulk}} = \frac{\Xi \mathbb{A}_E^2}{\Theta_E} \qquad [\nabla_\nu s^\nu]_{\text{heat}} = \frac{Q_\nu Q^\nu}{k\, \Theta_E^2} \qquad \mathcal{R}^s_\rho = \Xi\, \mathbb{A}_E^2\, u_\rho - \frac{s^E}{k} Q_\rho$$

Viscous effects in neutron star mergers?

Duez+ PRD (2004), Shibata+ PRD (2017), Most+ PRL 2019, Hammond+ PRD 2021, Celora+ CQG 2022...

Previous understanding → viscous effects negligible

Based on the simulations/knowledge at that time: temperatures not so large, system very smooth, gradients too small



Example: Alford+ PRL (2018) → rough estimates of the importance of dissipation channels

Simulations (**ideal fluid!**): estimates for macroscopic scale L of fluid variables gradients

From microscopic arguments: estimate for the characteristic microscopic scales l in the system

Knudsen number $\sim l/L$ may not be small in some cases (viscosity may affect the GW signal)

Shear → Relevant for trapped neutrinos if T > 10 MeV and gradients at small scales ~ 10 m (turbulence)

Heat → Relevant for trapped electron neutrinos if T > 10 MeV and gradients at scales ~ 100 m

Bulk → Should affect **density oscillations after merger!** Alford, Harris, PRC (2019)

*"Effects of **bulk viscosity** should be consistently included in merger simulations. This has not been attempted before and requires a formulation of the relativistic-hydrodynamic equations that is **hyperbolic** and **stable**"*

First attempt (postmerger): Camelio+ **arXiv:2204.11809** and **arXiv:2204.11810** (to appear in PRD)

Bulk viscosity: theoretical approaches

Camelio+
arXiv:2204.11809
arXiv:2204.11810

Bulk viscosity = dissipative response to compression and expansion (thermal/chemical re-equilibration)

No shear, ho heat, no superfluidity
→ isotropy

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu}$$

"Expansion around equilibrium" approach → full "Israel-Stewart" by Hiscock-Lindblom (1983)

$$p = p^{\mathrm{eq}}(\rho, \epsilon) + \Pi$$

$$\nabla_\mu (\Pi u^\mu) = -\frac{\Pi}{\tau} - \left(\frac{1}{\chi} - \frac{\Pi}{2}\right) \nabla_\mu u^\mu - \frac{\Pi}{2} u^\mu \nabla_\mu \left(\log \frac{\chi}{T^{\mathrm{eq}}}\right)$$

"Multifluid" approach → no "expansion", only assumes "separation of timescales"

Meaning: each independent "reaction coordinate" that evolves slowly goes into the EOS

$$p = p(\rho, \epsilon, \{Y_i\}_i)$$

$$\mathrm{d}u = \frac{p}{\rho^2} \mathrm{d}\rho + \frac{T}{m_\mathrm{n}} \mathrm{d}s - \sum_i \frac{\mathbb{A}^i}{m_\mathrm{n}} \mathrm{d}Y_i$$

$$\mathbb{A}^i = 0$$ Chemical equilibrium

$$\nabla_\mu (\rho u^\mu) = 0$$

$$\nabla_\mu (T^{\mu\nu}) = -\mathcal{Q} u^\nu$$

$$\nabla_\mu (\rho Y_i u^\mu) = m_\mathrm{n} \mathcal{R}_i$$

Hydro-bulk-1D

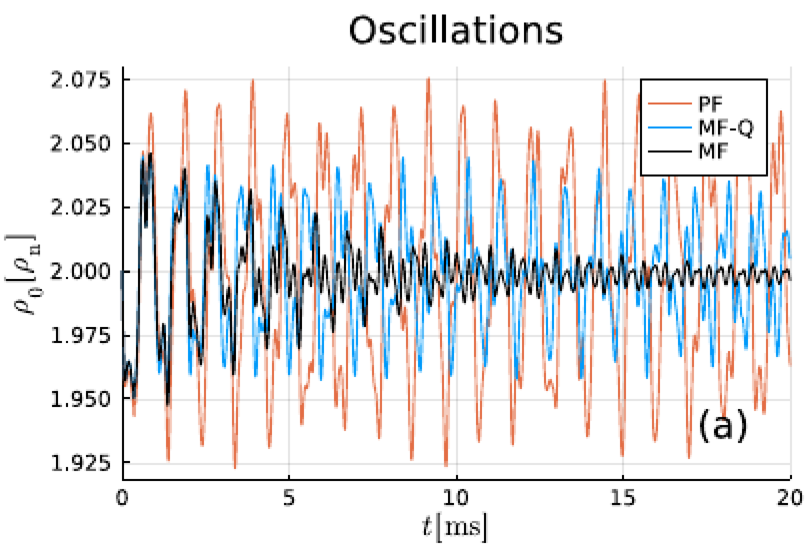
Camelio+
arXiv:2204.11809
arXiv:2204.11810

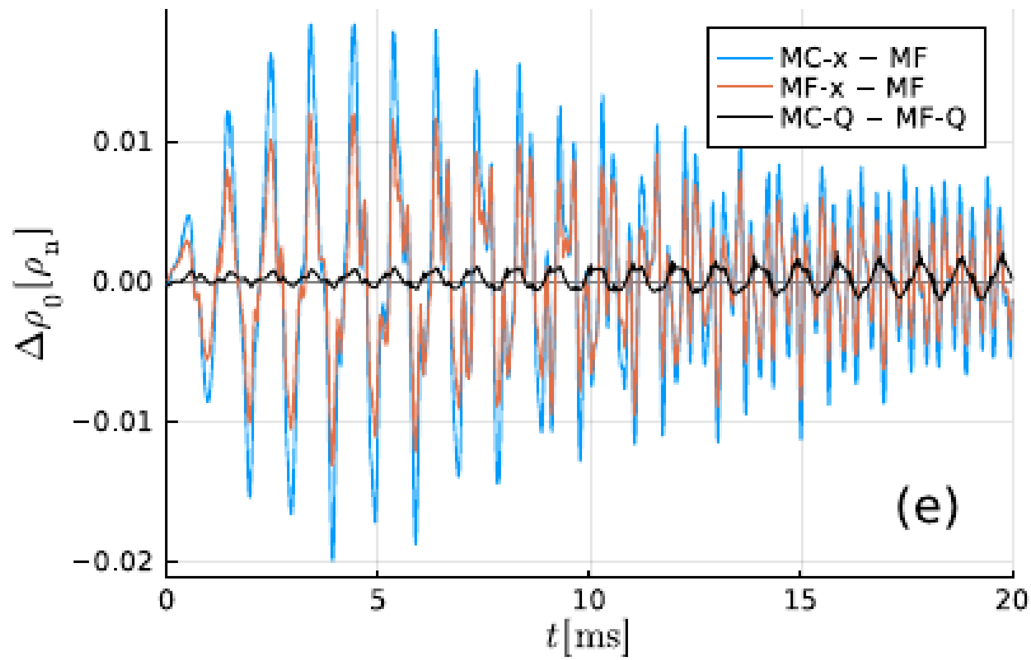
First simulation of a NS with the **complete Hiscock-Lindblom model** of bulk viscosity.

One-dimensional, GR code, publicly available: Giovanni Camelio, `hydro-bulk-1D` (2022)

We include the **energy loss due to the luminosity of the reactions** in the bulk stress formulation.

Bulk & luminosity should be consistent (the same reactions are responsible for both)





PF = perfect fluid

MF = multifluid out of beta equilibrium (npe matter)

MF-Q = multifluid out of beta equilibrium (npe matter) + consistent neutrino luminosity

MC = “equivalent” Maxwell-Cattaneo

Final considerations

From the general considerations in **arXiv:2003.04609** (also, arXiv:2110.05546)

→ Dissipative Carter's multifluid can encore heat and bulk viscosity

→ Theoretically identical to Israel-Stewart close to equilibrium

→ Far from equilibrium: Israel-Stewart is pertubative, Carter is not

→ "Carter" = non perturbative generalization of "Israel-Stewart" without shear

Hydro-Bulk-1D: First simulation of proto-NS with the **complete Hiscock-Lindblom model** of bulk viscosity and neutrino luminosity. Comparison with Carter's formalism for bulk viscosity.

One-dimensional, GR code, publicly available: Giovanni Camelio, `hydro-bulk-1D` (2022)

Numerical check of the theoretical result (arXiv:2204.11810):

Israel-Stewart is a good approximations of the multi-component fluid when:

→ small perturbations

→ the equation of state of the fluid depends on only one independent particle fraction

For more than one independent particle fraction and for large perturbations (e.g. muons)

→ the bulk stress approximation is still valid but less accurate

Message: in mergers, isolated NS (cold and hot), supernovae... just use "Carter" for bulk viscosity!

→ arXiv:2003.04609 (general theory), arXiv:2204.11810 (comparison of 3 approaches to bulk and numerics)

Unified Irreversible Extended Thermodynamics

Principles of UEIT:
→ hydrodynamics as a **non-equilibrium thermodynamic theory** for non-homogeneous states
→ It unifies some EIT models (Israel-Stewart, Divergence-type) with Carter's multifluid scheme
→ Clarifies the common rationale behind EIT models (each EIT model evolved independently in different areas)

1. The **constitutive relations do not involve derivatives** of the fields ("ultra locality", like for the perfect fluid)

$$T^{\nu\rho} = T^{\nu\rho}(\varphi_i) \qquad s^\nu = s^\nu(\varphi_i) \qquad n^\nu = n^\nu(\varphi_i)$$

2. The hydrodynamic equations are all of the **first order** both in **space** and in **time**

$$\mathfrak{F}_h(\varphi_i, \nabla_\sigma \varphi_i) = 0$$

3. The **II Law** is enforced for any initial condition $\nabla_\nu \big[s^\nu(\varphi_i) \big] := \sigma(\varphi_i, \nabla_\sigma \varphi_i) \geq 0$

Multifluid → addresses 1,2,3 in a simple way.
→ Not based on near-equilibrium but ONLY on separation of timescales

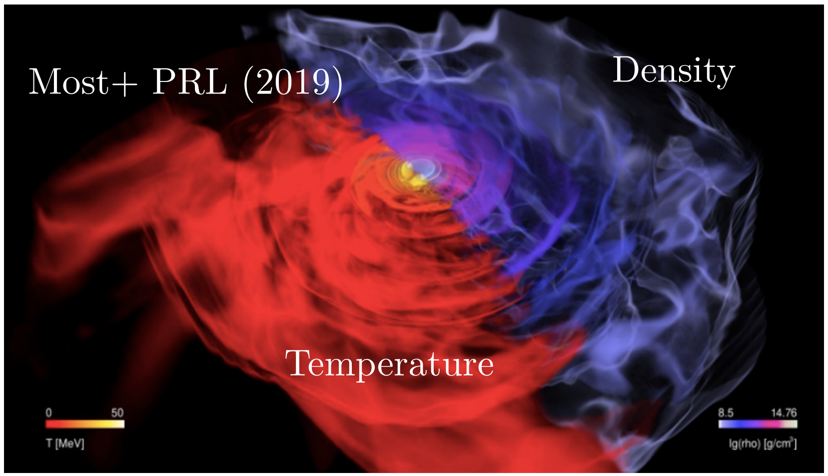
Deviations from equilibrium (think of usual derivation from kinetic theroy):
Zero order → perfect fluid
First order → usual Navier-Stokes-Fourier (diffusion of energy & momentum)
Second order → Grad 14 moments (non rel), Israel-Stewart (relativistic)

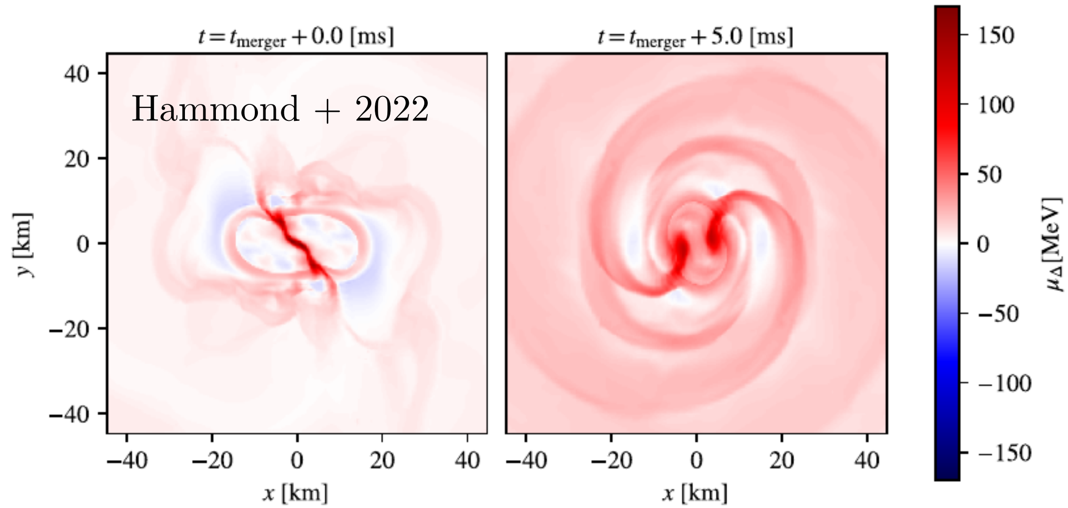
Review and UEIT formulation:
Gavassino & MA, (2021)
Unified Extended Irreversible Thermodynamics and the stability of theories for dissipation

Viscous effects in neutron star mergers?

Duez et al PRD (2004), Shibata et al. PRD (2017), Alford et al. PRL (2018)

Rarefaction/compression of the fluid elements → Chemical abundances are pushed out of chemical equilibrium.
Possibly relevant for: CC superovae, NS mergers, NS oscillations





*"The effects of **bulk viscosity** should be consistently included in future merger simulations. This has not been attempted before and requires a formulation of the relativistic-hydrodynamic equations that is hyperbolic and stable"*
Alford et al. PRL (2018)

$$n \rightarrow p + e^- + \bar{\nu}_e$$
$$p + e^- \rightarrow n + \nu_e$$

$$n + X \rightarrow p + X + e^- + \bar{\nu}_e$$
$$p + e^- + X \rightarrow n + X + \nu_e$$

Bulk viscosity: "deviation from eq." approaches

Bulk viscosity = dissipative response to compression and expansion (thermal/chemical re-equilibration)

No shear, ho heat, no superfluidity
$\rightarrow$ isotropy

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p g^{\mu\nu}$$

"Expansion around equilibrium" approach $\rightarrow$ from kinetic theory up to a certain order ***O(f–f$_o$)***

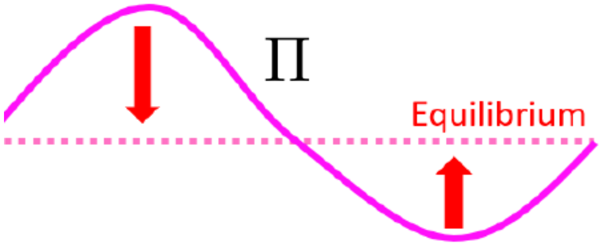
$$p = p^{\mathrm{eq}}(\rho, \epsilon) + \Pi$$

departure from perfect fluid

Zero order $\rightarrow$ relativistic perfect fluid: $\Pi = 0$

First order $\rightarrow$ Landau, Eckart (spurious gapped modes unstable, acausal) $\Pi = -\zeta \nabla_\lambda u^\lambda$

Second order $\rightarrow$ Israel-Stewart, Hiscock-Lindblom: stable and causal in the linear regime



$$\nabla_\mu(\Pi u^\mu) = -\frac{\Pi}{\tau} - \left(\frac{1}{\chi} - \frac{\Pi}{2}\right)\nabla_\mu u^\mu - \frac{\Pi}{2} u^\mu \nabla_\mu \left(\log \frac{\chi}{T^{\mathrm{eq}}}\right)$$

As time passes, full evolution becomes "first order".
Lindblom relaxation effect: tendency of dissipative fluids to lose DOF, by transforming dynamical equations into phenomenological constraints

Perfect multifluid in relativity

We have seen the perfect fluid:
- local equilibrium (equilibrium thermodynamic variables defined in the local rest frame)
- 5 DOF: $\nabla_\nu n^\nu_n = 0 \qquad \nabla_\rho T^\rho{}_\nu = 0$ are enough to define the dynamics

Neutron stars are conductive!
Many flows are possible at the same time (electric current in MHD, superfluidity, heat conduction…)
…we typically need **more DOF**. Where do we get enough equations of motion?

Carter multifluid approach solves the problem of deriving the **equations of motion** for a **conductive mixture** of an arbitrary number of fluid species ("species": abstract concept, e.g. "entropy").

Important: *Carter's approach gives the equations of motion in the* ***inviscid*** *limit* (non dissipative).
Why? It is a variational approach → Liouville theorem is incompatible with relaxation to equilibrium.
Dissipative variational approaches exist but are of different nature (often they need a "DOF doubling").

Equations of motion

$$\underbrace{\mu^h_\rho \nabla_\nu n^\nu_h}_{\text{Transfusion}} + \underbrace{2 n^\nu_h \nabla_{[\nu}\mu^h_{\rho]}}_{\text{Single perfect fluid part}} = \mathfrak{R}^h_\rho$$

Transfusion between species gives rise to a force

Single perfect fluid part

Hydro force (e.g, **friction**, not specidied by the model, must be supplied)

Superfluid hydrodynamics

General Relativistic version of the two-fluid hydro: Gavassino, Antonelli, CQG 2019

Vortex core scale: "trunk"
~ 10 fm in a NS
(microscopic models)

Inter-vortex scale: "trees"
~ 10^{-3} cm in a NS
(vortex filament model)

Fluid element: "forest"
from mm to km in a NS
(macroscopic hydrodynamics)

We can not take into account each vortex (~10^{18} in a pulsar) → **"two-fluid"** smooth hydrodynamics

2 Euler-like equations + **entrainment** + **mutual friction**

$$\partial_t \rho_{\mathrm{x}} + \nabla_i(\rho_{\mathrm{x}} v^i_{\mathrm{x}}) = 0$$

$$(\partial_t + v^j_{\mathrm{x}} \nabla_j)(v^{\mathrm{x}}_i + \varepsilon_{\mathrm{x}} w^{\mathrm{yx}}_i) + \nabla_i(\tilde{\mu}_{\mathrm{x}} + \Phi) + \varepsilon_{\mathrm{x}} w^j_{\mathrm{yx}} \nabla_i v^{\mathrm{x}}_j = f^{\mathrm{x}}_i / \rho_{\mathrm{x}}$$

The **dynamics of vortices** in a fluid element gives the form and strength of the macroscopic **"mutual friction"**

Chemical label $\mathbf{X = n,p}$ **n**→superfluid neutrons **p**→normal component

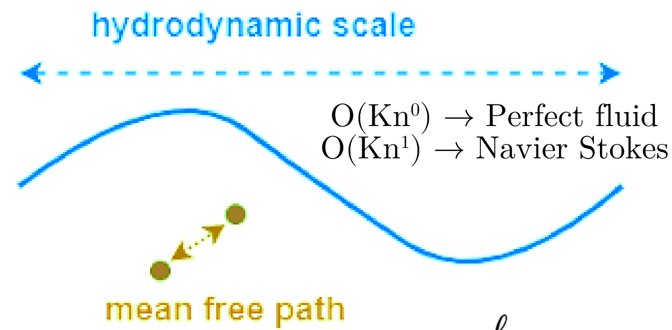
Hydrodynamics as a derivative expansion

Hydrodynamics may also be seen as a macroscopic treatment based on:

- **Separation of length scales** (Kundsen number)

Kn ~ "mean free path"/"system length scale" ~ Mach/Reynolds

Kn ~ 0.01 or smaller → continuum approximation



$$K_N \sim \frac{\ell}{L} \ll 1$$

- **Conservation laws** (energy-momentum, charges) + possibly external symmetries

Example: incompressible Navier Stokes equation for a viscous fluid
→ conservation of mass + Newton's II law + local isotropy

$$\underbrace{\partial_t \vec{V} + (\vec{V} \cdot \vec{\nabla})\vec{V} + \frac{\vec{\nabla} P}{\rho_0}}_{\text{Ideal fluid } \sim \mathcal{O}(K_n^0)} = \underbrace{\frac{\eta}{\rho_0} \nabla^2 \vec{V}}_{\sim \mathcal{O}(K_n)} + \underbrace{\mathcal{O}(K_N^2)}_{\text{Higher order}}$$

References

Work done with: L. Gavassino (Vanderbilt University), G. Camelio, B. Haskell (CAMK, Warsaw)

General theory:

Equilibrium thermodynamics of a multifluid → arXiv:1906.03140
Stability and causality of Carter's multifluid → arXiv:2202.06760
Multicomponent fluid with bulk viscosity → arXiv:2003.04609
Dissipation in superfluids → arXiv:2012.10288 (vortices), arXiv:2110.05546 (heat & bulk viscosity)
Radiation hydrodynamics (M1) as a Carter's multifluid → arXiv:2007.09481

Some applications:

Glitches in pulsars → arXiv:1710.05879 (glitch amplitude), arXiv:2001.08951 (glitch timescale)
Effect of bulk viscosity due to chemical reactions in neutron star oscillations
→ arXiv:2204.11809 (formalism)
→ arXiv:2204.11810 (simualtions)

Reviews:

Andersson & Comer "Relativistic Fluid Dynamics" arXiv:gr-qc/0605010
Gavassino & Antonelli "Unified Extended Irreversible Thermodynamics and the stability of theories for dissipation"