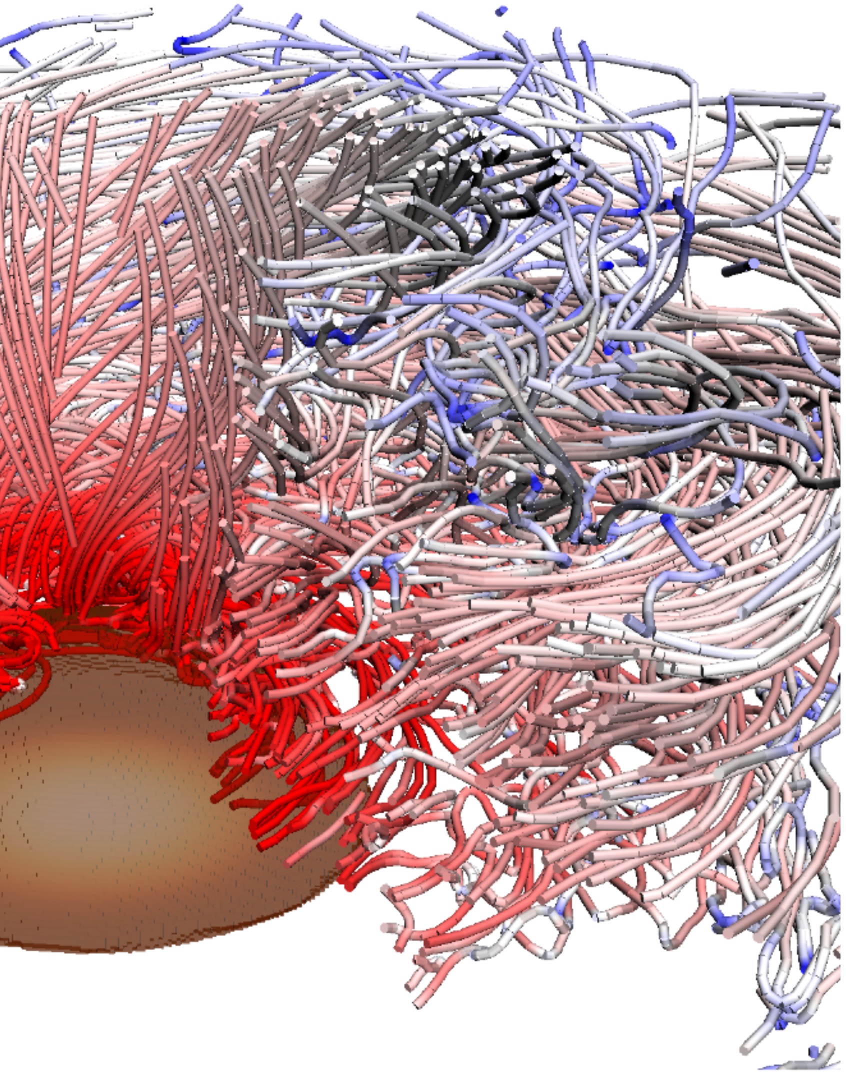
Max Planck Institute for Gravitational Physics
ALBERT EINSTEIN INSTITUTE

MRI-driven alpha-Omega dynamo in binary neutron star mergers

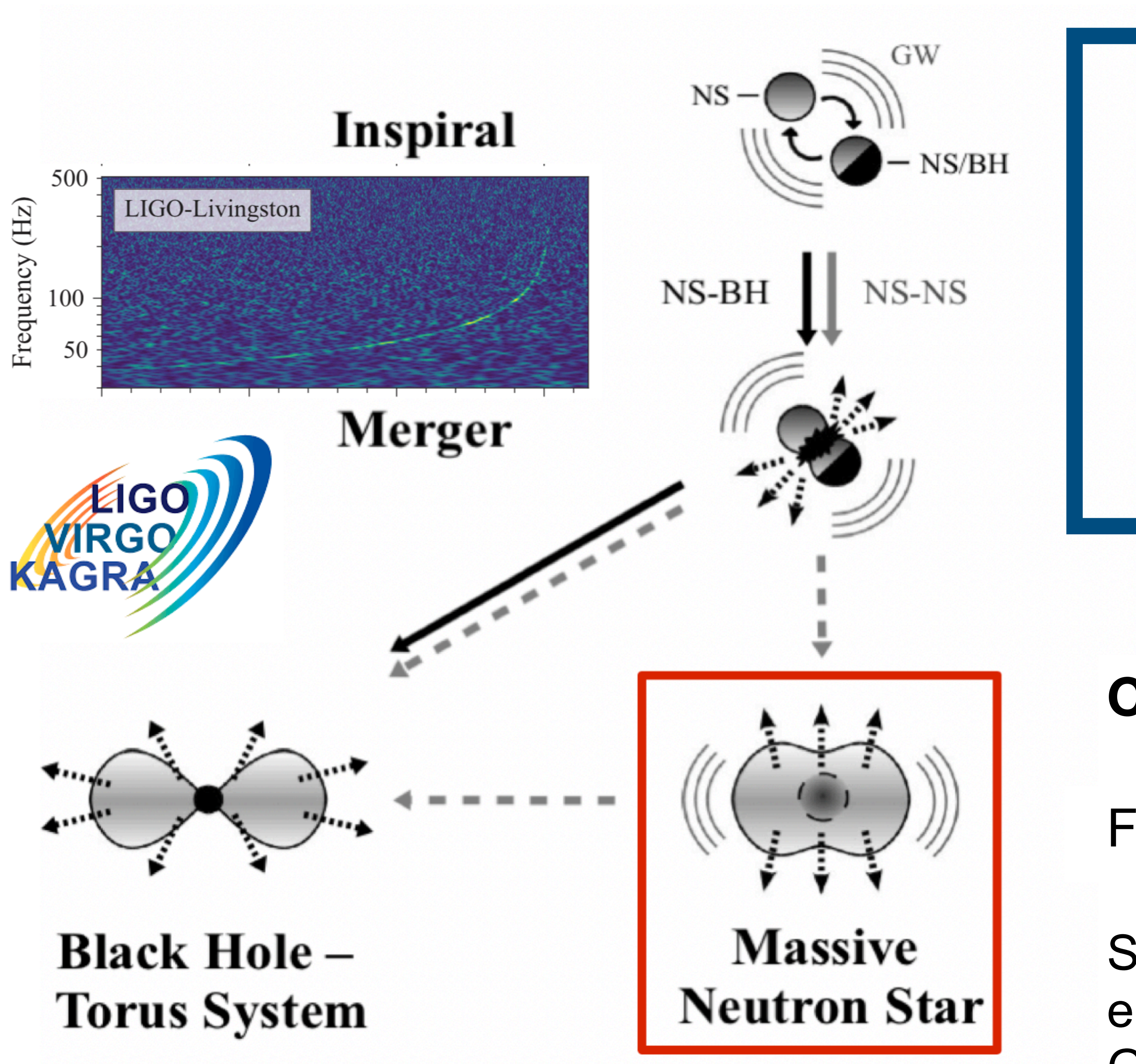


Alexis Reboul-Salze

in collaboration with
Kenta Kiuchi[1,2], Masaru Shibata[1,2], Yuichiro Sekiguchi[2,3]

[1]Max Planck Institute for Gravitational Physics, Potsdam, Germany
[2]Center for Gravitational Physics and Quantum Information, Yukawa Institute for Theoretical Physics, Kyoto, Japan
[3]Department of Physics, Toho University, Chiba, Japan

Motivation: Binary neutron star mergers



3 possibilities :

- direct collapse to a black hole
- hypermassive NS stabilized by rotation : delayed collapse
- stable neutron star

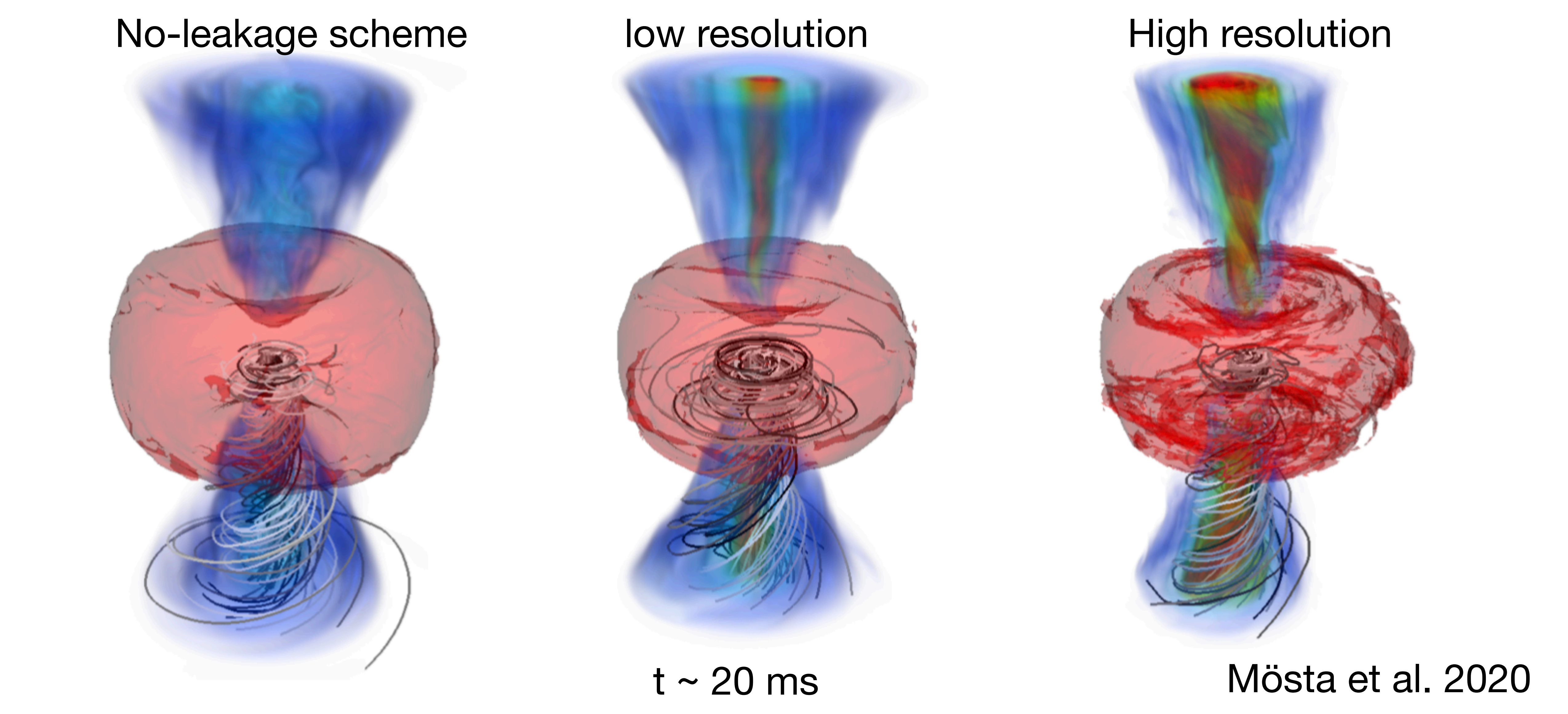
Central Engine of Short GRBs ?

Formation of a magnetar ?

Signature in future joint gravitational wave – electromagnetic observations as GW170817

Models of the sGRB central engine

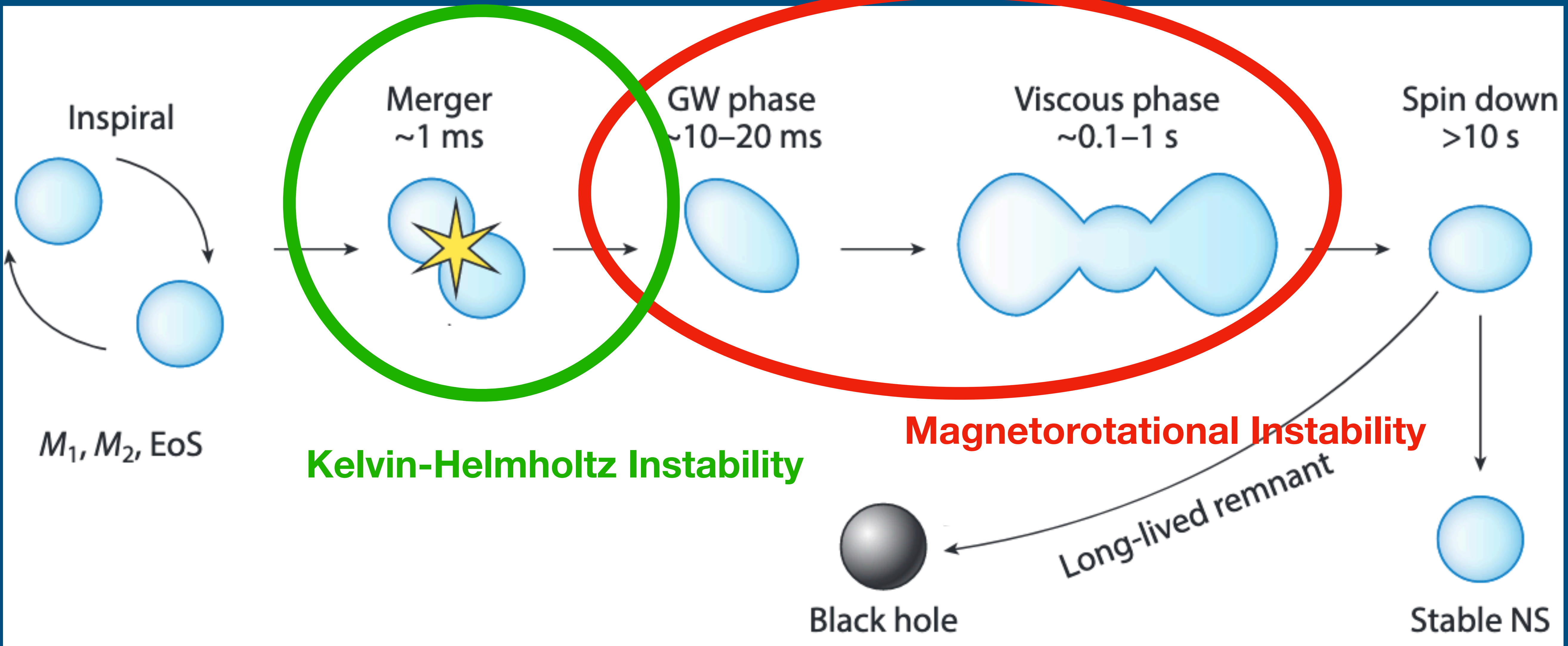
Initial dipole field in the remnant:



Requires a strong large-scale magnetic field
-> Two amplifications mechanisms in BNS merger,
Kelvin-Helmholtz instability and Magnetorotational instability

Amplification mechanisms

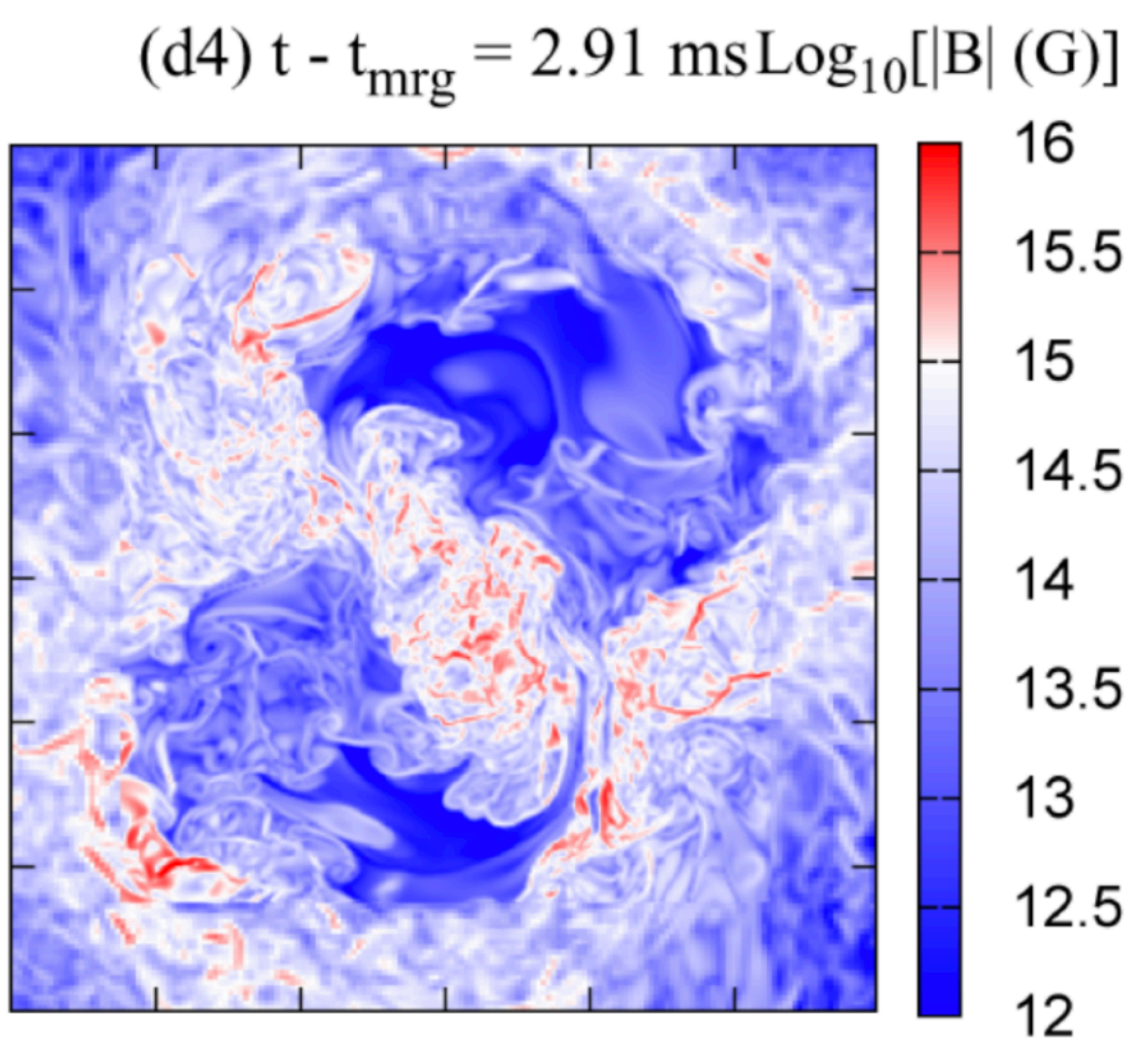
Evolution of the merger



Radice et al. 2020

Magnetic field amplification on small scales

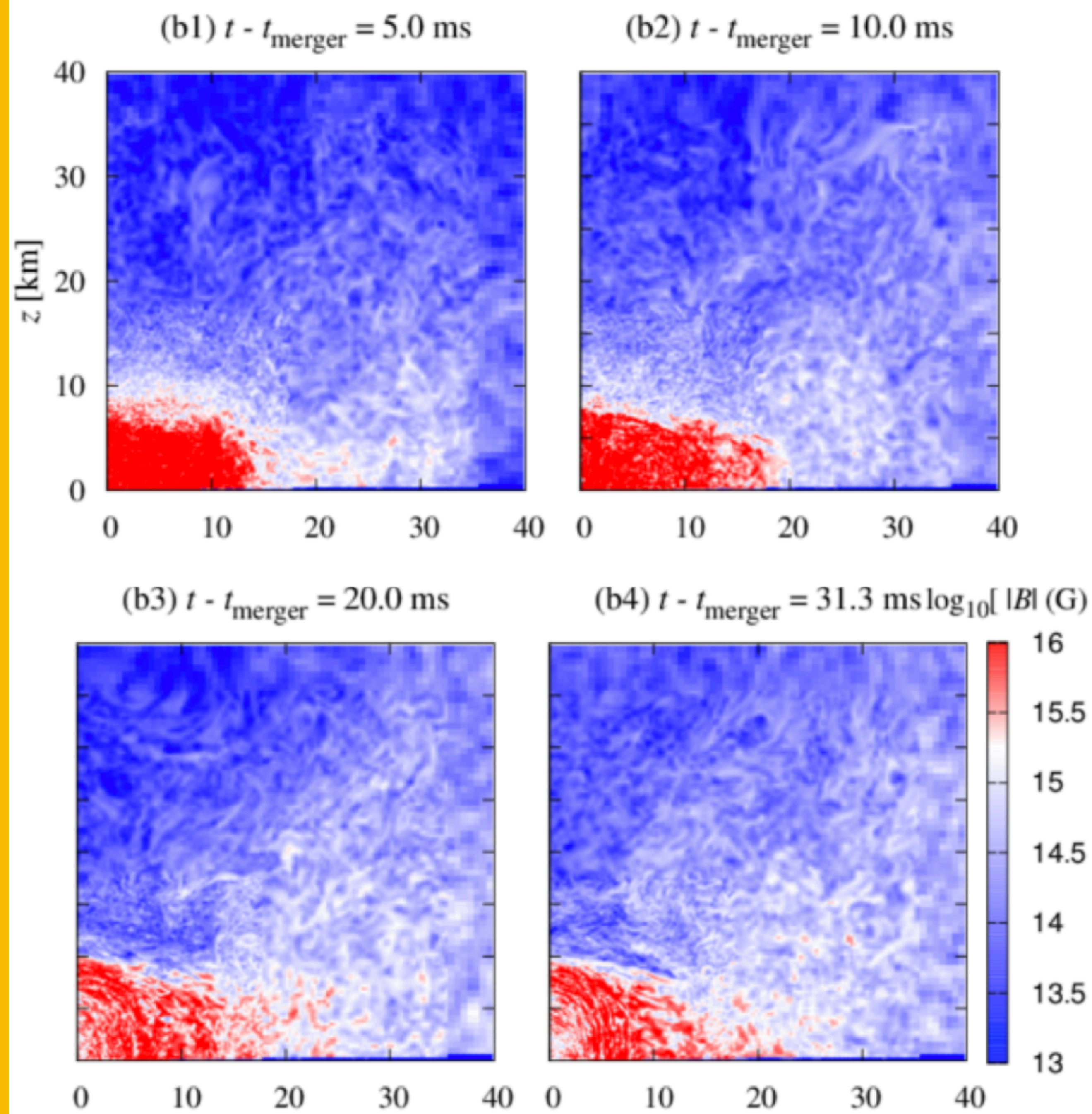
Kelvin-Helmholtz in NS merger



Kiuchi et al. 2015

Amplification from 10^{13} G to 10^{16} G on small scales in a few milliseconds

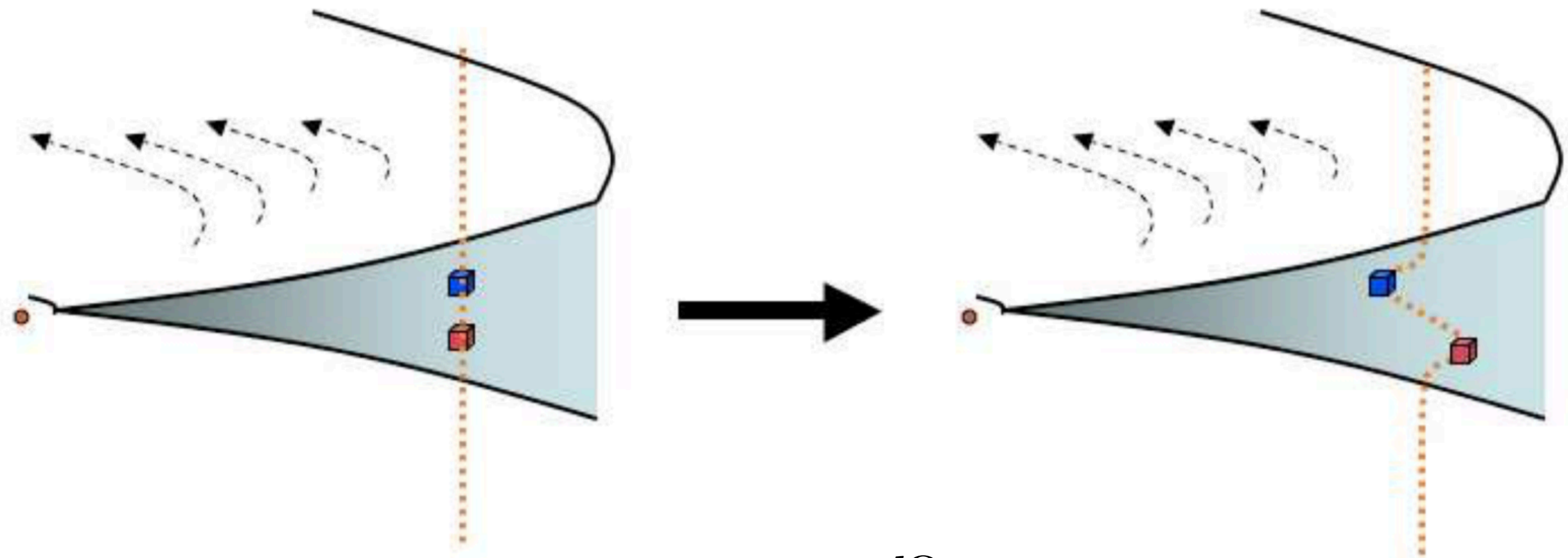
MRI in NS merger



Kiuchi+2018

Magneto-rotational instability (MRI)

MRI mechanism in a simple case:



Credit : Fromang

Instability criterion: $\frac{d\Omega}{dr} < 0$

Growth rate: $\sigma = \frac{q\Omega}{2}$ with $\Omega \propto r^{-q}$

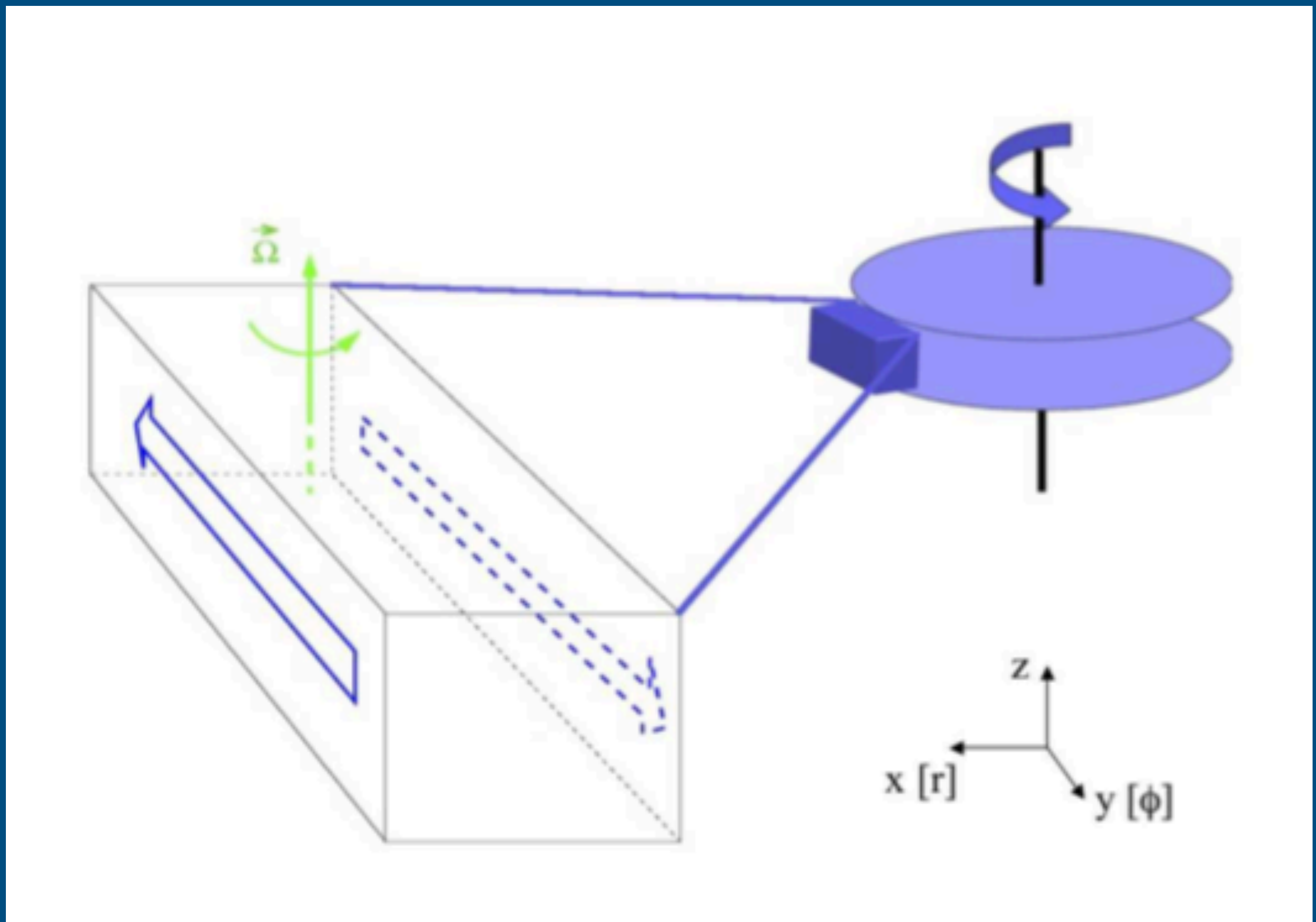
-> Fast growth for fast rotation

Wavelength: $\lambda_{MRI} \propto \frac{B}{\sqrt{\rho}\Omega}$

-> Short wavelength for weak magnetic fields

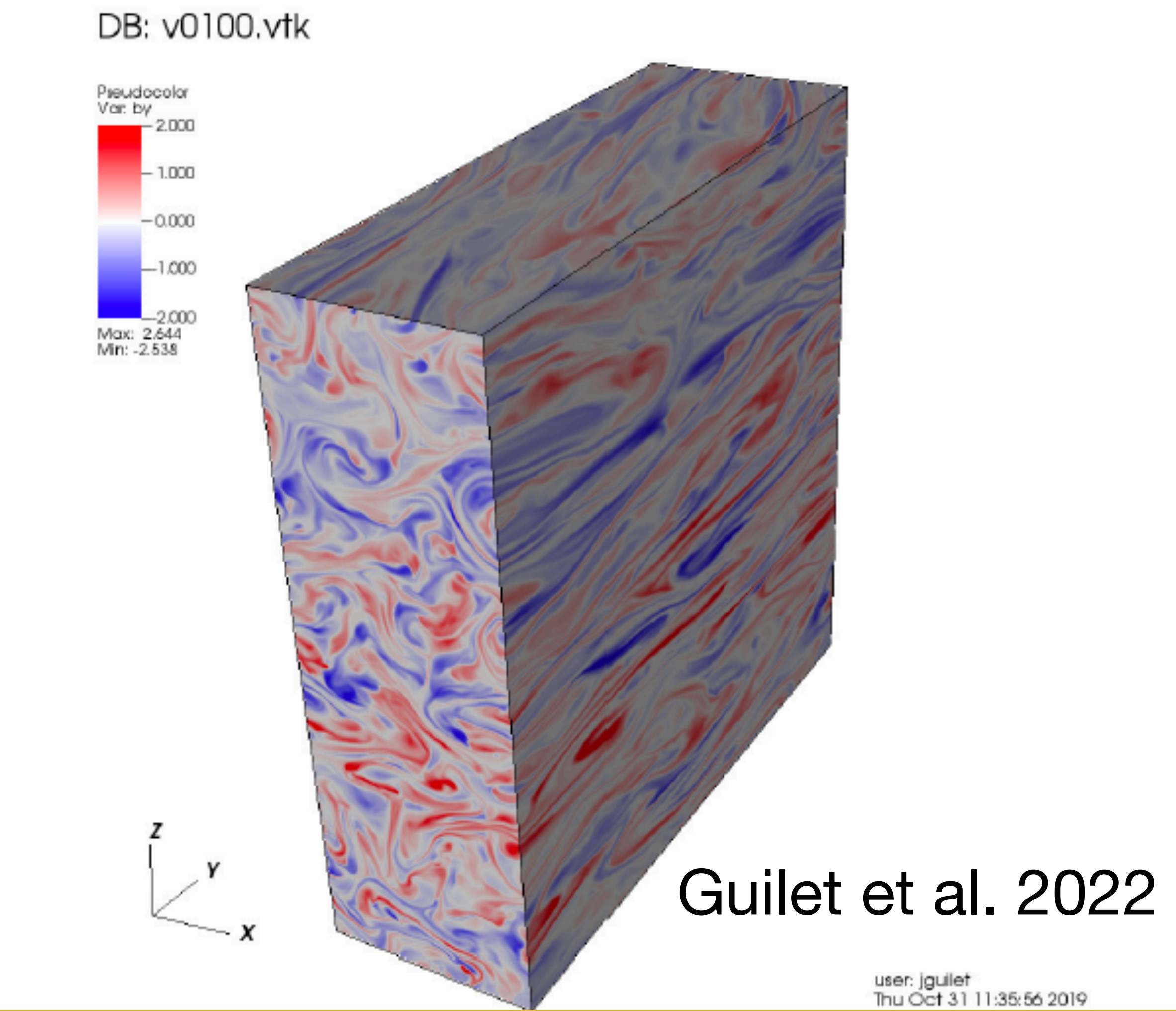
Local models in PNS

“Shearing box” models



Credit: G. Lesur

Turbulence MHD : toroidal field



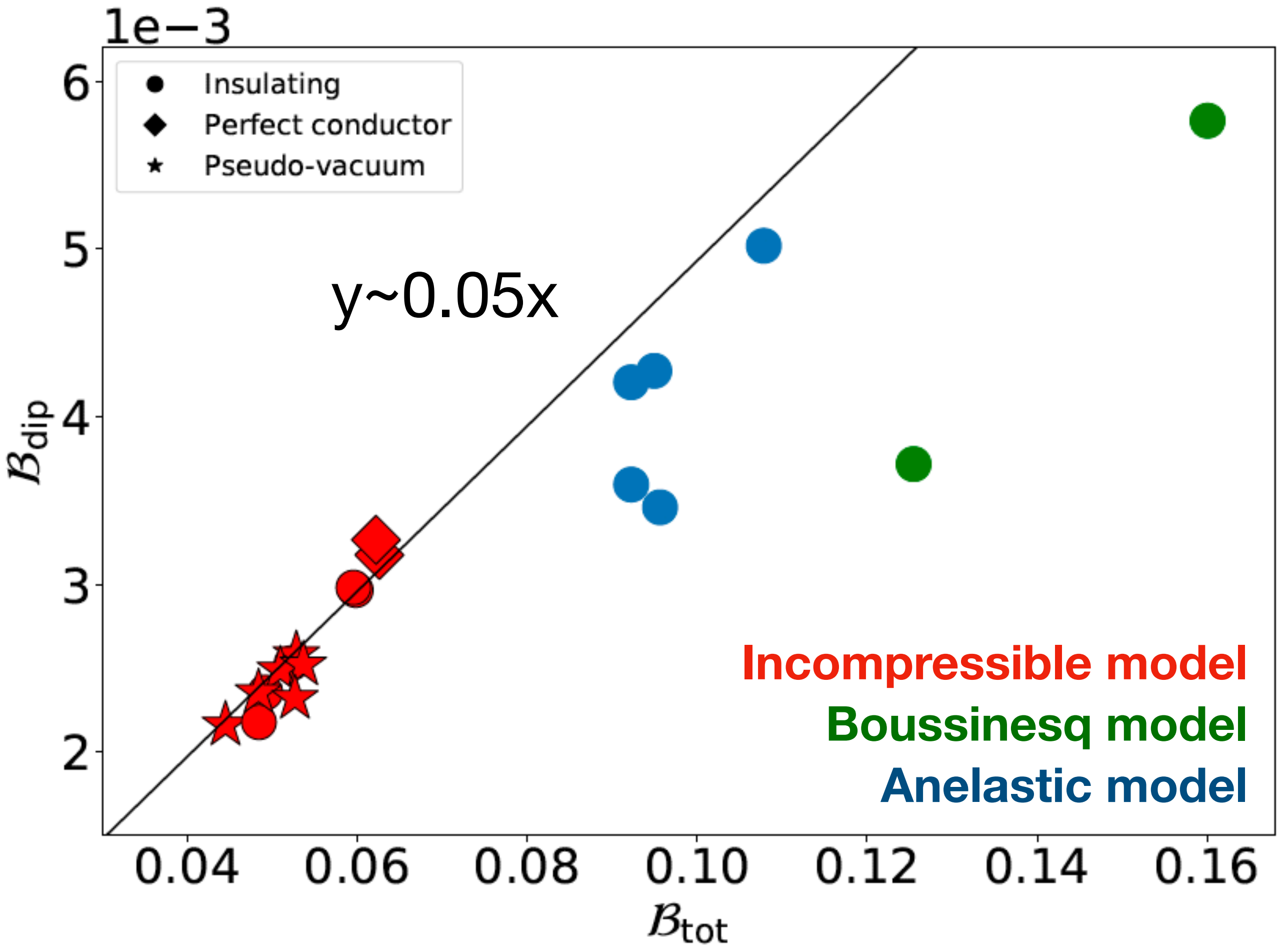
Important parameters in HMNS (and Proto-neutron stars)

Neutrino viscosity: Viscous impact on growth rate (Guilet et al. 2015,2017)
+ Magnetic Prandtl number Pm = viscosity/resistivity (Guilet et al. 2022, Held et al. 2022)

Buoyancy: Brunt-Vaisala Frequency N/Ω (Guilet et al., 2015)

Large-scale magnetic generation by the MRI in PNS

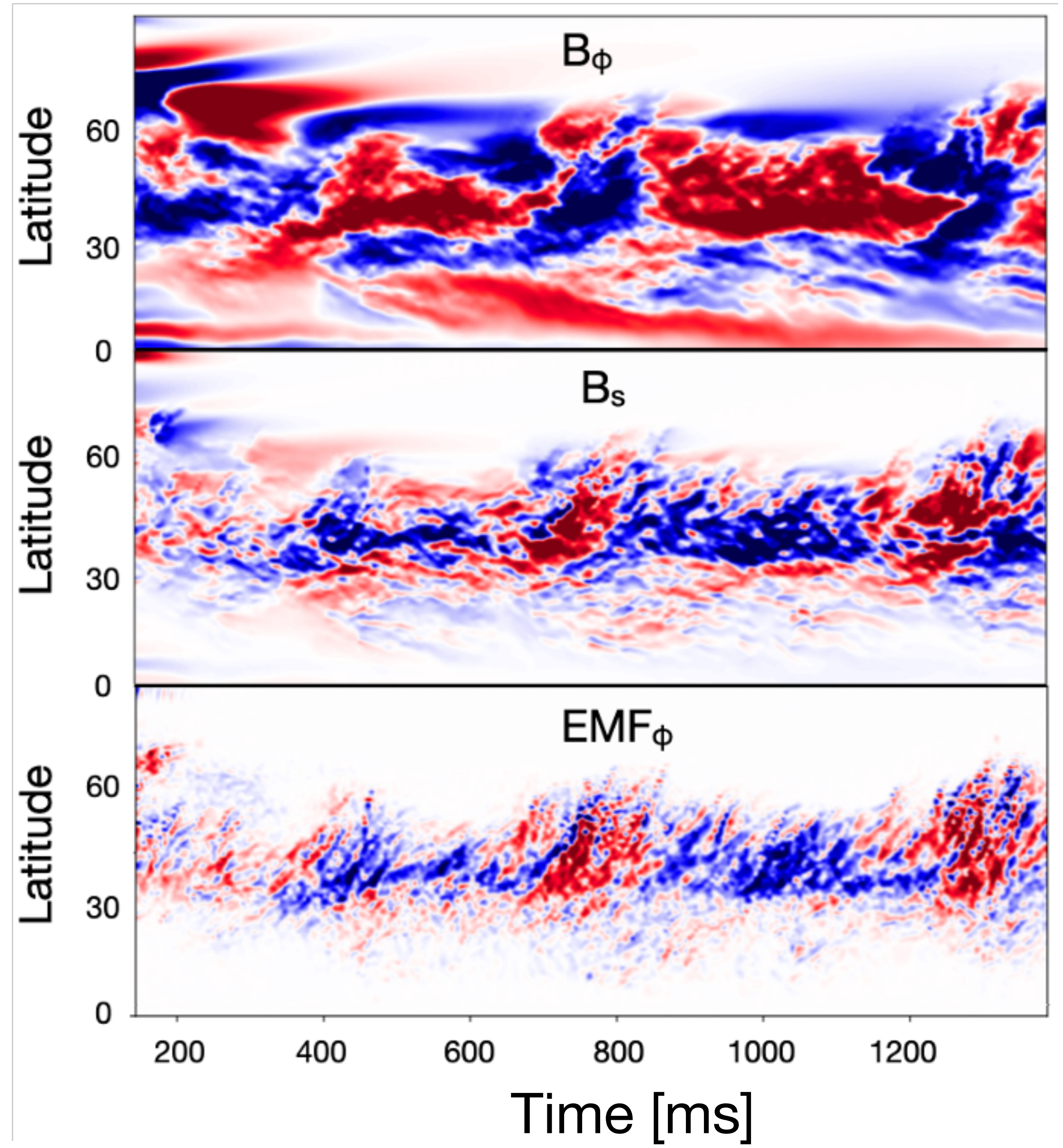
Dipole generation in simplified models



$$\mathcal{B} = \frac{B}{\sqrt{\rho_o \mu_0} D\Omega}$$

(Reboul-Salze+2021,2022)

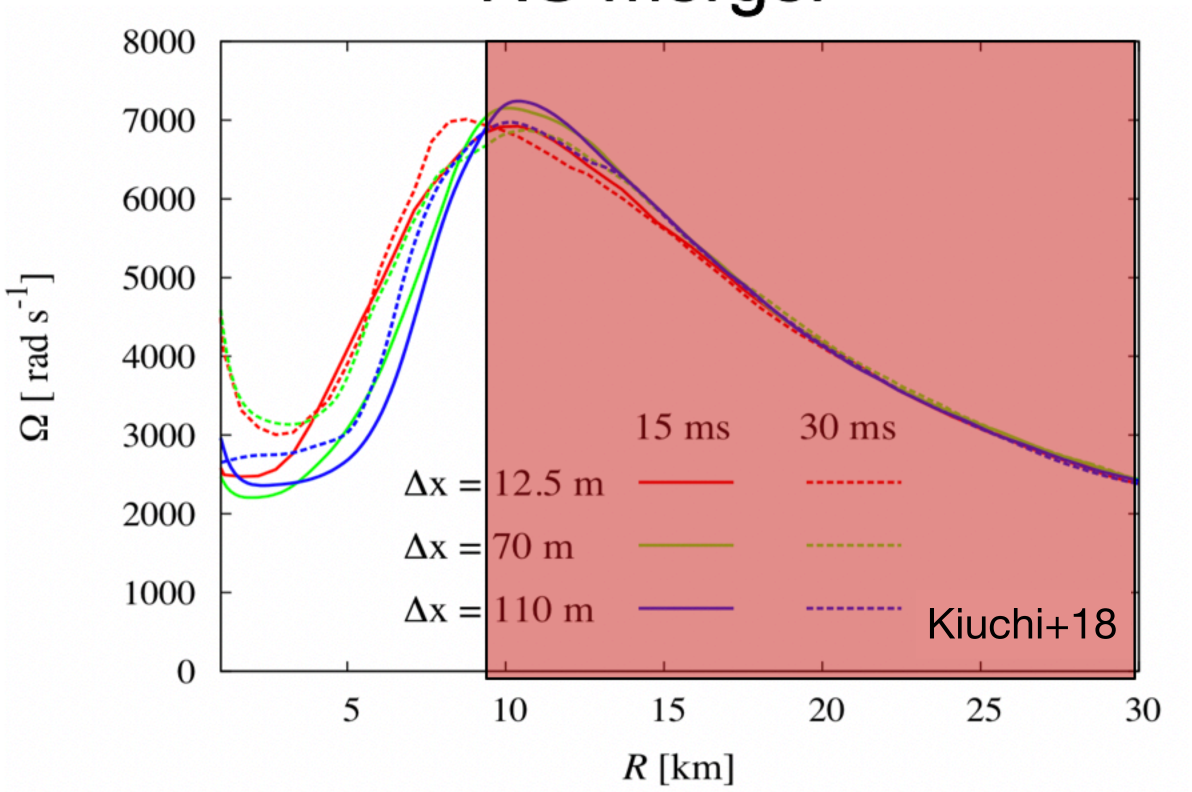
alpha-Omega dynamo in PNS



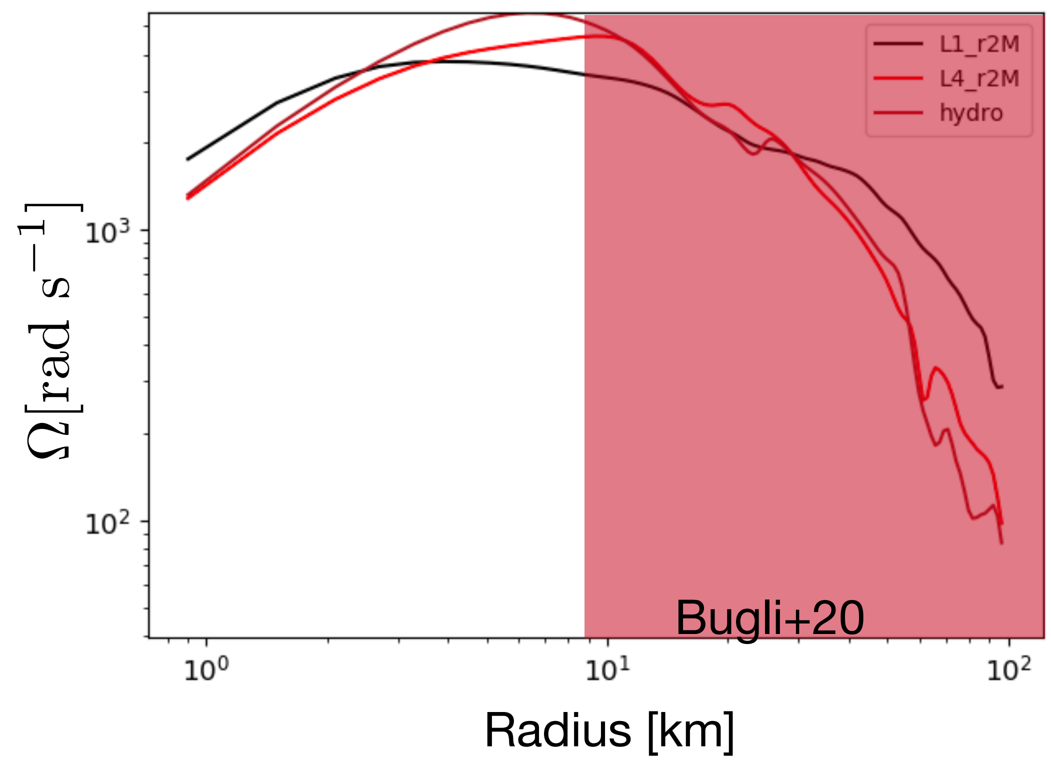
(Reboul-Salze+2022)

Core-Collapse supernovae vs NS mergers

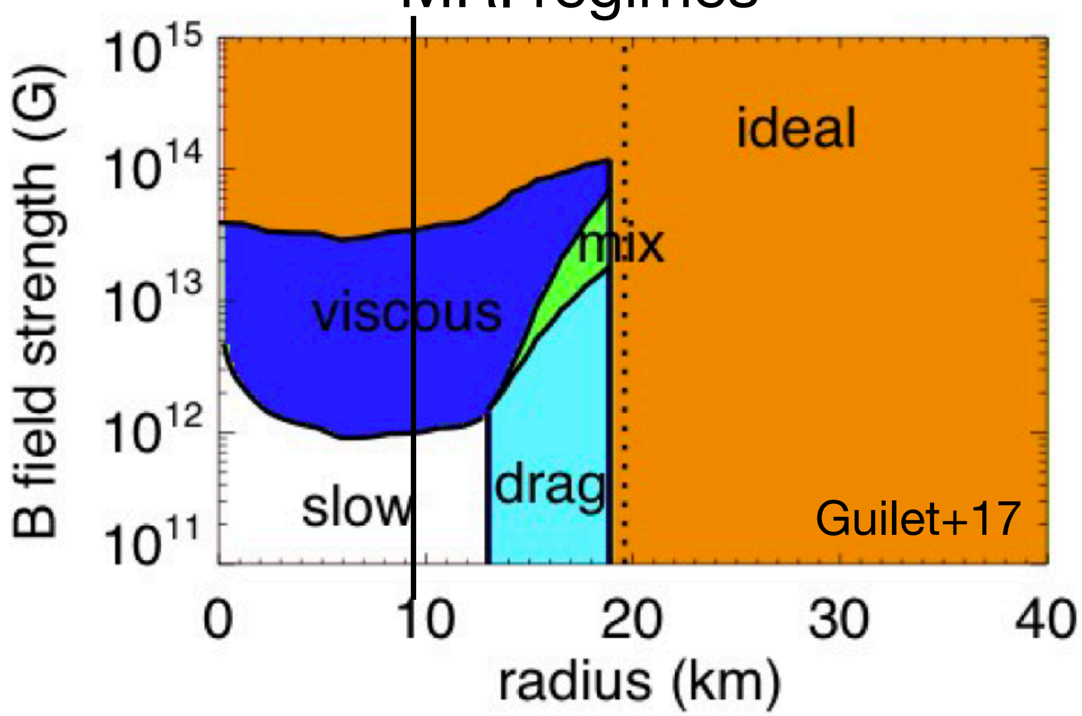
NS merger



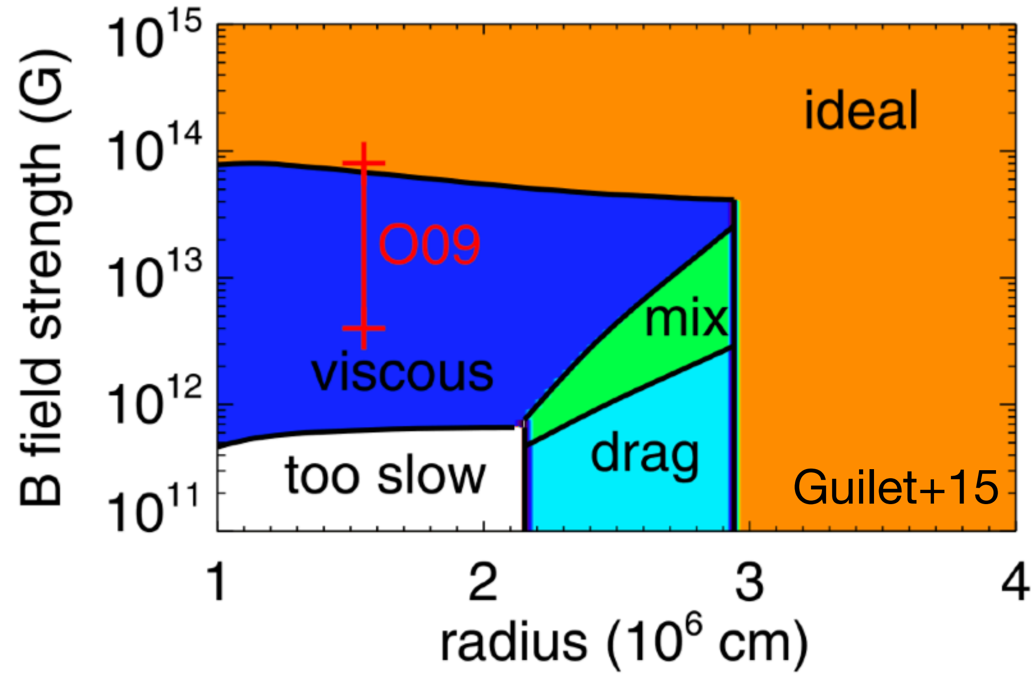
Supernovae



MRI regimes



MRI regimes



Setup for 3D GRMHD simulations (Kiuchi, R-S, et al., in prep)

Initial conditions

Neutron star binary of
1.35-1.35 solar mass,
initialised with LORENE code

Irrotational neutron stars
with initial orbital separation of ≈ 44 km

DD2 equation of state
-> Long-lived remnant for > O(1) s

Initial poloidal loop of $10^{15.5}$ G
to resolve the MRI in the simulation

Numerical methods

Latest version of
3D ideal GRMHD radiation code

Advanced Riemann HLLD solver
(Kiuchi et al 2022)

Fixed-Mesh Refinement

Gray M1+GR-Leakage scheme

Equatorial plane symmetry

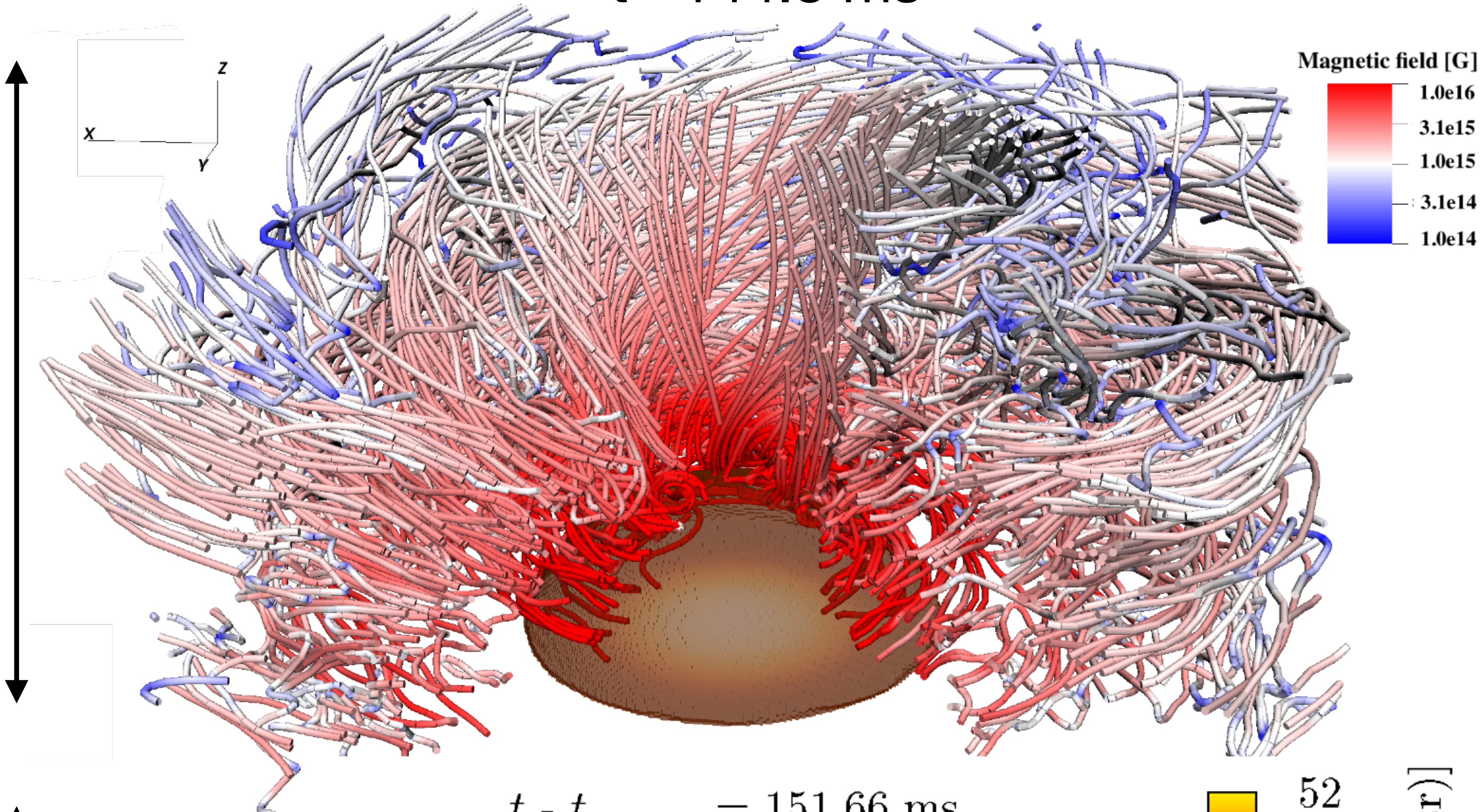
Resolution $2N \times 2N \times N$ grid
with $N = 361$
$\Delta x\_finest = 12.5$ m then relaxed to $\Delta x = 50$m
and $\Delta x = 100$m

Simulation performed by Kenta Kiuchi

Magnetic field lines and jet

t= 144.8 ms

35 km

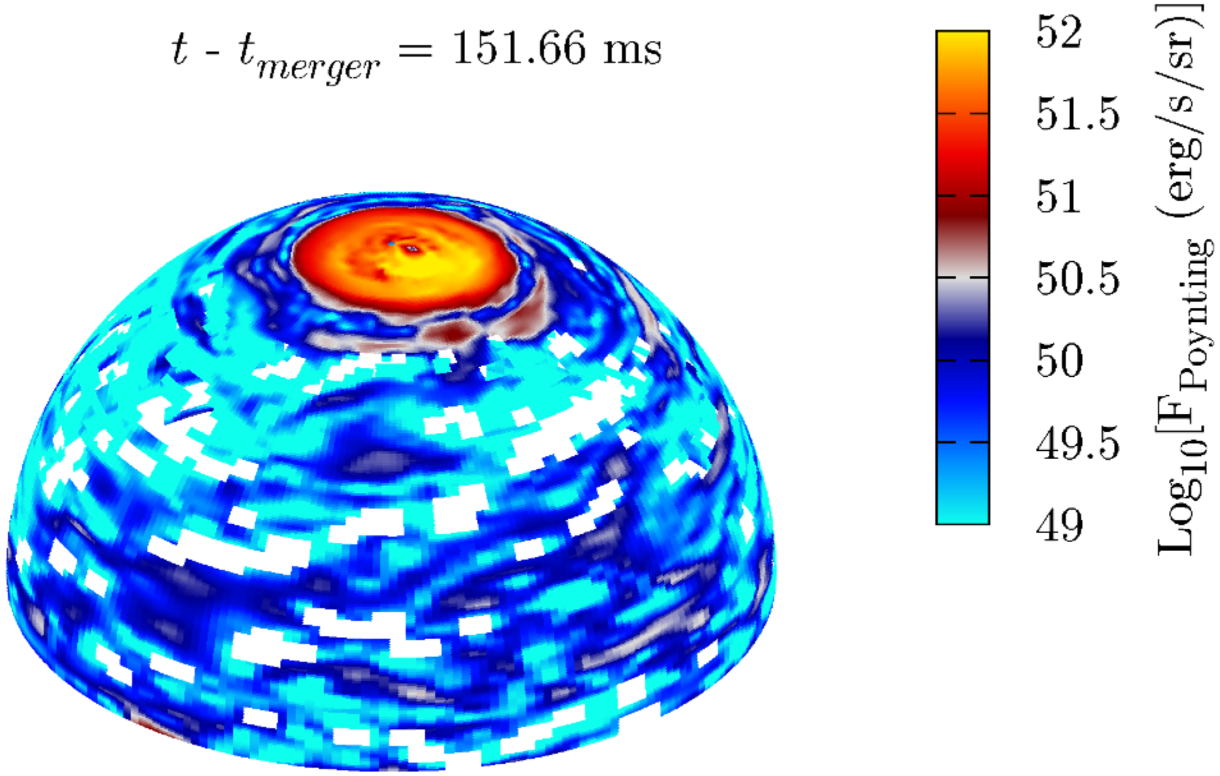


Turbulence dominated by the toroidal field

Jet starts from ~10 km

t - t_{merger} = 151.66 ms

500 km



Pointing-flux luminosity ~ 10^{51}erg/s

Jet angle $\theta < 12^{\circ}$

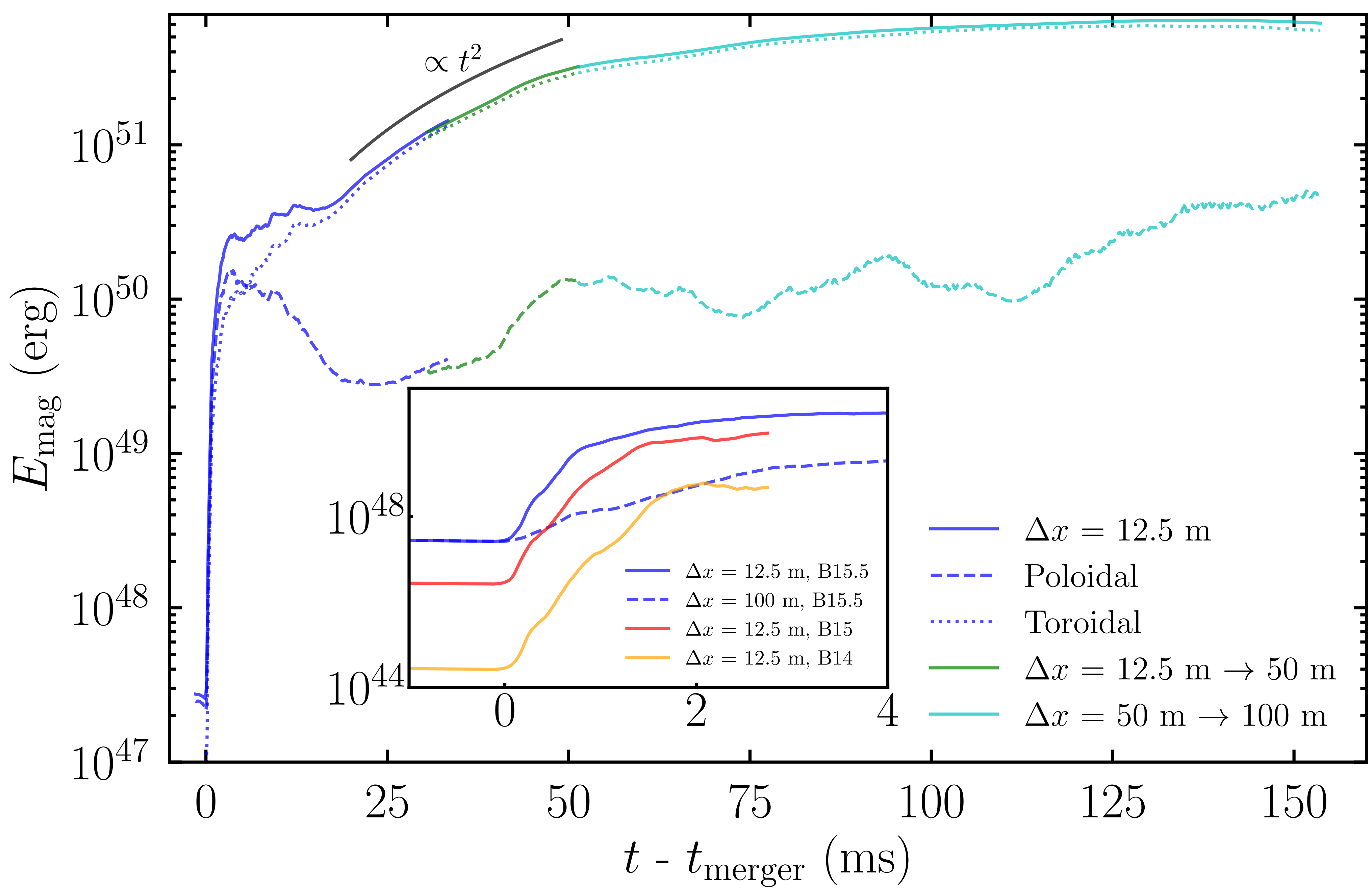
Terminal Lorentz factor Γ_{∞}~ 2-2.3

Jet+winds:

Post-merger ejecta Mass $> 0.1 M_{\odot}$

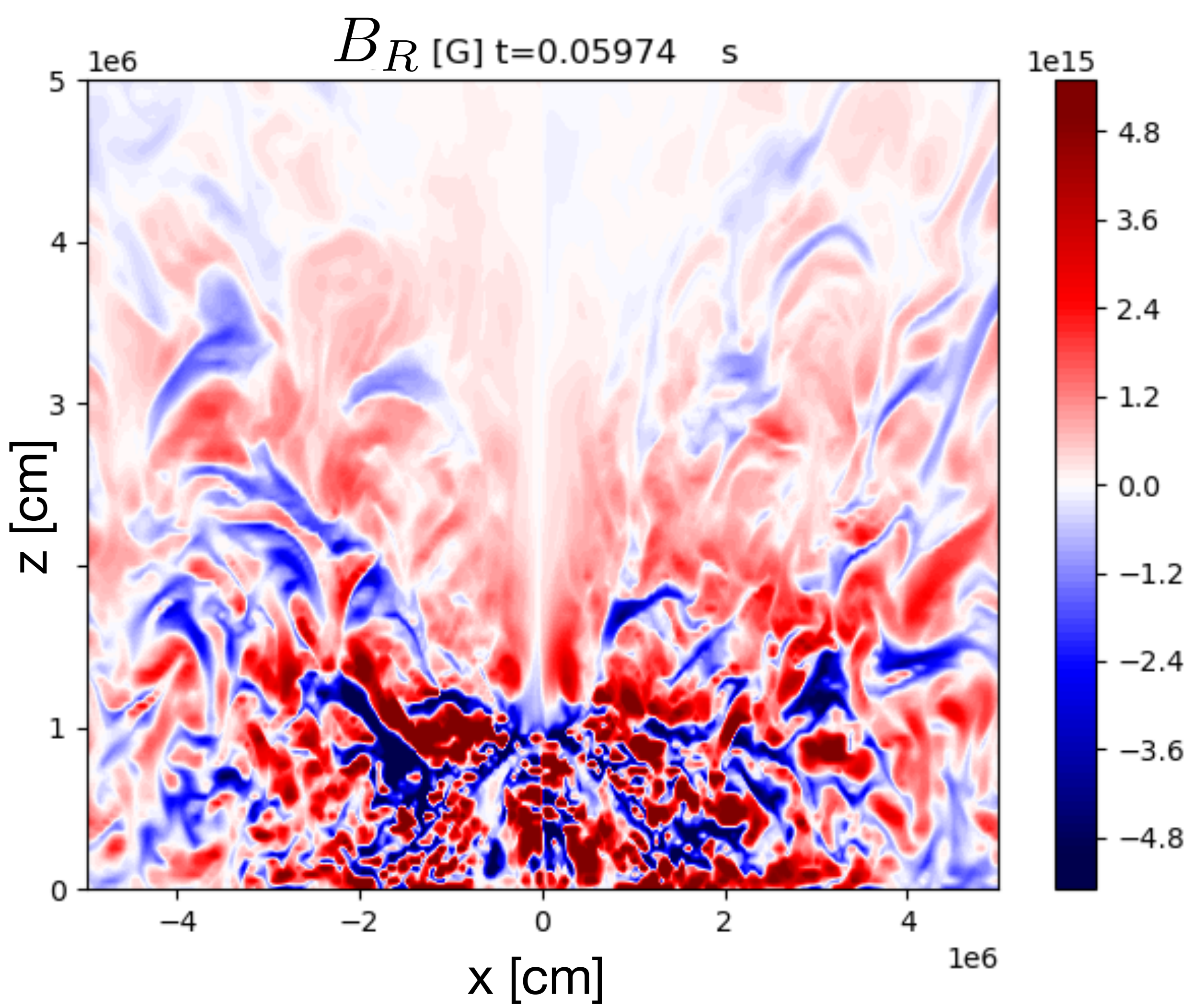
Time evolution of magnetic energies

Different phases of evolution



Turbulent field vs axisymmetric field

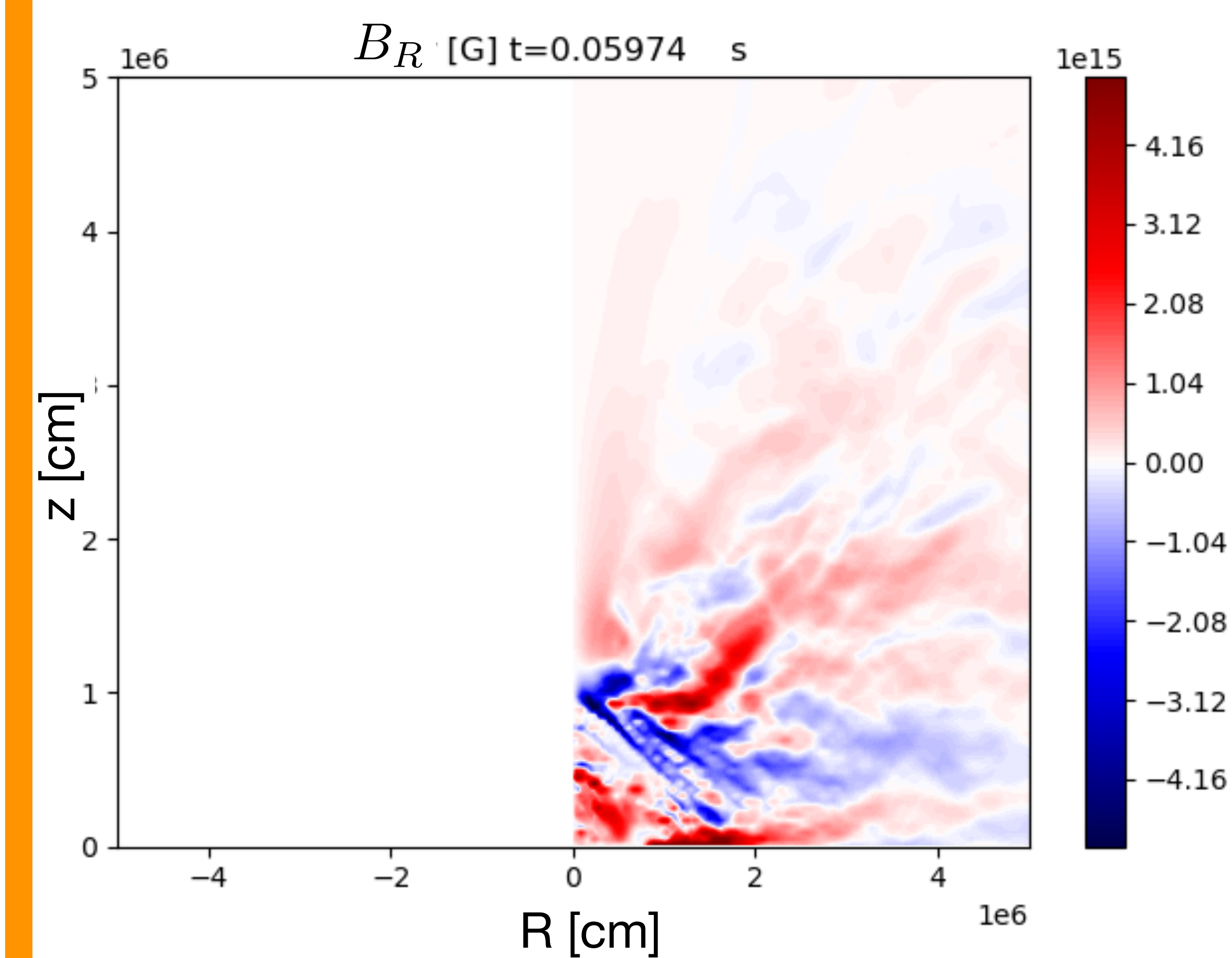
Total radial magnetic field



Turbulence strength decrease with radius

"small-scale" turbulence

Axisymmetric magnetic field

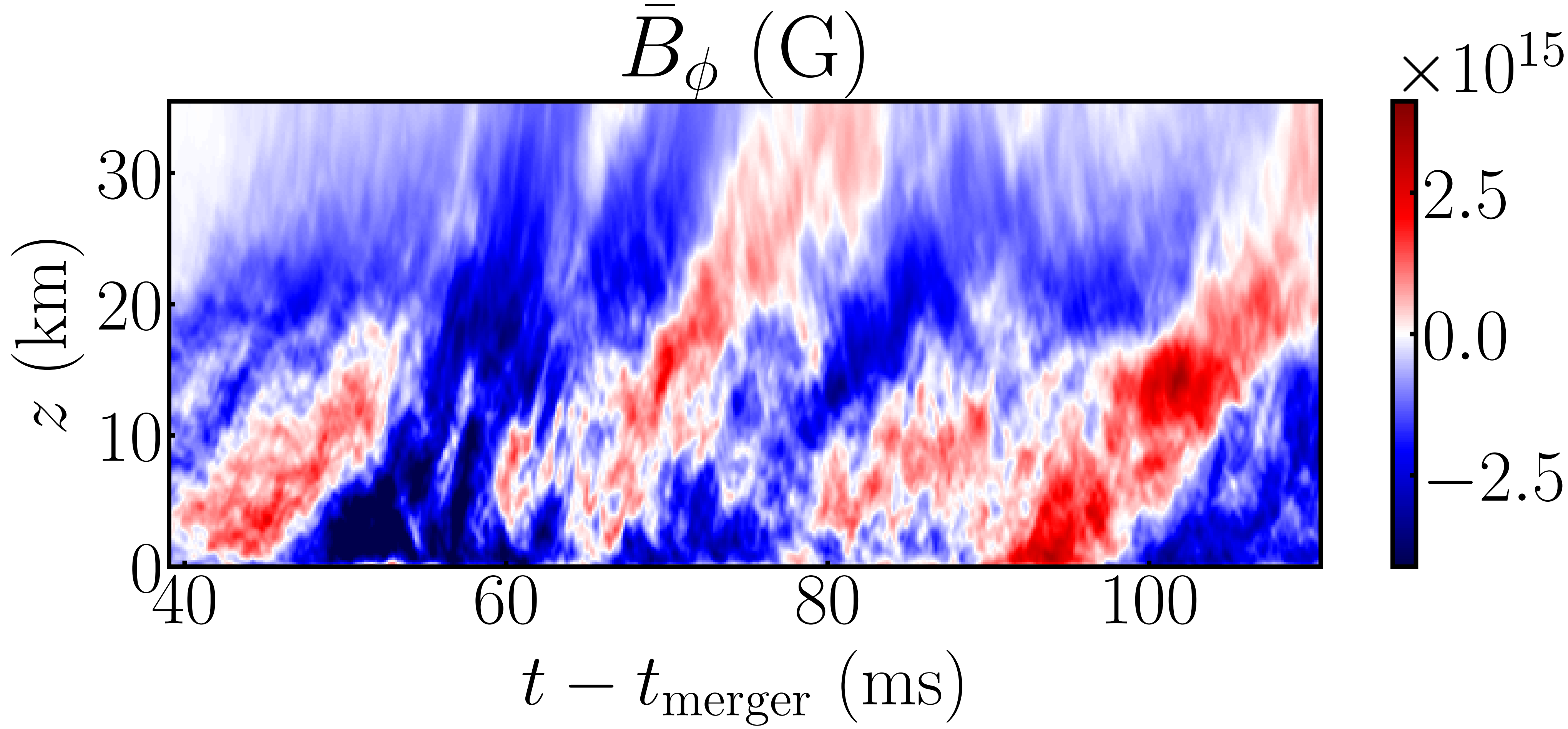


weaker axisymmetric field

larger scale axisymmetric field

Axisymmetric field time evolution

Butterfly diagram for R= 30 km



Mean-field theory

Principle

$$\vec{B} = \overline{\vec{B}} + \vec{b} \quad \text{et} \quad \vec{U} = \overline{\vec{U}} + \vec{u}$$

Induction equation for mean field

in ideal GRMHD

$$\frac{\partial \overline{\vec{B}}}{\partial t} = \vec{\nabla} \times \left(\overline{\vec{U}} \times \overline{\vec{B}} + \vec{\mathcal{E}} - \eta \vec{\nabla} \cancel{\times \overline{\vec{B}}} \right)$$

with $\vec{\mathcal{E}} = \overline{\vec{u} \times \vec{b}}$ the electromotive force (EMF)

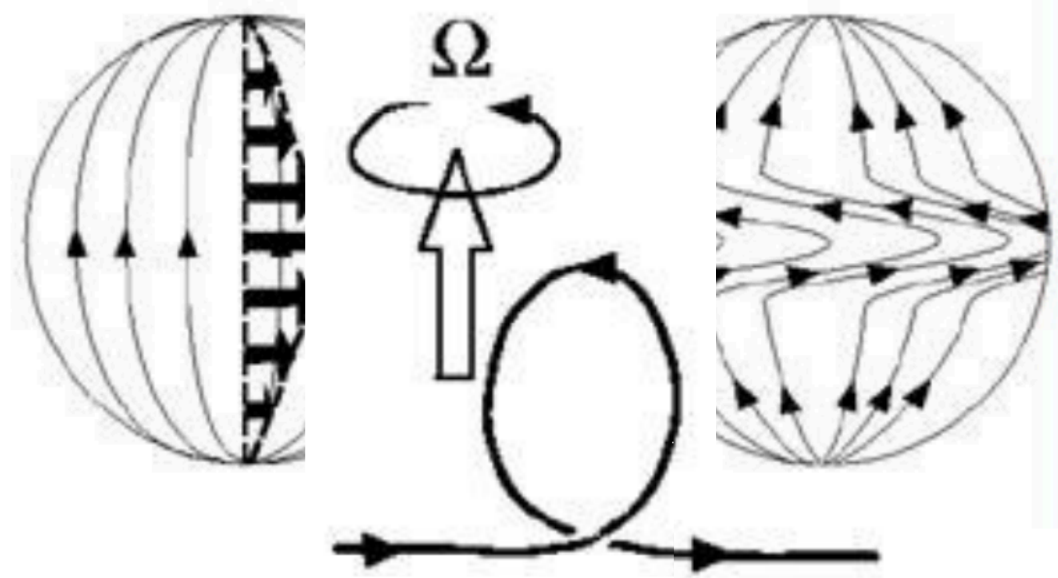
Dynamo loop

Omega effect $\quad \dfrac{\partial B_\phi}{\partial t} = r \sin\theta \vec{B}_P \cdot \vec{\nabla}\Omega$

We can write $\quad \mathcal{E}_i = \alpha_{ij}\overline{B}_j + \beta_{ij}\left(\vec{\nabla} \times \overline{\vec{B}}\right)_j + \ldots$

Alpha effect $\quad \mathcal{E}_i = \alpha_{ij}\overline{B}_j$

Turbulent resistivity $\quad \mathcal{E}_i = \beta_{ij}\overline{J}_j$



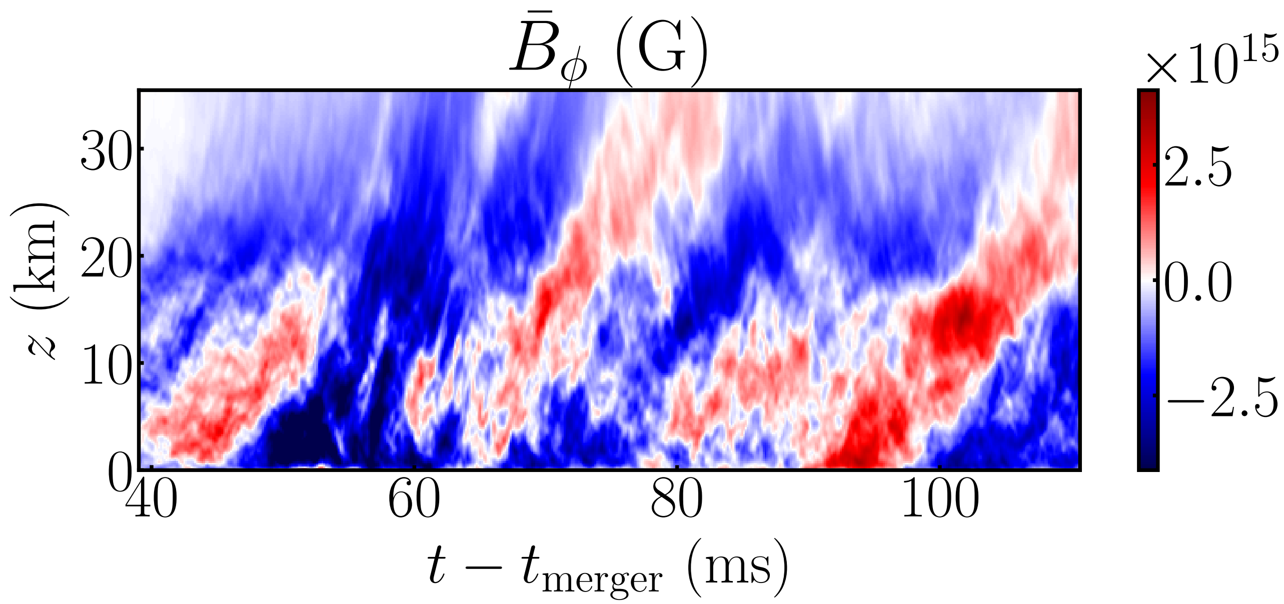
How to compute the coefficients: $\quad \alpha_{ij} = \dfrac{\langle \mathcal{E}_i \overline{B}_j \rangle}{\langle \overline{B}_j^2 \rangle}.$

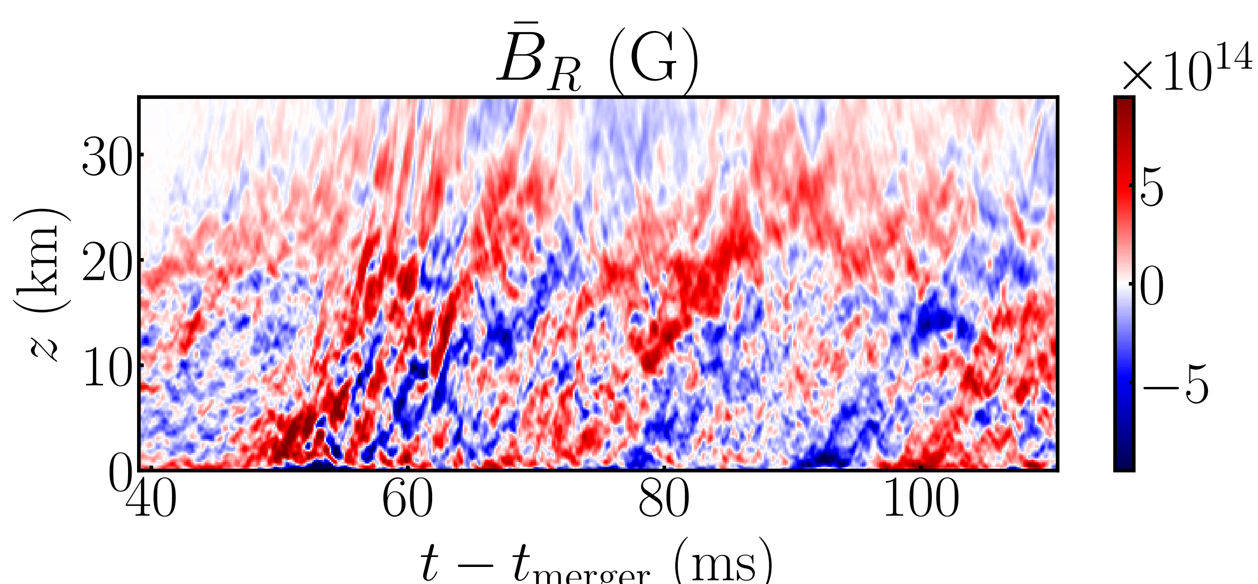
→ correlation estimations

→ Matrix estimation: Singular value decomposition (Racine+2011) $\quad \vec{\mathcal{E}} = \hat{\mathcal{A}}\overline{\vec{B}} + \hat{\mathcal{N}}$

Visual correlations at R = 30 km

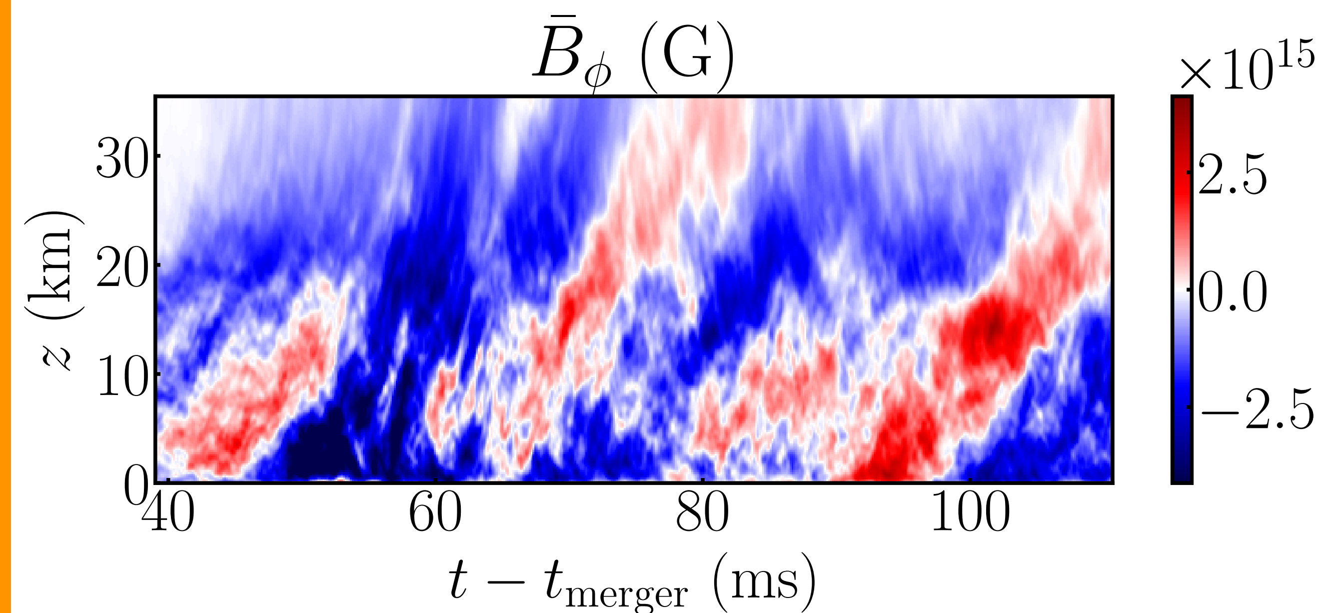
Omega effect

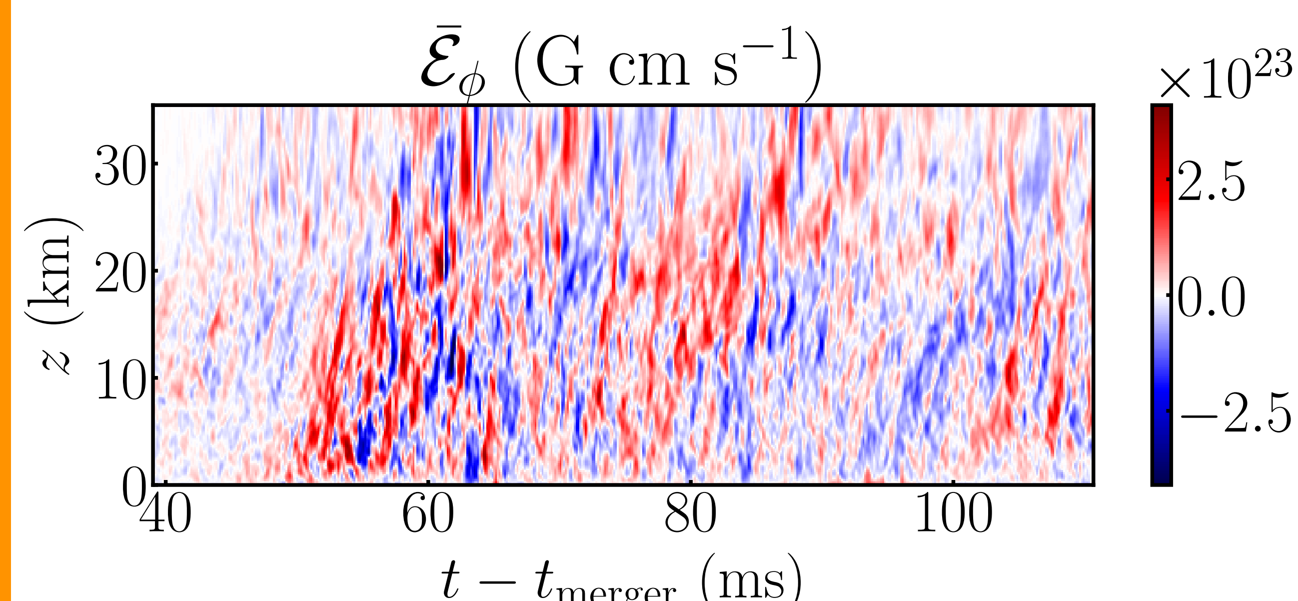




$$\frac{\partial \overline{B}_\phi}{\partial t} = \mathrm{R}\,\overline{B}_\mathrm{R}\frac{d\Omega}{ds}$$

alpha-effect

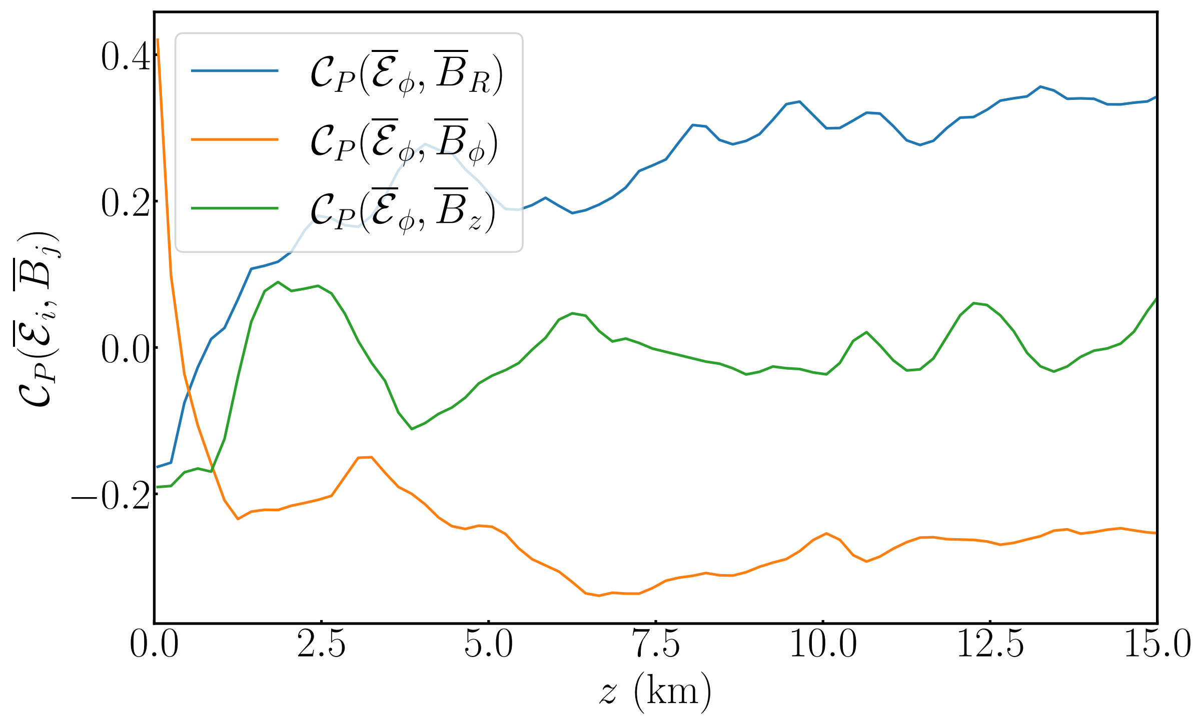




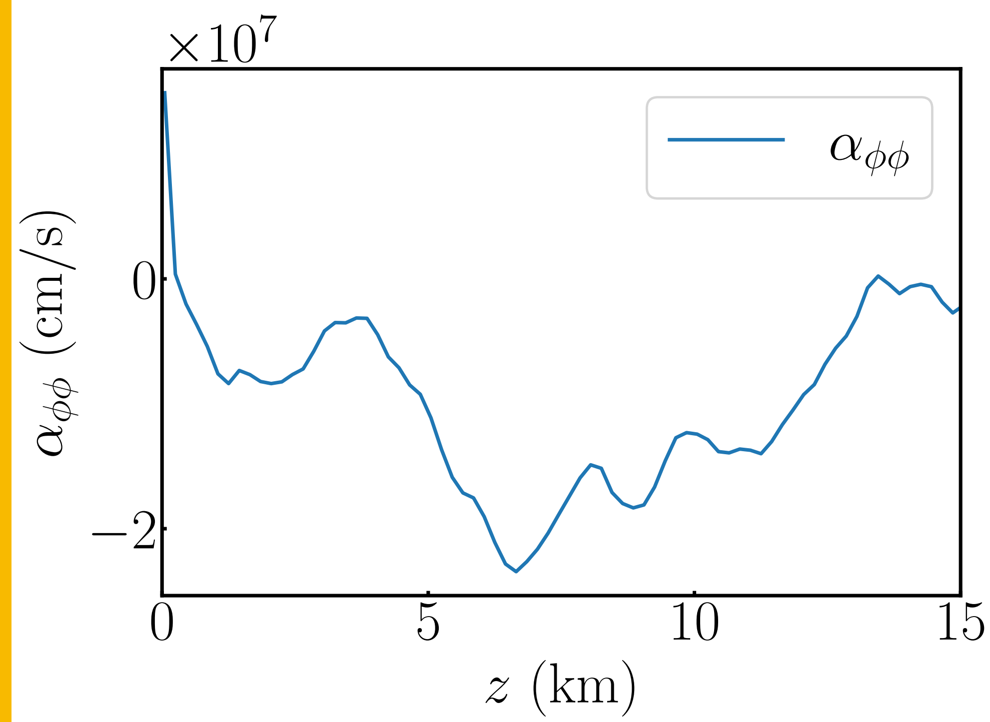
$$\mathcal{E}_\phi = \alpha_{\phi\phi}\bar{B}_\phi$$

Period estimation

Correlation for toroidal EMF



diagonal alpha component



SVD method

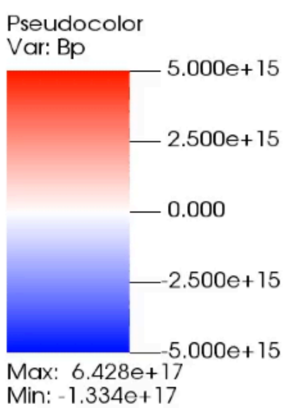
Dynamo period

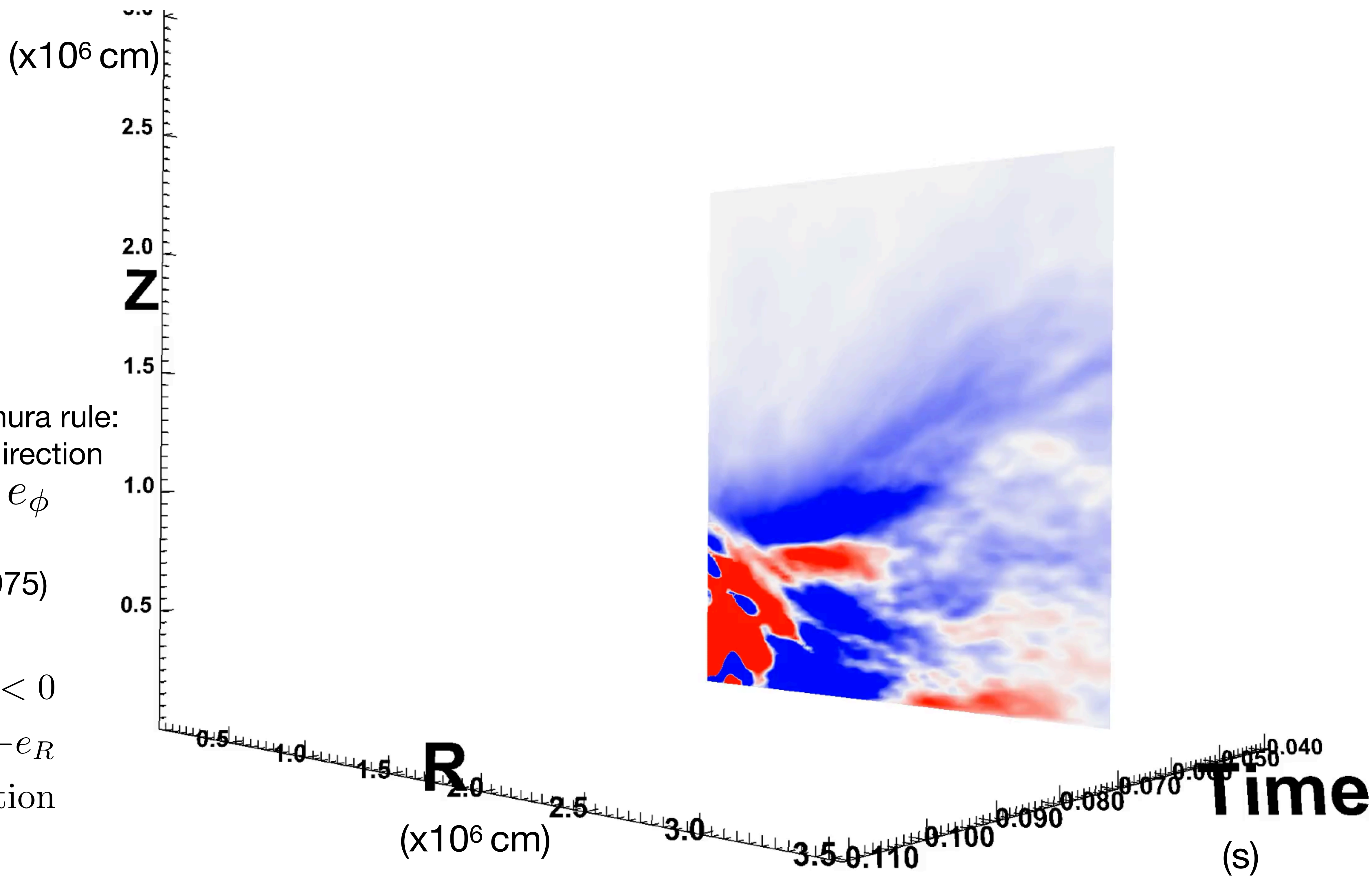
$\alpha\Omega$ dynamo period: $$P = 2\pi \left(\frac{1}{2} \alpha_{\phi\phi} \frac{d\Omega}{d \ln s} k_z \right)^{-1/2}$$ (Gressel+ 2015)

km	20	30	40
Omega [rad s^{-1}]	4200	2515	1688
shear rate q	-1.0	-1.34	-1.44
alpha$_{\phi\phi}$ [cm s^{-1}]	-8.1x10^6	-1x10^7	-1x10^7
Theoritical period [ms]	**20**	**24**	**37**
Simulation period [ms]	**18**	**19-24**	**20-30**

Averaged values over a scale height H (kz = 2*pi/H)
-> compatible with simulation periods

Dynamo wave propagation





Parker-Yoshimura rule:
Propagation direction
$\alpha_{\phi\phi} \nabla\Omega \times e_\phi$
(Parker 1955,
Yoshimura 1975)

$$\alpha_{\phi\phi} < 0$$
$$\nabla\Omega \propto -e_R$$
$$\rightarrow +e_z \text{direction}$$

A HMNS anelastic model (Reboul-Salze et al., in prep)

3D GRMHD 1.35-1.35 neutron star merger with a long-lived remnant (EOS DD2)

HMNS simulation at t = 17ms:

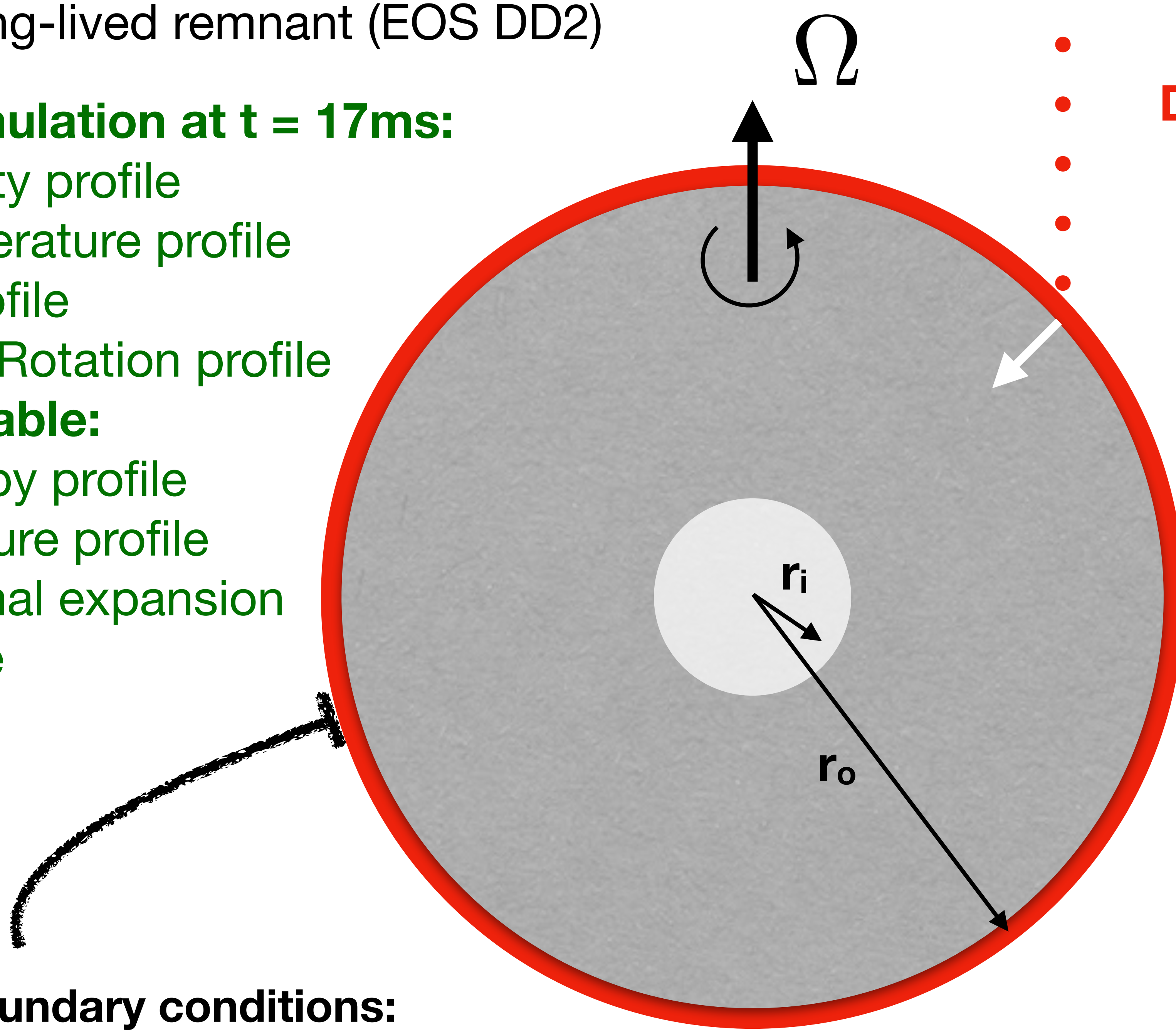
- Density profile
- Temperature profile
- Ye profile
- Initial Rotation profile

EOS table:

- Entropy profile
- Pressure profile
- Thermal expansion profile

Hypotheses:

- **Spherical geometry**
- **Thermal stratification**
- **Diffusive regime for neutrino**
- **Constant diffusivities**
- **Low Mach number**
- **Hydrostatic equilibrium**



Boundary conditions:

- **Mechanical : Inner/no slip, outer/no slip**
- **Magnetic : Insulating**

Parameters

$\rho_0 = 1.12 \times 10^{12}$ g cm^{-3}

$\Omega_0 = 4913$ rad s^{-1}

$r_0 = 30$ km

nu = 2.3×10^{12} cm^2 s^{-1}

Pr = 0.0037

N/Omega = 1.4

Pm = 16

Anelastic MHD equations: volume forcings

Separation of variables: $\vec{V} = \vec{U} + \vec{V_f}$

$$
\begin{cases}
\vec{\nabla} \cdot \left(\bar{\rho}\vec{U}\right) = 0 \quad \text{et} \quad \vec{\nabla} \cdot \vec{B} = 0, \\
\dfrac{\partial U}{\partial t} + \left(\left(\vec{U} + \vec{V_f}\right) \cdot \vec{\nabla}\right)\left(\vec{U} + \vec{V_f}\right) + 2\Omega_0 \vec{e}_z \times \left(\vec{U} + \vec{V_f}\right) = -\vec{\nabla}\left(\dfrac{\Pi'}{\bar{\rho}}\right) + \dfrac{g_0 r_0^2}{C_p r^2} S' \vec{e}_r \\
\qquad - \tau U_{\phi, m=0} + \dfrac{1}{\mu_0 \bar{\rho}}\left[\left(\vec{\nabla} \times \vec{B}\right) \times \vec{B}\right] + \dfrac{\nu}{\bar{\rho}} \vec{\nabla} \cdot \vec{\vec{\sigma}}, \\
\dfrac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left[\left(\vec{U} + \vec{V_f}\right) \times \vec{B}\right] + \eta \vec{\nabla}^2 \vec{B}, \\
\bar{\rho}\bar{T}\left(\dfrac{\partial S'}{\partial t} + \left(\left(\vec{U} + \vec{V_f}\right) \cdot \vec{\nabla}\right) S' + \left(U_r + \vec{V_f}\right)\dfrac{\mathrm{d}\bar{S}}{\mathrm{d}r}\right) = \kappa \vec{\nabla} \cdot \left(\bar{\rho}\bar{T}\vec{\nabla} S'\right) + \nu Q_\nu + \dfrac{\eta}{\mu_0^2}\left(\vec{\nabla} \times \vec{B}\right)^2
\end{cases}
$$

Coriolis — Pressure — Buoyancy

“Dissipation” Forcing — Lorentz — Viscosity

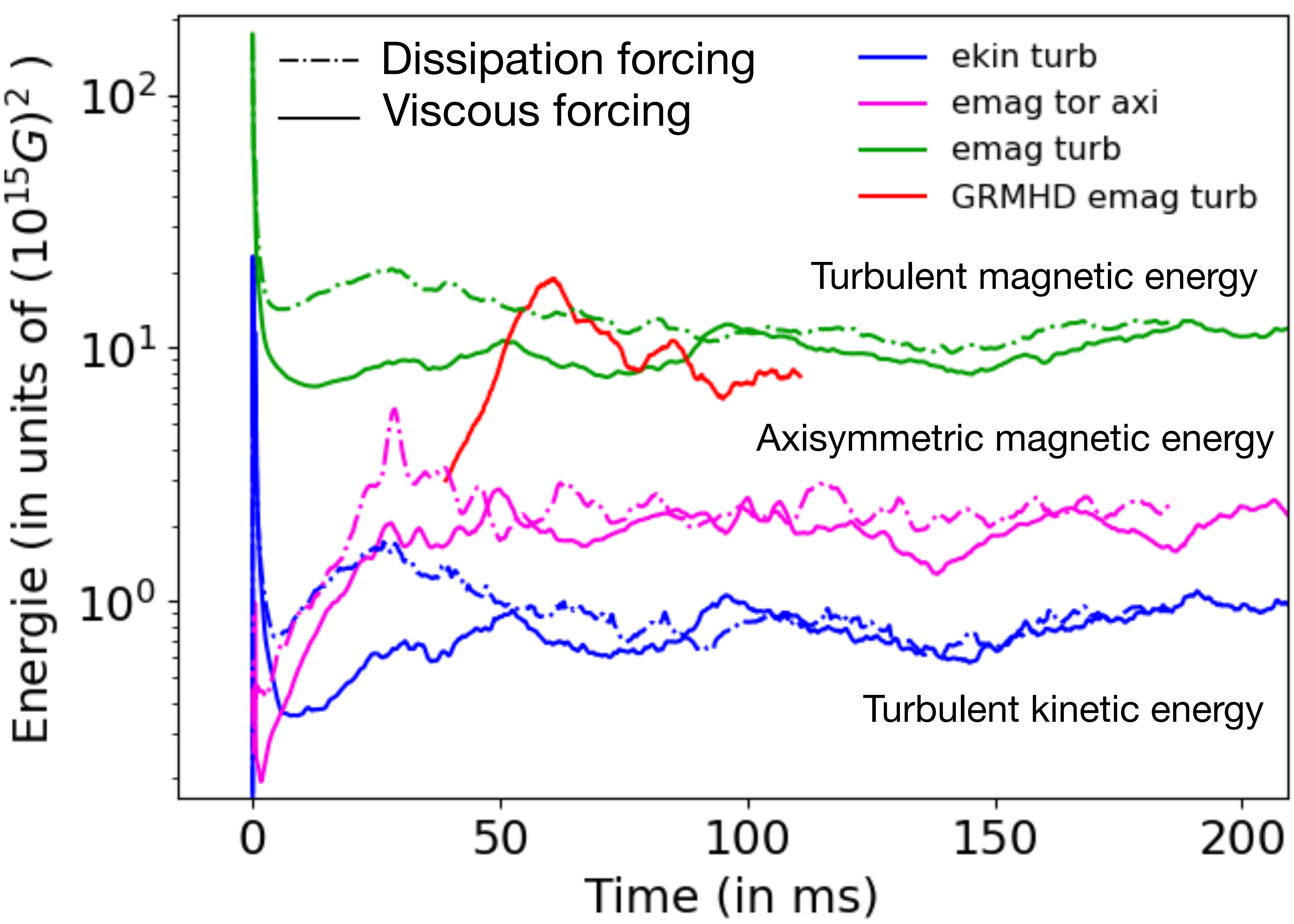
Induction — Resistivity

Advection — Heat flux — Heating

→ More efficient forcing than just forcing at the outer boundaries

In the following $\tau = 6.7 \times 10^3 s^{-1}$

Time evolution



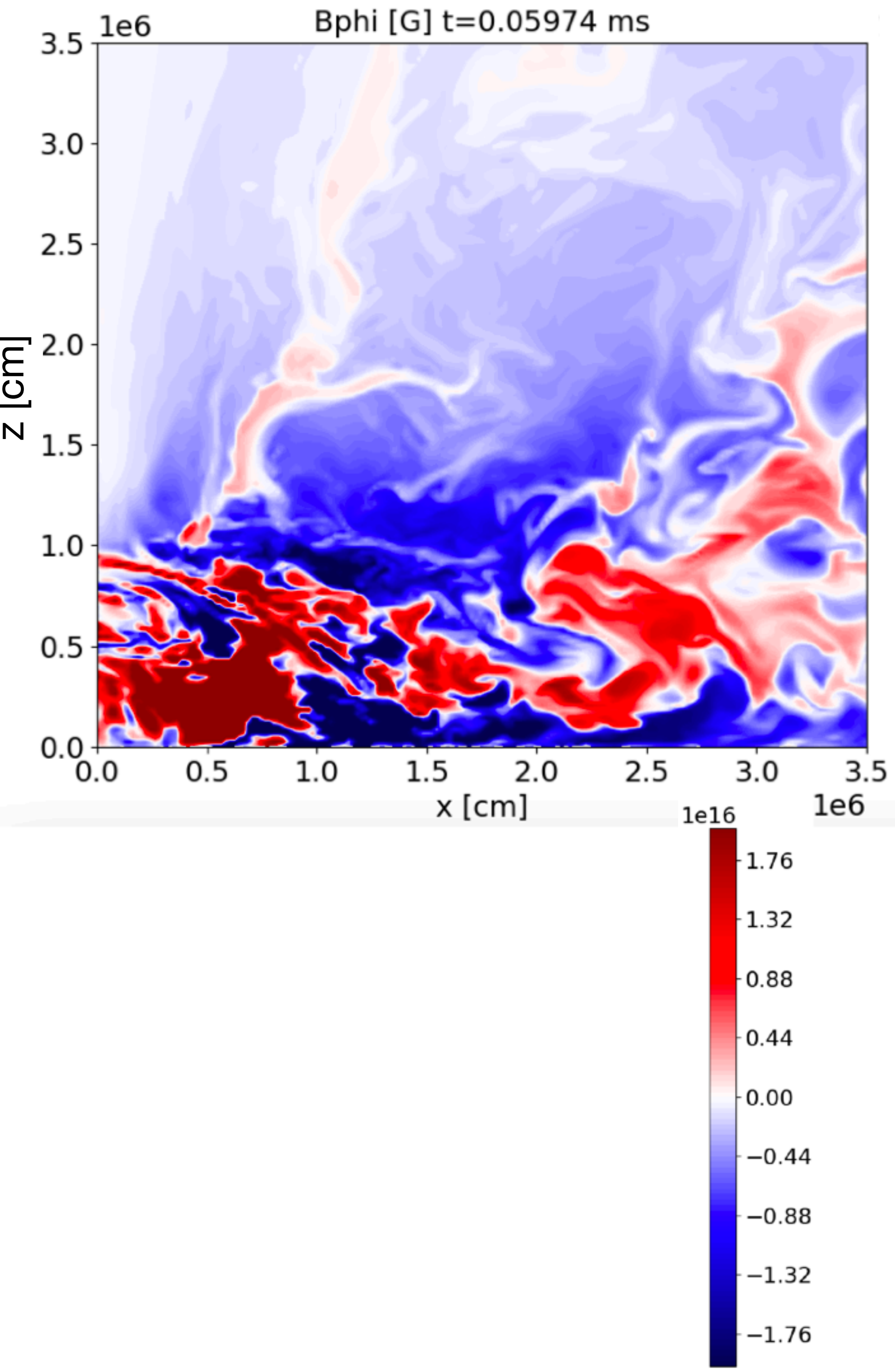
GRMHD energy with the inertial magnetic field

B ~ 3×10^{15} G

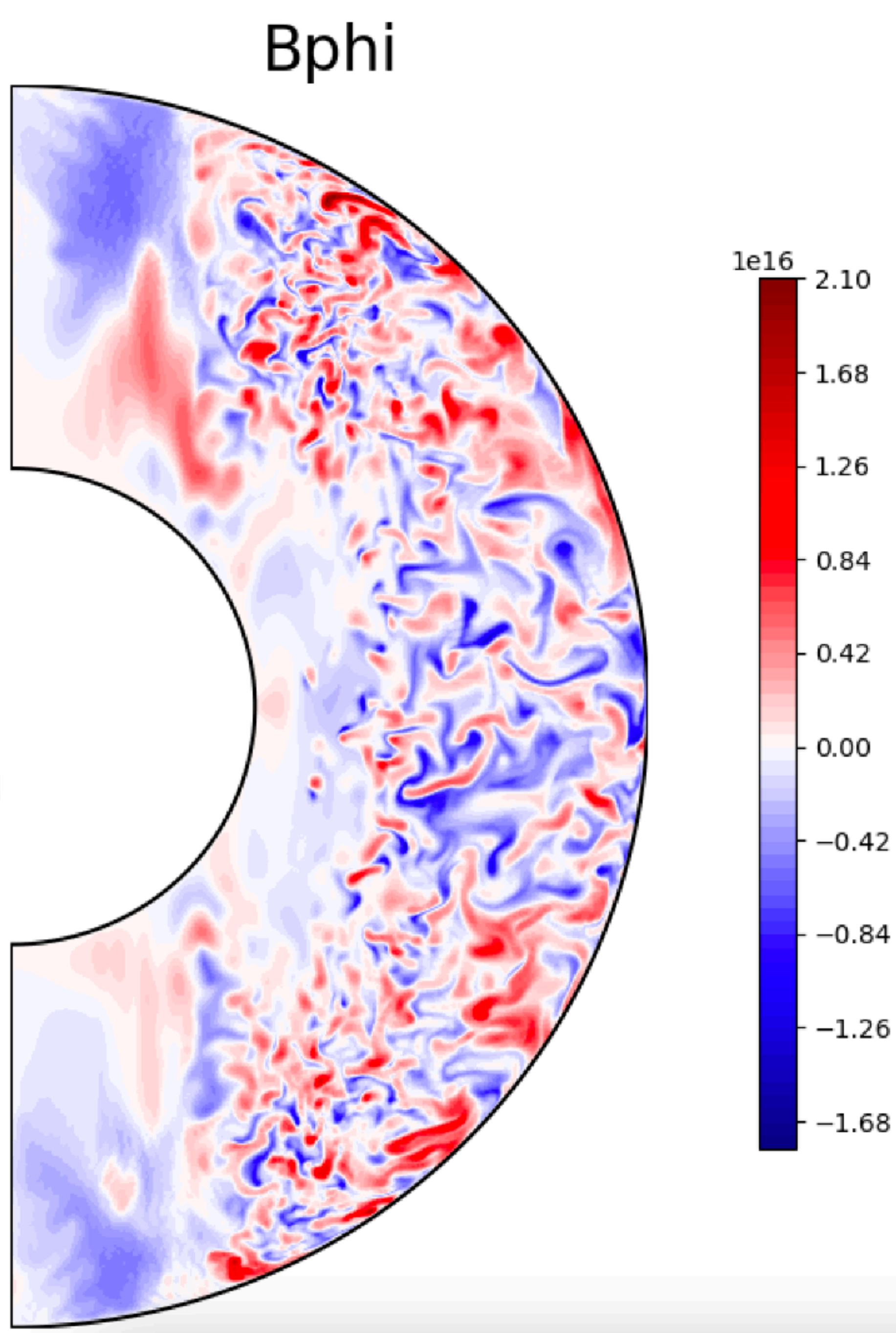
B ~ 1.5×10^{15} G

Magnetic field comparison

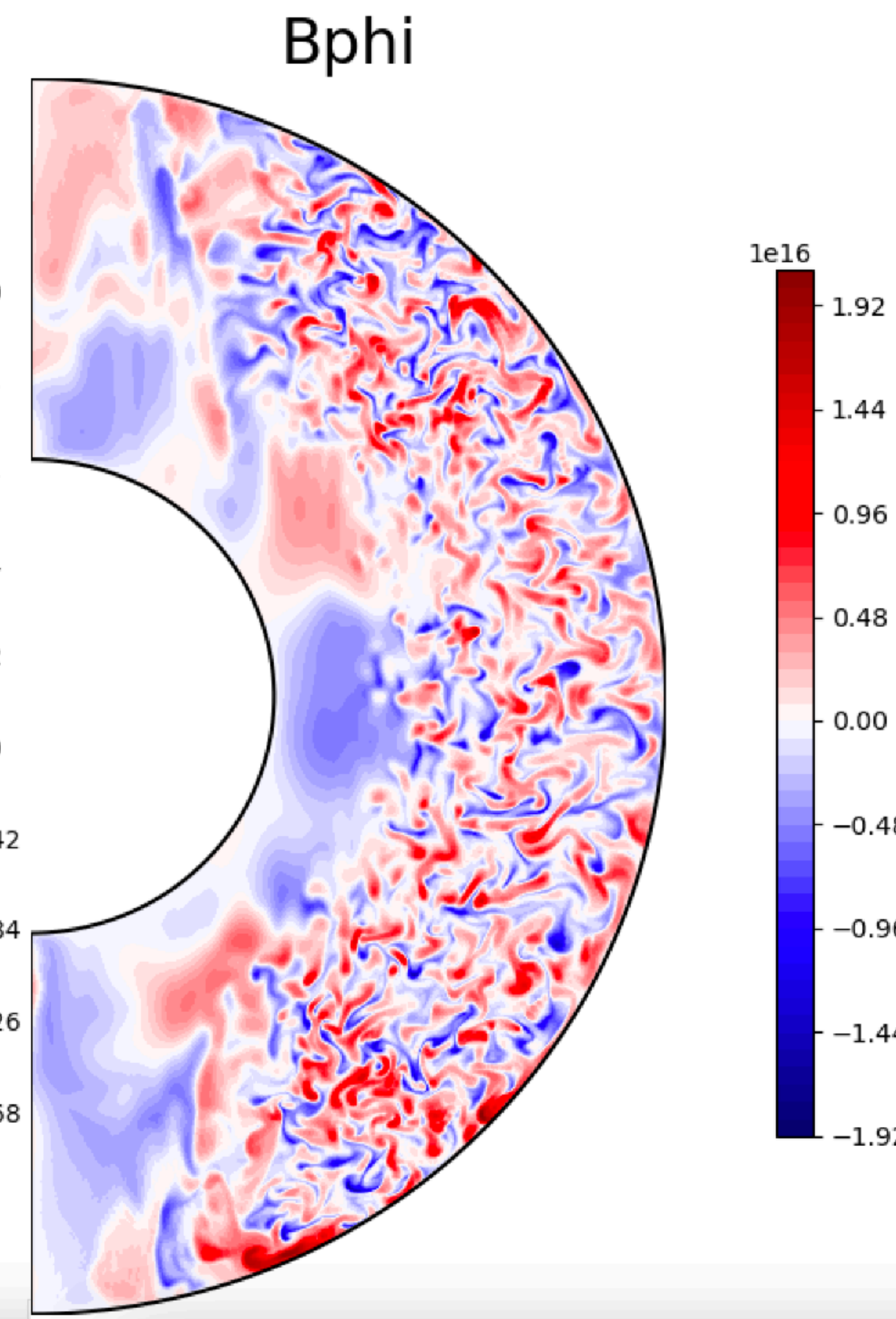
GRMHD simulation
t=60 ms



Viscous forcing t=166 ms

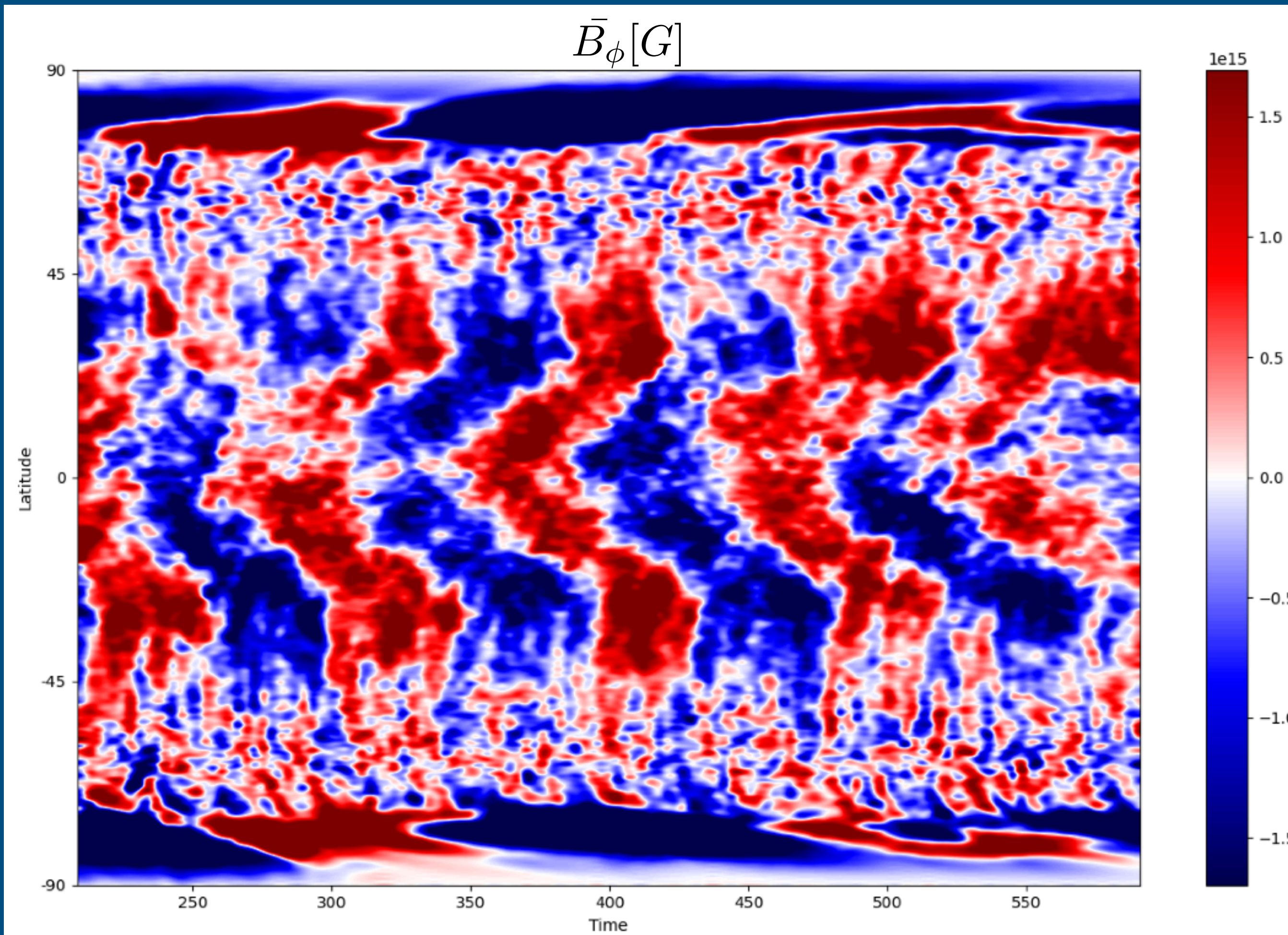


Dissipation forcing t=164ms



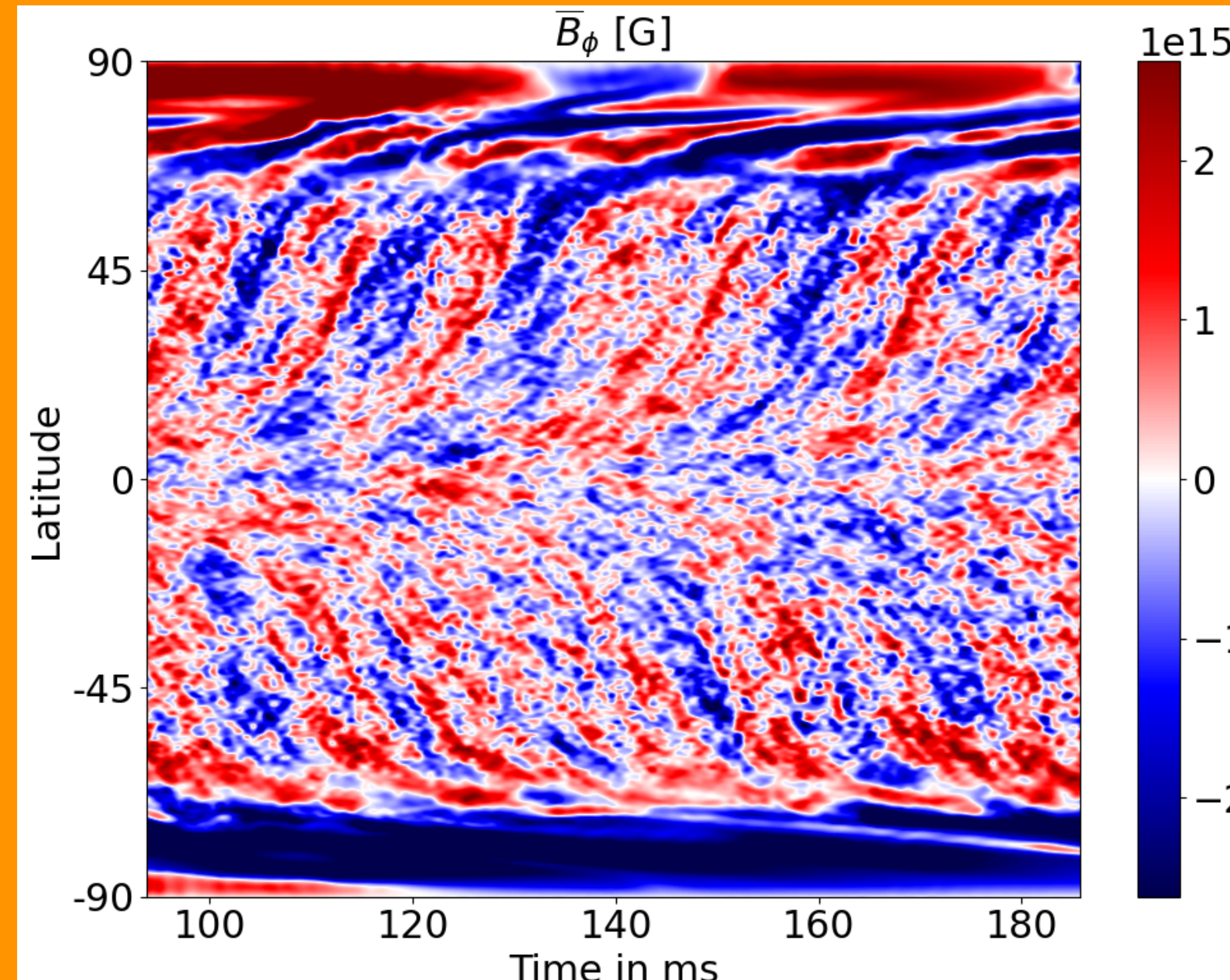
Comparison of the Forcing : Butterfly diagram at r=24 km

Viscous forcing



Period ~ 100 ms

Dissipation forcing



Period ~ 20 ms

	Viscous forcing	Dissipation forcing	GRMHD sim
Omega [rad s^{-1}]	4200	3600	2515
shear rate q	-0.2	-1.2	-1.34
alpha$_{\phi\phi}$ [cm s^{-1}]	2x 10^6	7x10^6	1x10^7
Theoritical period	**122**	**27**	**24**

Summary and perspectives

- **3D GRMHD model**

→ self-consistent MRI-driven alpha-Omega dynamo with 20 ms period.

→ Launch of a jet with properties comparable to sGRBs

→ Supports the magnetar engine of sGRBs

Kiuchi, R-S et al, in prep

- **simplified MHD models**

→ Comparable alpha-Omega dynamo with GRMHD simulation with **explicit diffusivities and no initial large-scale field**

→ **Forcing and shear rate are important parameters for comparison**

Reboul-Salze et al, in prep

- Perspectives

→ **Investigate the impact of initial magnetic field**

→ **Further analysis on the jet/ejecta and core magnetic amplification**

→ **Impact of magnetic Prandtl number on the alpha-Omega dynamo**

→ **Modelisation of alpha-Omega for larger scale simulations?**