Minimally implicit Runge-Kutta methods: RRMHD equations and neutrino transport equations in supernovae simulations

CoCoNuT Meeting 2023

Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Potsdam, Germany, April 3-5, 2023

Isabel Cordero Carrión

@FuturaConjetura

VNIVERSITAT DE VALÈNCIA

GWG

VIRGO

G2NET

COST ACTION CA17137
A NETWORK FOR GRAVITATIONAL WAVES, GEOPHYSICS AND MACHINE LEARNING

Goal: Present the application of the Minimally-Implicit Runge-Kutta (MIRK) methods in some systems relevant for astrophysical scenarios.

Contents:

→ MIRK methods for the RRMHD equations.
[I. Cordero-Carrión, S. Santos-Pérez, C. Martínez-Vidallach. AMC 443, 127774 (2023)]

→ MIRK methods for the neutrino transport equations (M1 scheme) in supernovae simulations.
[S. Santos-Pérez, M. Obergaulinger, I. Cordero-Carrión. arxiv:2302.12089]

→ General idea and potential future applications.

MIRK methods for the RRMHD equations

·· Magnetic fields are key in accretion disks, AGN, relativistic jets, compact objects.

·· A consistent treatment is necessary to avoid numerical resistivity.

·· Hyperbolic equations + constraints (divergence of magnetic and electric fields) → augmented system of hyperbolic equations [Komissarov 2007] (velocity, density, electric and magnetic fields, two additional scalar equations).

·· Structure of the equations:

$$\partial_t E^j = S_E^j - \sigma W[E^j + (v \times B)^j - (v_l E^l) v^j] = \tilde{S}_E^j,$$

$$\partial_t B^j = S_B^j,$$

$$\partial_t Y = S_Y,$$

·· Avoid numerical instabilities due to stiff source term in the evolution equation for the electric field for high conductivities.

MIRK methods for the RRMHD equations

·· PIRK methods to deal with wave-like equations (electric and magnetic fields) for low-order methods.

·· Ideal limit: infinite conductivity and $E^i = -(\boldsymbol{v} \times \boldsymbol{B})^i$.

·· Implicit / Semi-implicit methods include additional recoveries of primitive variables from conserved ones [Palenzuela et al. 2009] → potential convergence problems, additional computational cost.

·· First-order MIRK method (stability criteria to select coefficients): pure explicit method with an effective time step.

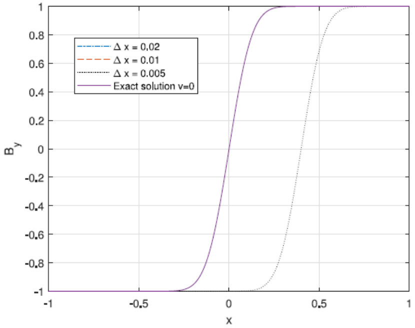
$$E^i|_{n+1} = E^i|_n + \frac{\Delta t}{1 + \Delta t\, \overline{\sigma}|_n} \left[S_E^i|_n + \overline{\sigma}|_n E^l|_n \left(v^i|_n v_l|_n - \delta_l^i \right) - \overline{\sigma}|_n (v|_n \times B|_{n+1})^i \right]$$

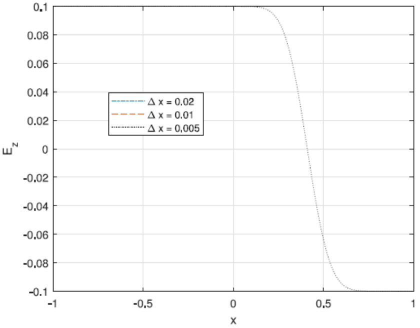
·· Analogous derivation for the two-stage second-order MIRK method.

MIRK methods for the RRMHD equations

·· Applications: Self-similar current sheet: 1D problem; CFL=0.8; initial data at t=1:

$$\boldsymbol{v} = (v^x, 0, 0),\ \boldsymbol{E} = (0,0,0),\ \boldsymbol{B} = (0, B^y(x, t=1), 0),\quad B^y_e(x,t) = \mathrm{erf}\left(\frac{x}{2}\sqrt{\frac{\sigma}{t}}\right) \qquad \sigma = 10^3$$





Stable simulations with zero and non-zero velocities ($v_x = 0.1$),
first and second-order methods.

MIRK methods for the RRMHD equations

·· Applications: Circular Polarized Alfvén waves: 1D; full system (including matter); EoS for an ideal fluid, $\Gamma = 4/3$; $\rho(x,0) = p(x,0) = 1$; CFL=0.3 → 0.7; $\sigma = 10^8$; KO term;

$$\boldsymbol{B}(x,0) = B_0\,(1, \cos(kx), \sin(kx)),$$

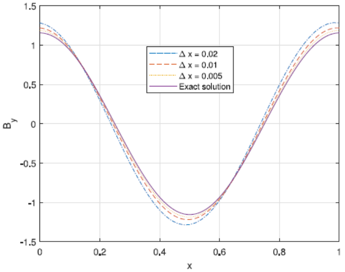
with $k = 2\pi$ and $B_0 = 1.1547$, and

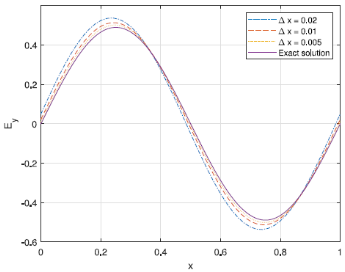
$$\boldsymbol{E}(x,0) = -\boldsymbol{v}(x,0) \times \boldsymbol{B}(x,0),$$

with $\boldsymbol{v}(x,0) = \frac{v_A}{B_0}(0, B^y(x,0), B^z(x,0))$ and $v_A = 0.423695$

Stable simulations, first and second-order methods.

Exact solution refers to the one in the stiff limit.





MIRK methods for the M1 neutrino transport equations

·· The explosion mechanism of CCSNe cannot be understood without a detailed account of the generation and transport of neutrinos.

·· Boltzmann equation → momentum-space integration of the distribution function. Truncation at a finite n: n=0 or diffusion; n=1, quite used – M1 scheme.

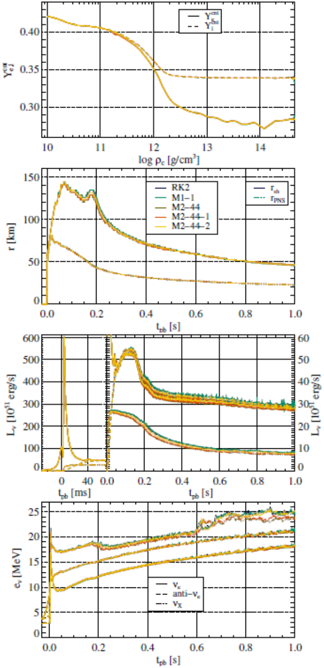
·· Optically thick regime → very different timescales of different interactions and stiff source term for very high opacities.

·· Structure of the equations:

$$\partial_t E = S_E + C^{(0)}, \qquad C^{(0)} = c\,\kappa_a (E_{\text{eq}} - \boxed{E}),$$
$$\partial_t F^i = S_F^i + C^{(1),i}, \quad C^{(1),i} = -c\,\kappa_{\text{tra}} \boxed{F^i}.$$

·· IMEX-like method [Just et al. 2015].

·· Similar derivation of MIRK methods, taking into account stability in the stiff limit.



MIRK methods for the M1 neutrino transport equations: application

·· Core-collapse simulation including all the important interactions that dominate the dynamics (see more details of initial data in arxiv reference).

·· Stable and accurate results using first and second MIRK order methods vs reference results.

·· Direct relation between the values of the coefficients in the first and second-order schemes and stability and correct values at the stiff limit.

·· Slight modifications from pure explicit methods (first and second-order) and similar computational cost.

General idea and potential future applications

·· Hyperbolic equations with stiff source terms which can be somehow linearized with respect to the conserved (evolved) variables:

$$\partial_t U + \partial_i F^i(U) = S(U), \qquad S(U) = S_E(U) + \frac{1}{\epsilon}[S_I(U) - U_0];$$

$$S_I(U) = \sum_{i=1}^{n} G_i(U) \boxed{U^i.}$$

Only the conserved variables are evaluated implicitly.
The factors multiplying these conserved variables are always evaluated explicitly.

·· Other examples: general relativistic force-free electrodynamics, rarefied gases problems, shallow water equations...

Thanks for your attention!