Lesson 1

Walrasian Equilibrium in a pure Exchange Economy.

General Model

General Model: Economy with n agents and k goods.

- **Goods.**
- Concept of good: good or service completely specified phisically, spacially and timely.
- **Assumption 1 :** There is a finite number **k** of goods.
- **Assumption 2 :** Goods can be consumed in any non-negative real number.
- Goods are perfectly divisible
- The space of goods is R_+^k

General Model. Goods.

- Al allocation of goods is a vector

$$x = (x_1, x_2, ..., x_k) \in R_+^k$$

- **Prices:** There exists a market for each good and then a price. (there exist future markets for all future goods)
- Let p_l be the amount paid (in terms of a “numeraire”) for each unit of good x_l, *l=1,2,….k.*
- *pl is*
  - + *(good or scarce commodity),*
  - - *(bas or bad commodity)*
  - *0 (free good)*

- The price system is a vector:

$p=(p_1,p_2,\ldots,p_l,..,p_k)$ *en* $\boldsymbol{R}^k$

General Model.

- Barter Economy

  The economy works out without the help of the good money (or any other good as accounting measure)

  The model is of perfect information (or perfect forecast)

  The model is static. Steady state (the dynamic structure is not in the model).

  Agents choose “consumption plans for all their lives)

General Model. Agents.

- **Assumption 1**. A finite number, **n**, of consumers.
- Consumer's aim= To choose a consumption plan according to her choice-criterion and her survival (**consumption set**) and wealth constraints.
- **Choice criterion**: An economic decision maker always chooses her most preferred alternative from the set of available alternatives.
- **Assumption 2 :** consumers are price-takers.
- **Description of an economic agent**: Each consumer i defined by:
  - Her consumption set $\mathbf{X}^{\mathbf{i}}$
  - Her initial endowments $W^i = (w_1^i, w_2^i, \ldots, w_k^i) \in R_+^k$
  - Her preferences over baskets of goods.

General Model. Agents.

- **The consumption set**: $X^i \subset R_+^k$

  Example: Survival.

- **Assumption C1:**

$$X^i = R_+^k$$

- **Assumption C2**: ***X^i*** is convex and closed (perfect divisibility)

$$x^i = (x_1^i, x_2^i, \ldots, x_k^i) \in X^i = R_+^k$$

General Model. Preferences: Binary relationship between bundles of goods.

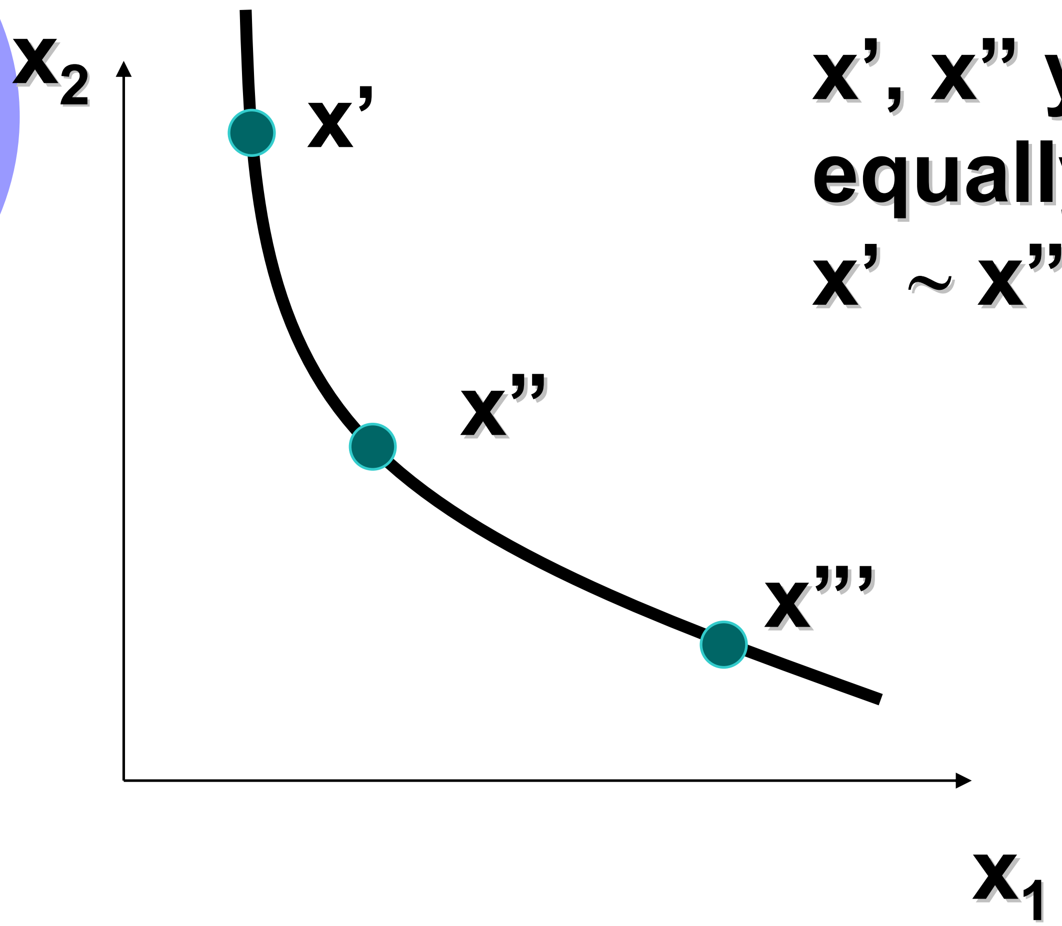
- $\succ$ denotes *strict preference:*
- x $\succ$ y means that x is *strictly preferred* (o is strictly better than) to y.
- $\sim$ denotes *indiference:*
- x $\sim$ y means that x and y *equally preferred (*is exactly as good as)*.*
- $\succcurlyeq$ denotes *weak preference*
- x $\succcurlyeq$ y means that x is *at least as preferred as* (or is at least as good) as y.

Assumptions on preferences

- Assumption 2. *Reflexivity*: x $\succcurlyeq$ x.
- Assumption 3. *Transitivity*: if x $\succcurlyeq$ y and y $\succcurlyeq$ z, then x $\succcurlyeq$ z.
- Assumption 1. *Completeness:* either x $\succcurlyeq$ y, or y $\succcurlyeq$ x, or both.

- Pre-order in X. The indiference relationship partitions X in equivalence clases, which are disjoints and exhaustives =Indiference sets with at least one element (by reflexivity)

Indiference set



x', x'' y x''' are equally preferred; x' ~ x'' ~ x'''.

General Model. Utility function.

Utility function: Rule associating a real number to each vector of goods in X:

$$U: \quad X \rightarrow \mathbf{R}, \quad \text{such that}$$

$$x \succ y \leftrightarrow u(x) > u(y) \; ; \; x \sim y \leftrightarrow u(x) = u(y).$$

An isomorphism is need (an order preserving relationship betwen sets) between X y **R**, in order that preferences are represented by an utility function.

Let **I** be the set of equivalence classes.

(I, $\succ$) is isomorphic to (**Q**, $\succ$), where **Q** is the set of rational number, whenever set I is finite.

If **I** is not finite, and additional assumption is need to guarantee that preferences are representable by utility functions.

General Model. Preferences

- Assumption 4. *Continuity.* For all x and y εX, the sets {x: x $\succsim$ y } and {x: x $\precsim$ y} are closed.

Theorem: If $\succsim$ satisfies completeness, reflexivity, transitivity and continuty, then there exists an utility function U: X→**R** representing these preferences. U is continuos and satisfies that u(x)≥u(y) if and only if x $\succsim$ y.

U(.) is **ordinal** and uniquely represents $\succsim$ if all the positive transformations of U(.) are included.

If U(.) represents $\succsim$ and if f:**R**→**R** is a monotone increasing function, then f(u(x)) also represents $\succsim$ since f(u(x))≥f(u(y)) if and only if u(x)≥u(y).

General Model. Preferences

If we want U(.) to be increasing:

- Assumption 5. ***Strong Monotonicity*** *(no-satiation)*: If $x \geq y$ and $x \neq y \rightarrow x \succ y$("More is preferred to less")

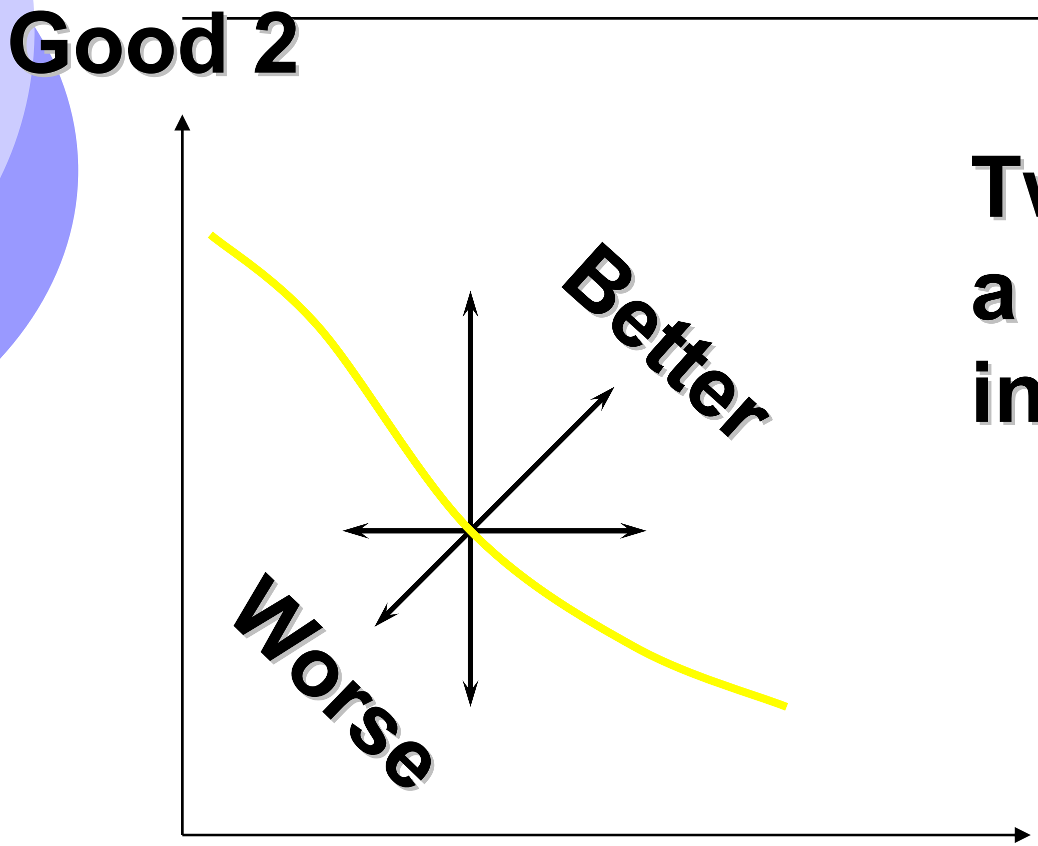
A weaker assumption is: ***Local no-satiation***:

- $x \, \varepsilon X$ and $e>0$, there exists an $y \, \varepsilon \, X$, such that $|x-y|<e \rightarrow y \succ x$.

To guarantee well-behaved demand functions.

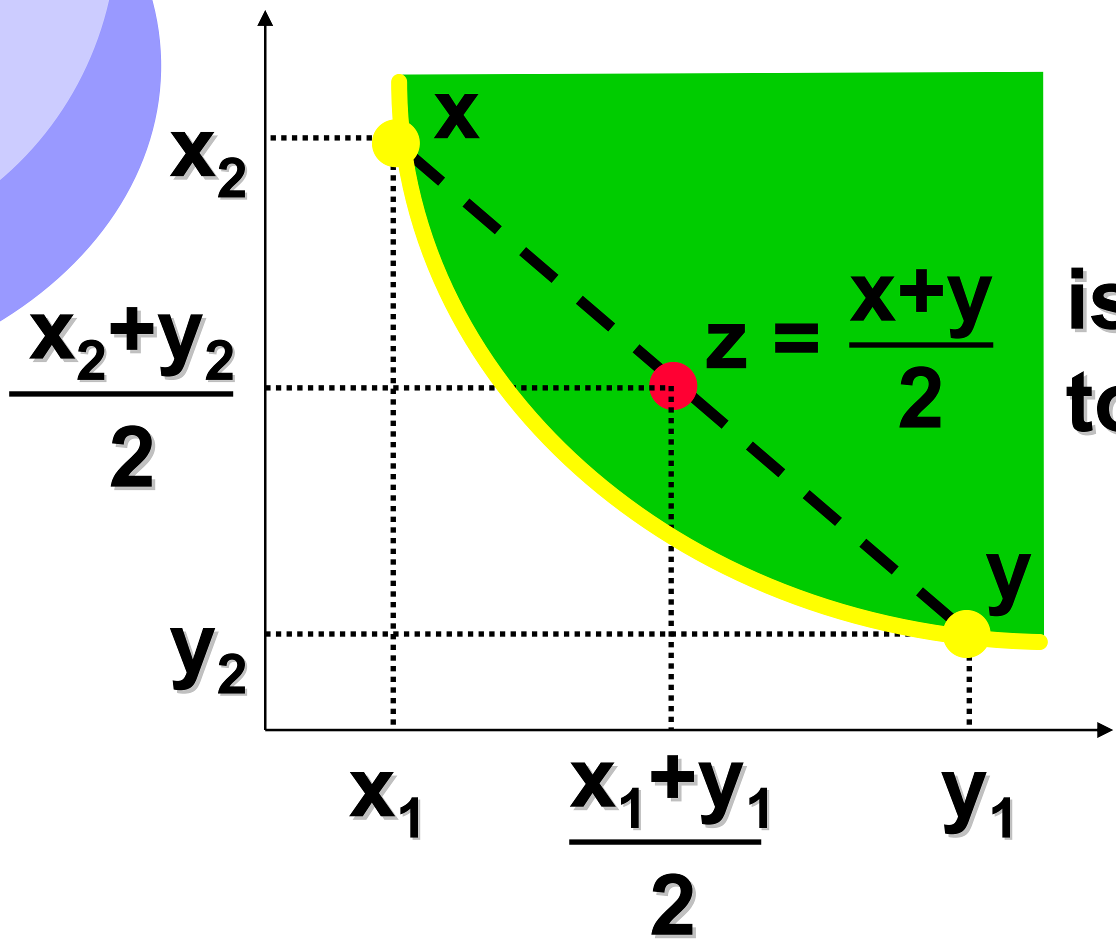
- Assumption 6. ***Stric convexity***. Given $x \neq y$ and $z \varepsilon X$, if $x \succcurlyeq z$ and $y \succcurlyeq z \rightarrow tx+(1-t)y \succ z$, for all $t \, \varepsilon(0,1)$.
- Assumption 6': ***Weak convexity***: If $x \succcurlyeq y$, then
- $tx+(1-t)y \succcurlyeq y$, for all $t \, \varepsilon(0,1)$.

Slopes of Indifference Curves



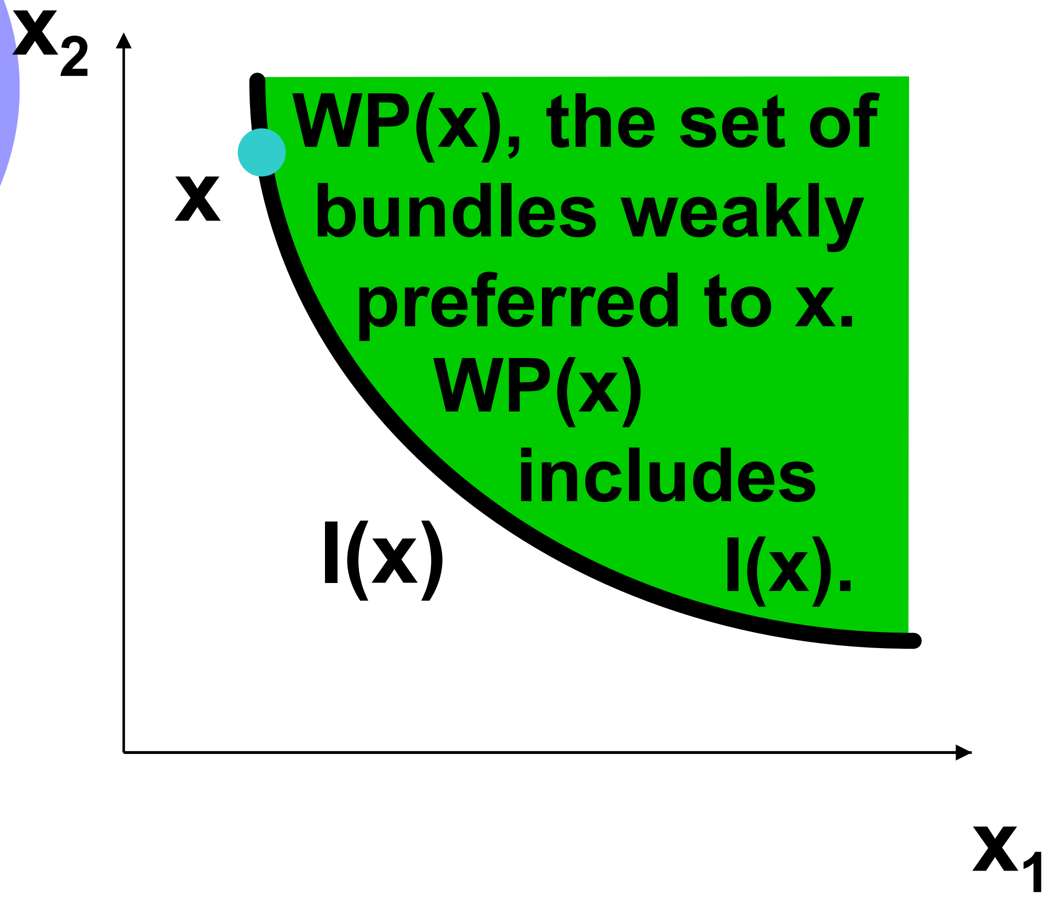
Two goods ➡ a negatively sloped indifference curve.

Well-Behaved Preferences -- Convexity.

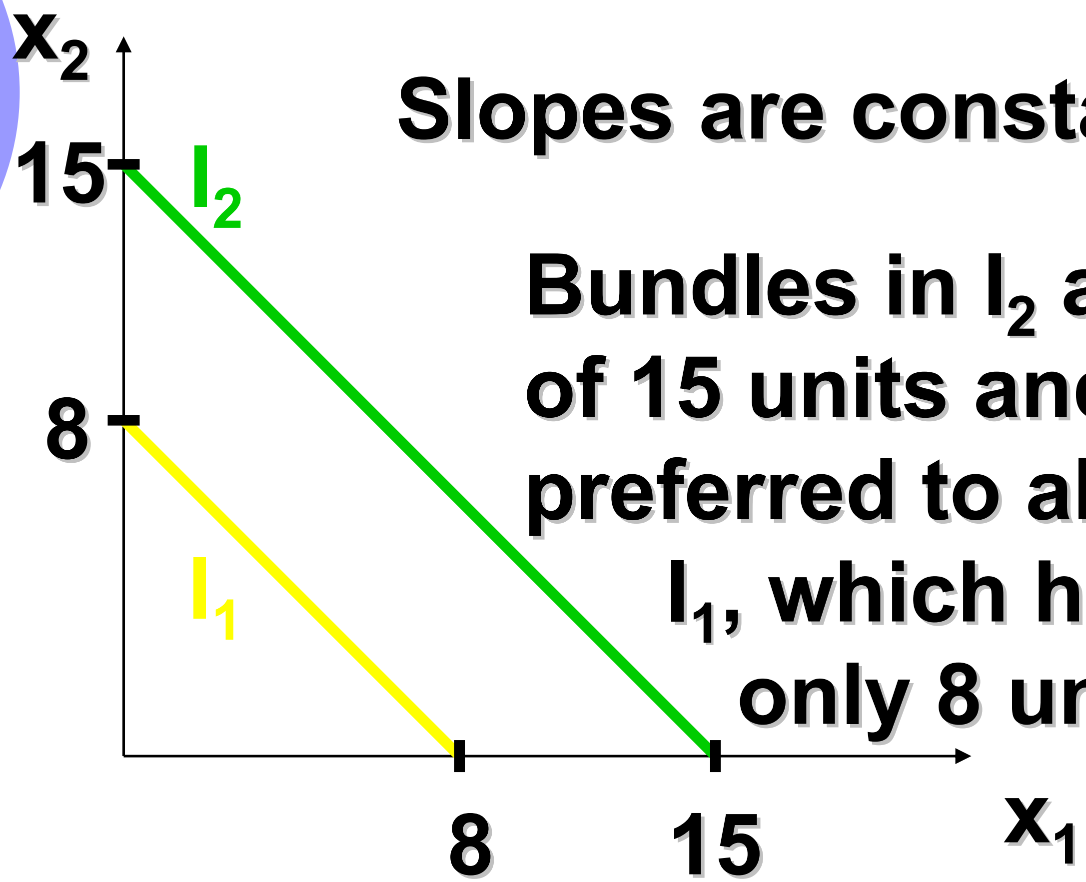


is strictly preferred to both x and y.

The weackly preferred set



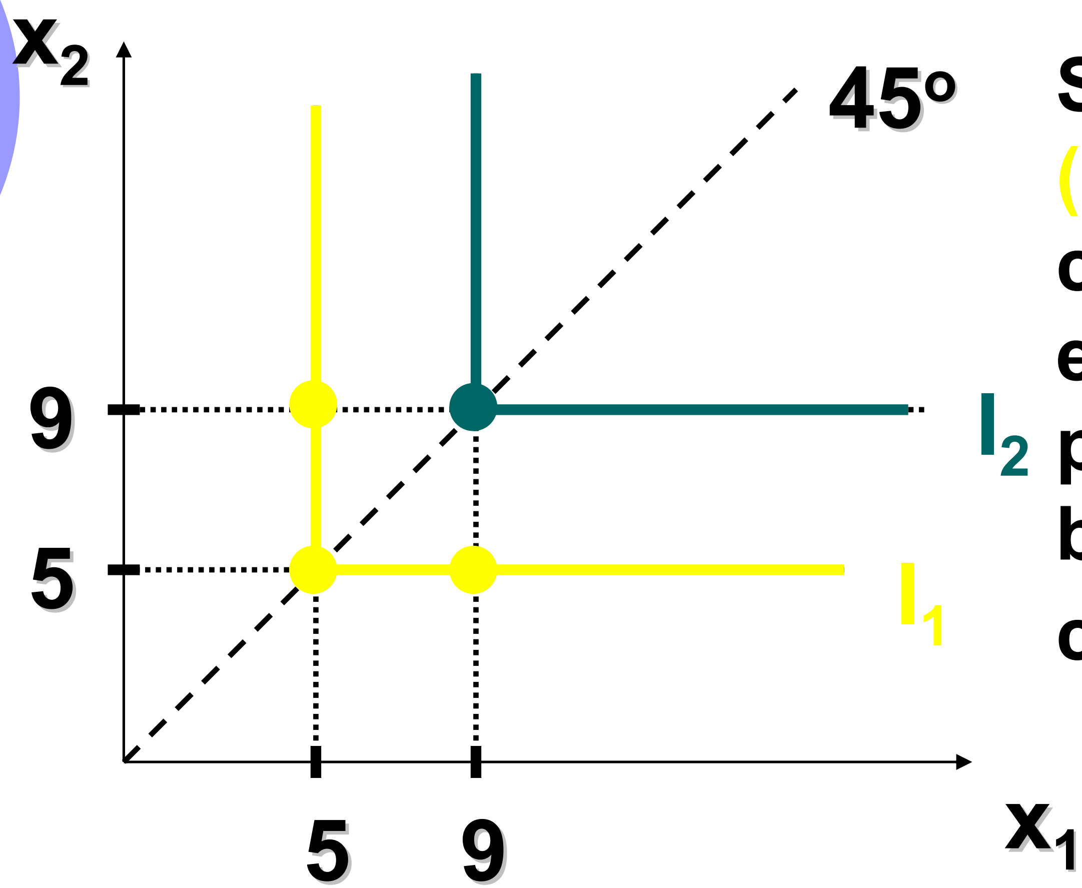
Convex Preferences. Perfect Substitutes. $U=x_1+x_2$



Slopes are constant at - 1.

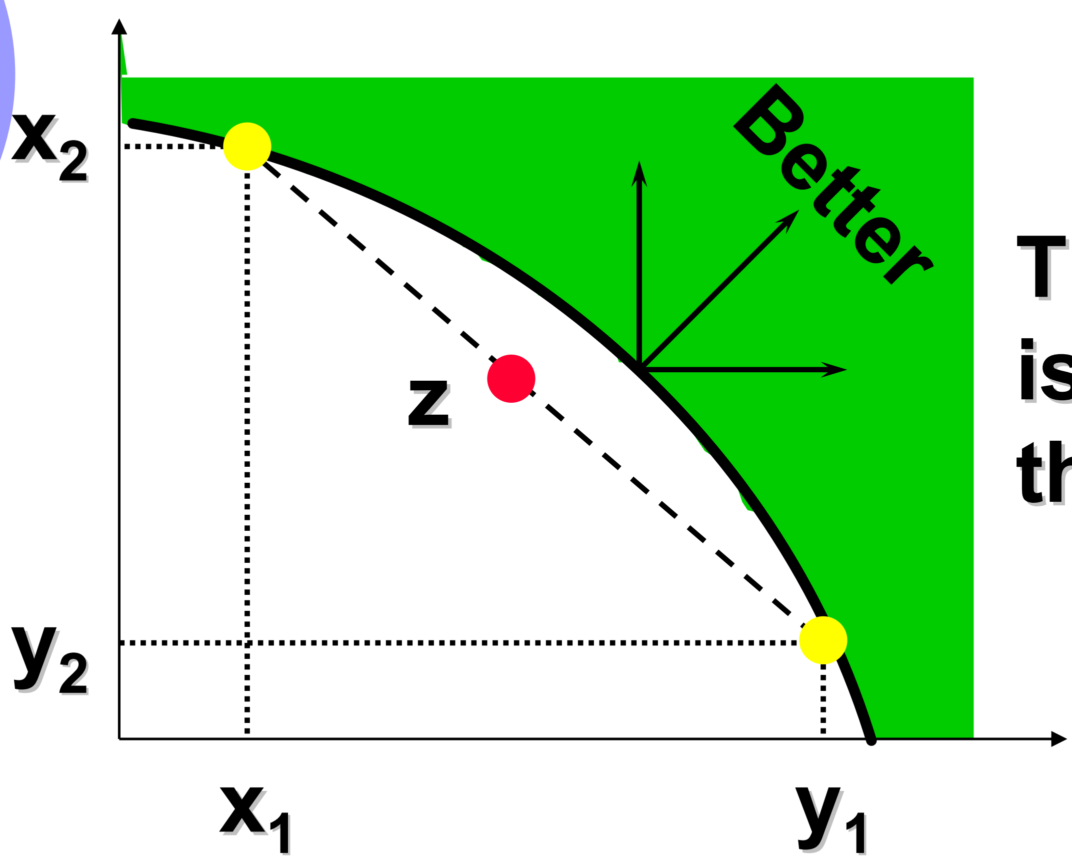
Bundles in I_2 all have a total of 15 units and are strictly preferred to all bundles in I_1, which have a total of only 8 units in them.

Convex Preferences: Perfect Complements: $U=\mathrm{Min}\{x_1,x_2\}$



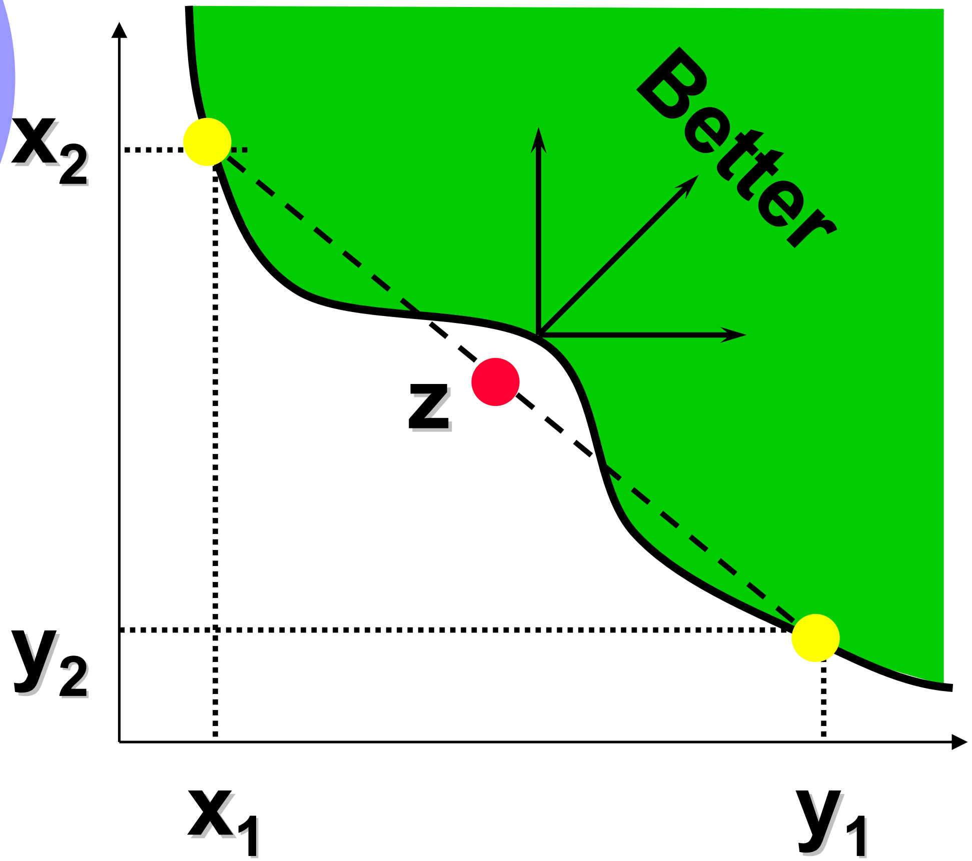
Since each of (5,5), (5,9) and (9,5) contains 5 pairs, each is less preferred than the bundle (9,9) which contains 9 pairs.

Non-Convex Preferences



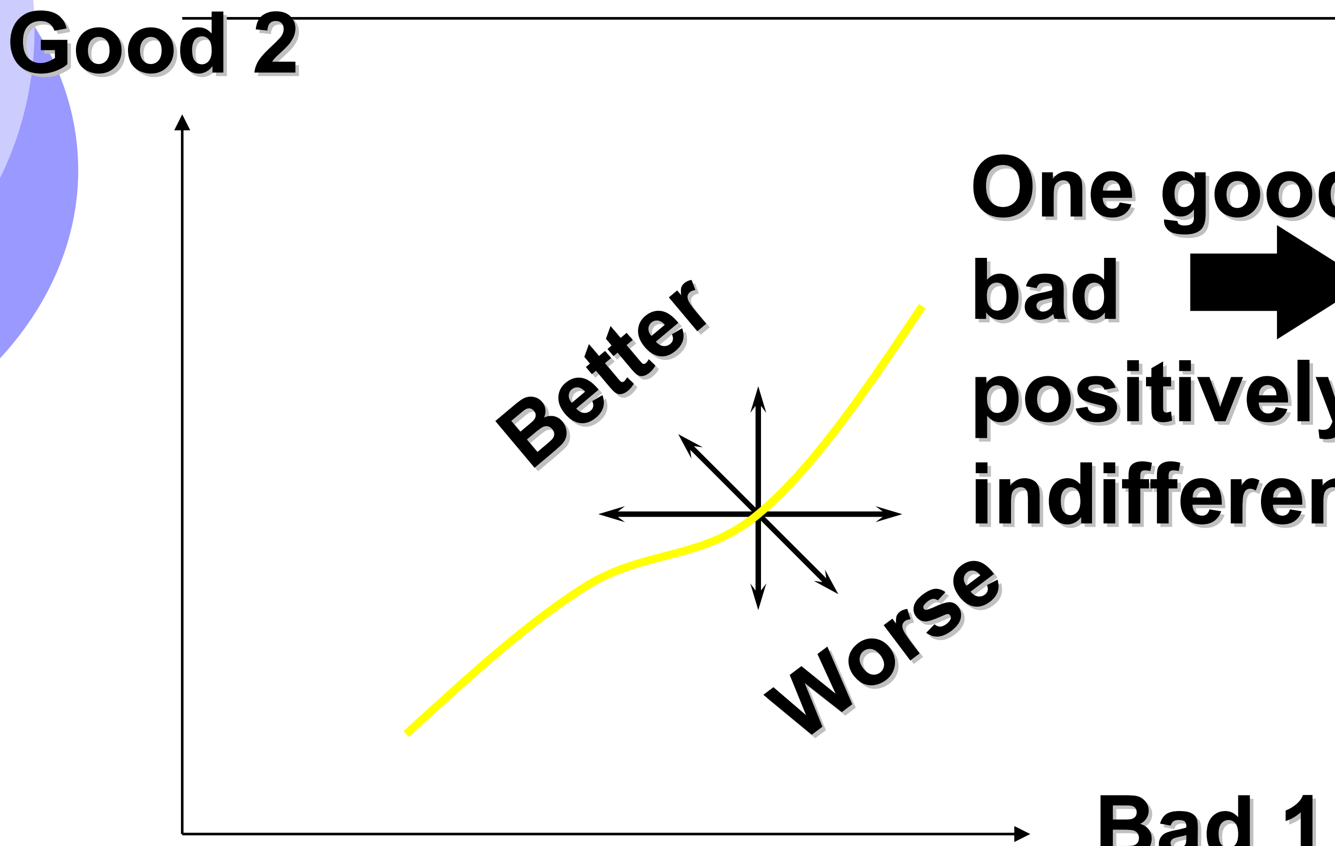
The mixture z is less preferred than x or y.

More Non-Convex Preferences



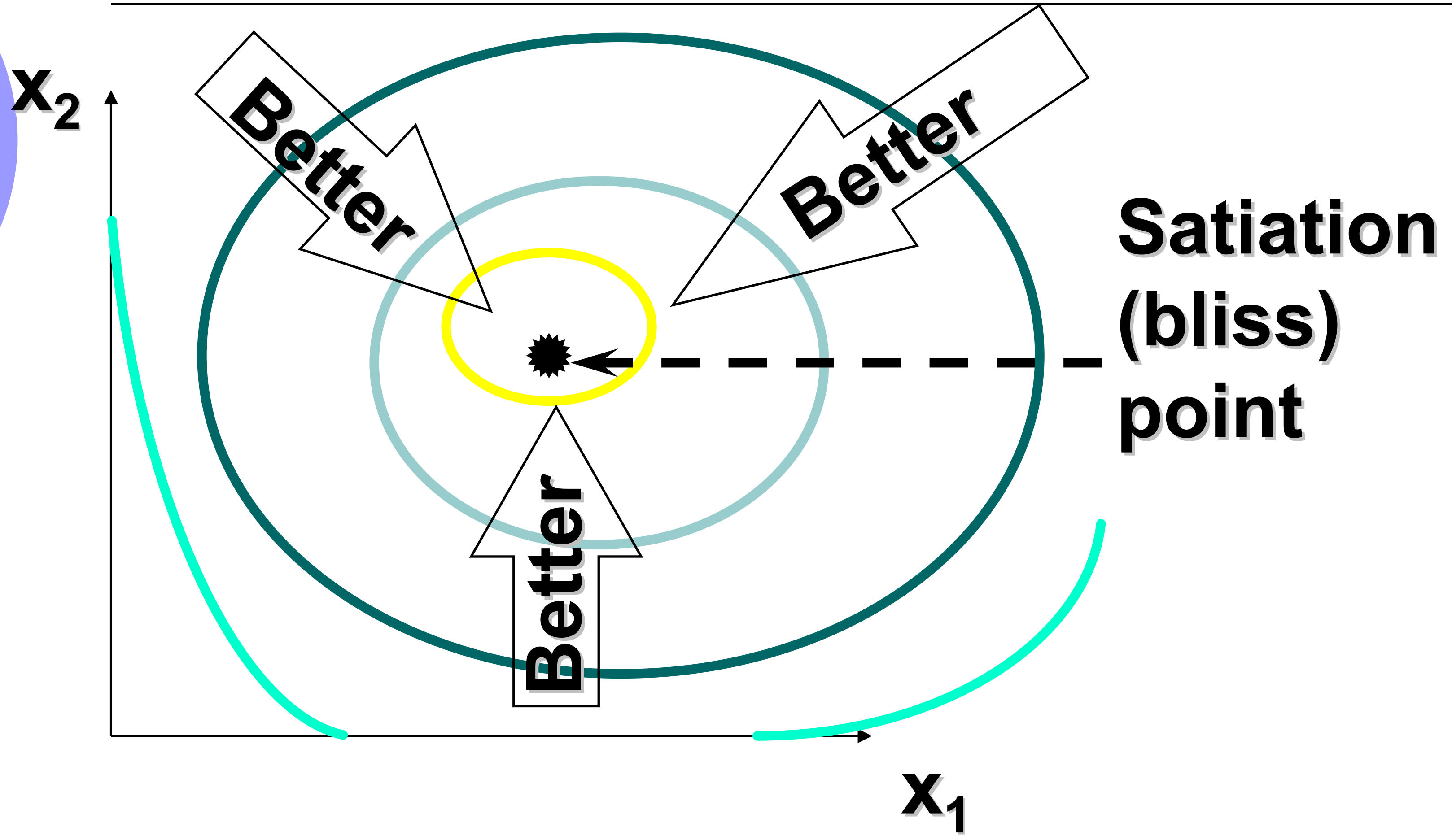
The mixture z is less preferred than x or y.

Slopes of Indifference Curves



One good and one bad ➡ a positively sloped indifference curve.

Indifference Curves Exhibiting Satiation



Preferences and Utility

- **Theorem**: If $\succcurlyeq$ satisfy the assumptions (1)-(6), then $\succcurlyeq$ can be represented by a utility function U(.), which is continuos, increasing and strictly quasi-concave.

- Consumer *i* is characterized by
  - a) Her consumption set: X^i
  - b) Her preferences $\succcurlyeq_i \rightarrow u^i(.)$
  - c) Her initial endowments w^i .

- ***Allocation***: $x=(x^1,x^2,\ldots,x^n)$ a collection of *n* consumption plans.
- ***Feasible allocation***: $\sum_{i=1}^{n} x^i = \sum_{i=1}^{n} w^i$

Demand functions and aggregate demand. Existence and characterization.

- **Question:** Is there a price vector *p* such that: *1*) each consumer *i* maximizes her $u^i(.)$ and *2*) the n-consumption plans are compatible?

- 1. Demand functions (Existence)
- **Weiertrass' Theorem**: Let *f* be a real continuous function defined in a compact set of an n-dimensional space, then *f* achieves its extreme values (maximum and minimum) in some points of the set→*Implies:*

- *Let f:* $\mathbf{R}^n \rightarrow \mathbf{R}$ *be continuous and* $A \subset \mathbf{R}^n$ *a compact set, then there exits a vector x* solving:*
  - *Max f(x)*
  - *subject to* $x \varepsilon A$

Demand functions. Existence

Consumer's problem:

- *Max*$_{\{x\}}$ $u^i(x^i)$
- *subject to*

$$\sum_{l=1}^{k} p_l x_l^i \leq \sum_{l=1}^{k} p_l w_l^i$$

- Notice:
  - 1. If $p \gg 0$, then the budget set is compact.
  - 2. If u^i is continuous and 1. is satisfied, then by Weiertrass' Theorem there exists at least a $x^{*i}=x^i(p, pw^i)$ maximizing the consumer's problem and which is continuous.
  - If u^i is strictly quasi-concave, then $x^{*i}=x^i(p, pw^i)$ is unique.
  - ***Demand function of agent i*** *(vector)*→ $\boldsymbol{x^{*i}=x^i(p, pw^i)}$.

Then: If $\succcurlyeq$ is strictly convex, continuous and monotone, then the demand function exists (is well-defined) and is continuous in all its points.

Demand functions. Characterization.

- Max$_{\{x\}}$ $u^i(x^i)$
- *s.t.* $$\sum_{l=1}^{k} p_l x_l^i \leq \sum_{l=1}^{k} p_l w_l^i$$
- Associate Lagrangian:
- $\Lambda(x_1^i, x_2^i, \ldots, x_k^i, \lambda) = u^i(x_1^i, x_2^i, \ldots, x_k^i) - \lambda[\sum_{l=1}^{l} p_l x_l^i - \sum_{l=1}^{l} p_l x_l^i]$
- Khun-Tucker Conditions.
- For an interior solution: C.P.O.:

$$\frac{\delta L}{\delta x_l^i} = \frac{\delta u^i}{\delta x_l^i} - \lambda p_l = 0, \qquad l = 1, 2 \ldots . k$$

$$\frac{\delta L}{\delta \lambda} = -[\sum_{l=1}^{k} p_l x_l^i - \sum_{l=1}^{k} p_l w_l^i] = 0$$

Marshallian or ordinary demand functions. Net demand functions. Comparative Statics.

- Marshallian or **ordinary** demand functions of agent *i* (vector): $x^{*i}=x^i(p, pw^i)$
- **Net** demand functions of *i* (vector):
- $x^{*i}=x^i(p, pw^i)-w^i$

- Comparative Statics:
  - 1. Effect of a change of a good initial endowment :
  - Let $pw^i=M^i$.
- $$\frac{\delta x_l^{*i}}{\delta w_l^i}=\frac{\delta x_l^i}{\delta M}\times\frac{\delta M}{\delta w_l^i}=\frac{\delta x_l^i}{\delta M}p_l$$

Comparative Statics. Slutsky's equation.

- 2.Effect of a change in good *l*'s price:

$$\frac{\delta x_l^{*i}}{\delta p_l} = \frac{\delta x_l^i}{\delta p_l}\Big|_M + \frac{\delta x_l^i}{\delta M}\frac{\delta M}{\delta p_l}$$

$$\frac{\delta x_l^{*i}}{\delta p_l} = \frac{\delta x_l^i}{\delta p_l}\Big|_M + \frac{\delta x_l^i}{\delta M} w_l$$

$$\frac{\delta x_l^{*i}}{\delta p_l} = \frac{\delta h_l^i}{\delta p_l} - x_l^{*i}\frac{\delta x_l^i}{\delta M} + \frac{\delta x_l^i}{\delta M}\frac{\delta M}{\delta p_l}$$

- And rearranging the above terms: Modified ***Slutsky's equation:***

$$\frac{\delta x_l^{*i}}{\delta p_l} = \frac{\delta h_l^i}{\delta p_l} - \frac{\delta x_l^i}{\delta M}(x_l^{*i} - w_l^i)$$

Comparative Statics.

- Slutsky's equation: $$\frac{\delta x_l^{*i}}{\delta p_l} = \frac{\delta h_l^i}{\delta p_l} - \frac{\delta x_l^i}{\delta M}(x_l^{*i} - w_l^j)$$
- **Normal good**:
  - Positive net demand → $\frac{\delta x_l^{*i}}{\delta p_l} < 0$
  - Negative net demand → $\frac{\delta x_l^{*i}}{\delta p_l} = ?$

Inferior good:

Positive net demand → $\frac{\delta x_l^{*i}}{\delta p_l} = ?$

- Negative net demand → $\frac{\delta x_l^{*i}}{\delta p_l} < 0$

Aggregate demand

Addition set: With the individualism hypotesis on preferences→aggregate quantities=sum of individual quantities (there is not externalities)

- $x^{*i}=x^i(p, pw^i)$ *vector of i's demand funtions.*
- $X(p)=\sum_i x^{*i}= \sum_i x^i(p, pw^i)$, ***aggregate demand*** function of the Economy.

- ***At each market l:***
- $x_l^{*i} =x_l^i(p, pw^i)$ *is i's demand function of good l, and*
- $X_l(p)= \sum_i x_l^i(p, pw^i)$ *is the market* ***aggregate*** *demand function of good l.*

Walrasian Equilibrium:

- Let $p=(p_1,p_2,\ldots,p_k)$ be a price vector.
- Each agent i: $Max_{\{x\}}\ u^i(x^i)$ *subjet to* $px^i=pw^i$
- *Solution: demand function of i:* $x^{*i}=x^i(p,\ pw^i)$
- *Aggregate demand:* $X(p)=\sum_i x^{*i}=\sum_i x^i(p,\ pw^i)$
- *Aggregate supply:* $\sum_i w^{i}$.

- Is there a price vector p^* *such that*
- $\sum_i x^i(p,\ pw^i)=\sum_i w^i$, and with free goods
- $\sum_i x^i(p,\ pw^i)\leqslant\sum_i w^i$?

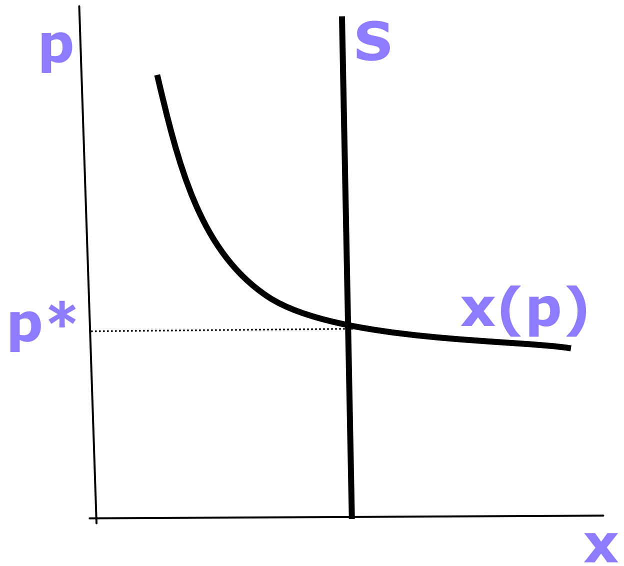
Walrasian equilibrium:

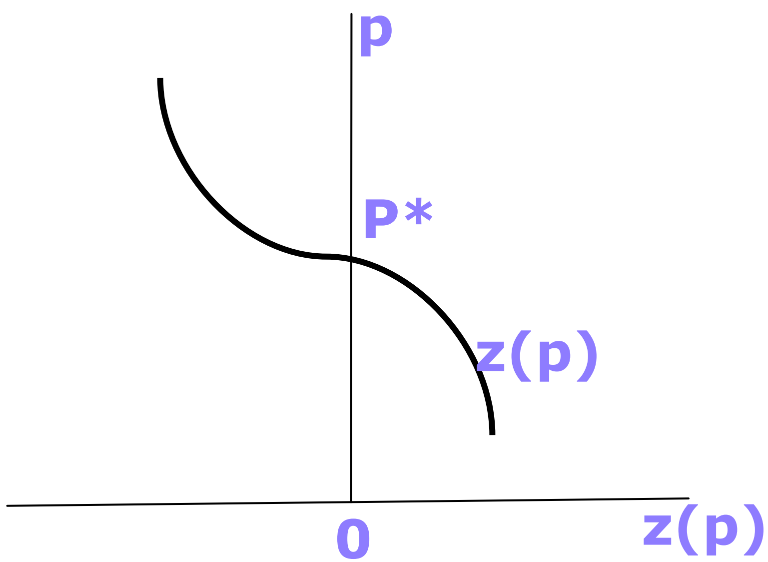
- Let $z(p)=\sum_i x^i(p, pw^i)- \sum_i w^i$ *be the* ***excess demand*** function of the economy, and

- *Let* $z_j(p)=\sum_i x_j^i(p, pw^i)- \sum_i w_j^i$, *be the* ***excess demand*** function of good (market) *j.*

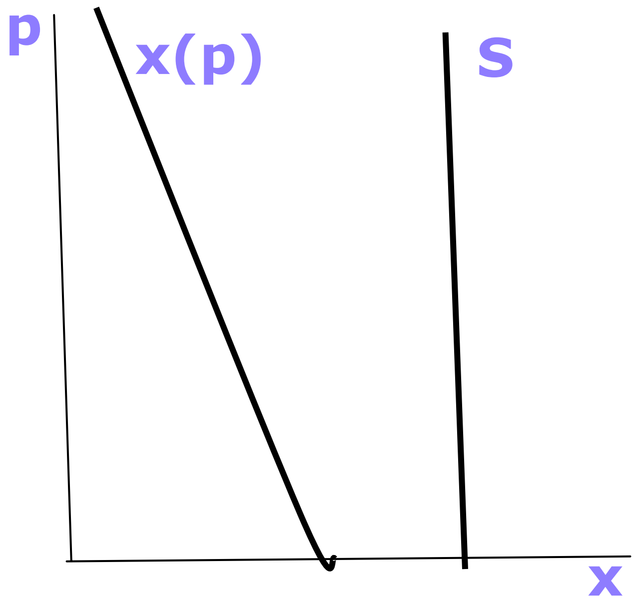
- A price vector $p^* \geq 0$, *is a* ***walrasian equilibrium*** (or ***competitive equilibrium***) *if:*
- $z_j(p^*)=0$, *if* j is scarce ($p_j^* > 0$)
- $z_j(p^*)<0$ *if* j is a free good ($p_j^* = 0$)

Examples of Walrasian equilibrium:

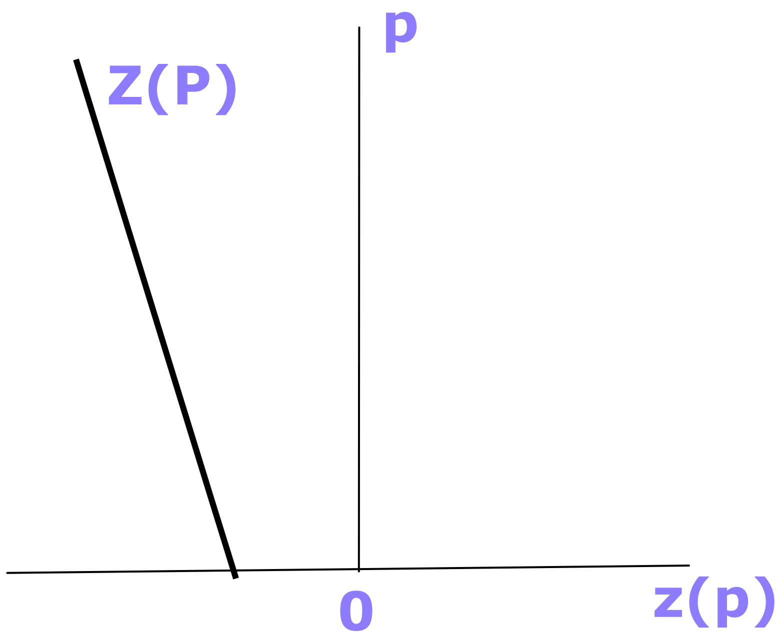




Examples of Walrasian equilibrium: Free goods

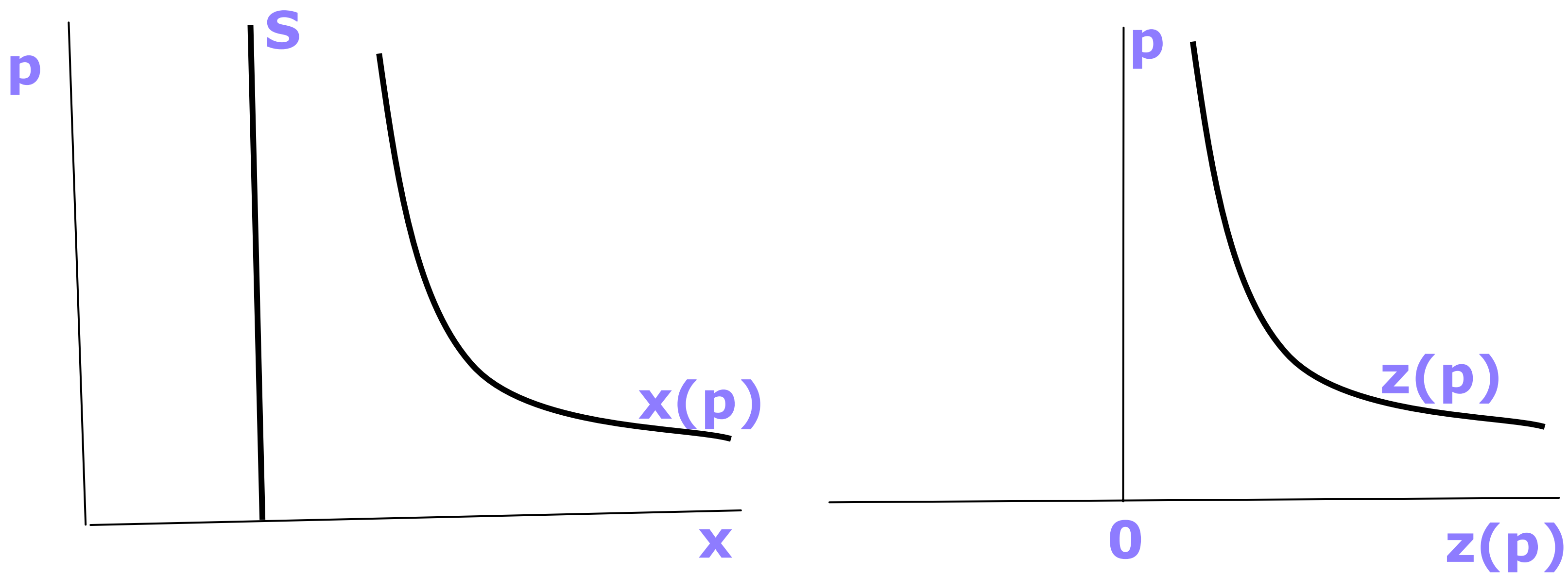


$p^*=0$

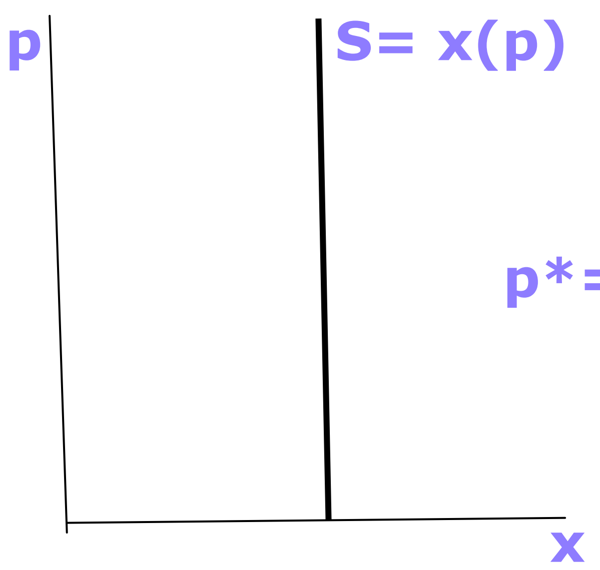


$z(p)<0 \rightarrow p^*=0$

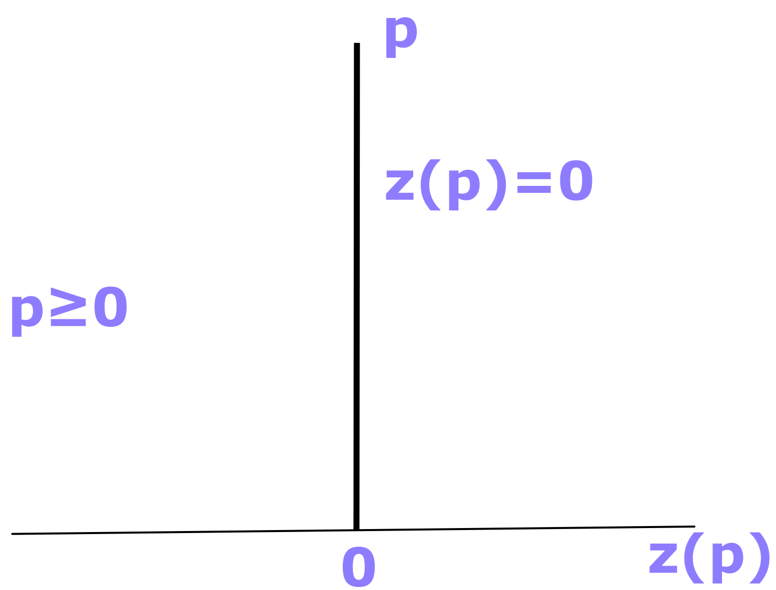
Examples of Walrasian equilibrium: Non existence case.



Examples of Walrasian equilibrium:



p*=all p≥0



Walrasian equilibrium: Properties of the excess demand function z(p).

- Let us come back to our model: Is there a price vector p* such that all markets clear? **First: properties of z(p).** Notice:
  - 1) The budget set of each agent *i* does not vary if all prices are multiplied by the same constant:→ **the budget set is homogeneous of degree zero in prices.**
  - 2) By 1)→**The demand function is homogeneous of degree zero in prices**: $x^i(kp,kpw^i)= x^i(p,pw^i)$.
  - 3) The sum of homogeneous functions of degree r is another homogeneous function of degree r:→**the aggregate demand function is homogeneous of degree zero in prices**: $\sum_i x^i(p,pw^i)$ is homogeneous of degree zero n prices.
  - 4) The excess demand function $\mathbf{z(p)= \sum_i x^i(p,pw^i)-\sum_i w^i}$ **is homogeneous of degree zero in prices**.
  - 5) By the assumptions on $\succcurlyeq$ (convexity) the demand function is continuous and since the sum of continuous functions is continuous→the aggregate demand function is continuous: z(p) is a continuous function.

  - **Z(p) is a continuous and homogeneous function of degree zero in prices.**

Walrasian equilibrium. Existence: Brower's fixed point Theorem.

- To show the existence of WE we will make use a fixed point theorem.
- The proof of the existence is made by modelling the behavior of "price-revision" by a "walrasian auctionier" until equilibrium prices are reached.

- $S \rightarrow p^0 \rightarrow \; z(p^0) \rightarrow S \rightarrow p^1 \rightarrow z(p^1) \rightarrow S\ldots$

auctionier agents S revises according to the sign of z(p) agents

- Transactions (exchanges) are only made at equilibrium prices.

Walrasian equilibrium. Existence: Brower's fixed point Theorem.

- Consider the mapping from a set into itself: *f: X→X*.
- Question: is there a point $\underline{x}$ such that $\underline{x}=f(\underline{x})$? If this point exists it is called a ***fixed-point***.
- The walrasian equilibrium is going to be defined as a fixed point of a mapping from the set of prices into itself:
- $S \rightarrow p^* \rightarrow z(p^*) \rightarrow S$

auctionier agents

S does not revise prices if excess demands are zero or negatives (free goods)

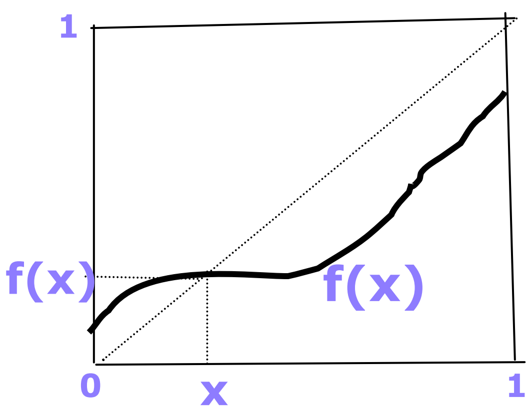
Walrasian equilibrium.
Brower's fixed point Theorem.

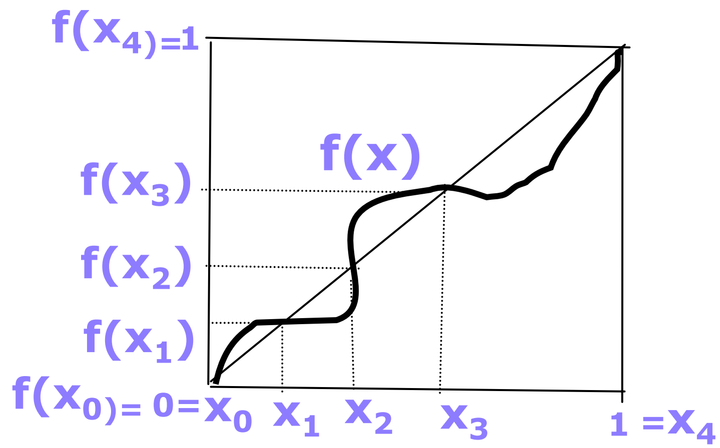
- ***Brower's fixed point Theorem:*** *Let* S be a convex and compact (closed and bounded) subset of some euclídean space and let f be a continuous function, f: $S \rightarrow S$, then there is at least an $\underline{x}$ in S such that $f(\underline{x}) = \underline{x}$.

- Notice:
- The Theorem is not about unicity.
- The Theorem gives <u>sufficient</u> conditions for the existence of fixed points.

Walrasian equilibrium. Brower's fixed point Theorem.

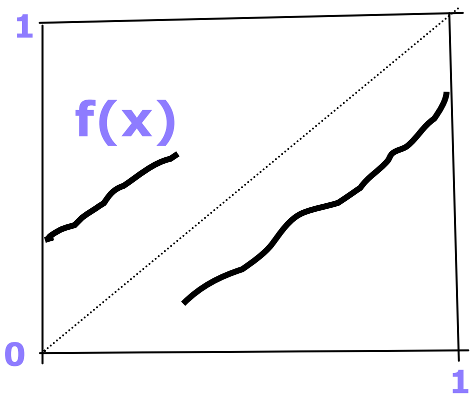
- Examp: Let S=[0,1] and let f: [0,1]→ [0,1]
- As S is a compact set, then if f is continuous there exists at least a point $\underline{x}$ such that $f(\underline{x})=\underline{x}$



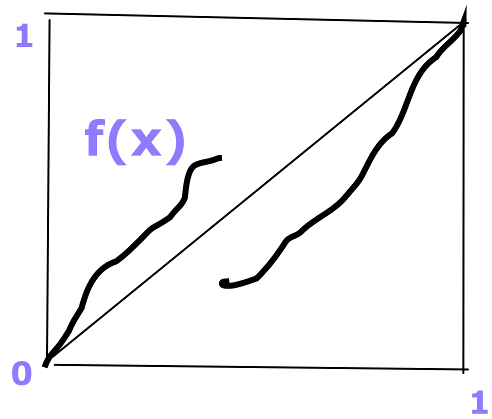


Walrasian equilibrium. Brower's fixed point Theorem. How important are the assumptions?

- 1. *f* not continuous



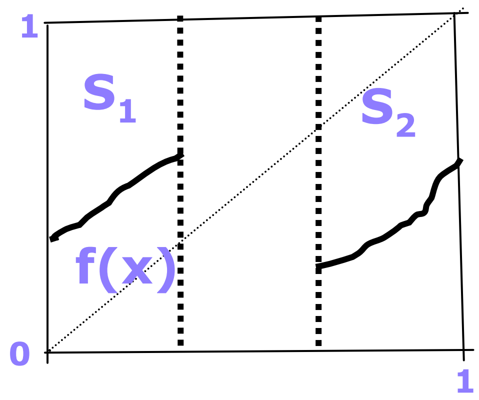
There is no fixed point

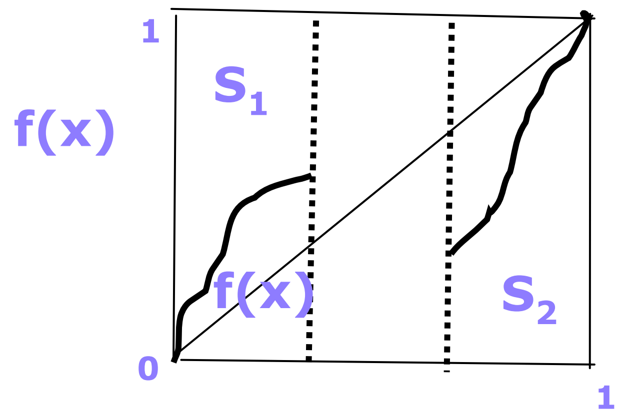


Two fixed points:
one in x=0
and another in x=1

Walrasian equilibrium. Brower's fixed point Theorem. How important are the assumptions?

- 2. S not convex: $S = S_1 \cup S_2$



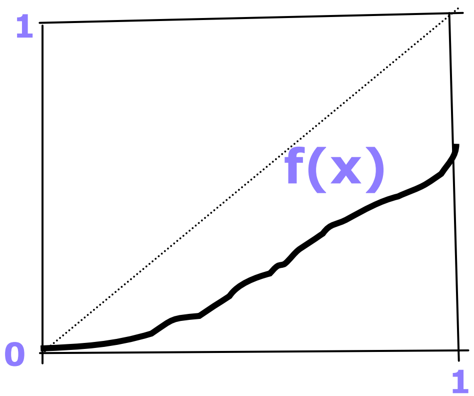


Theres is no fixed point

**Two fixed points:
one in x=0
and another in x=1**

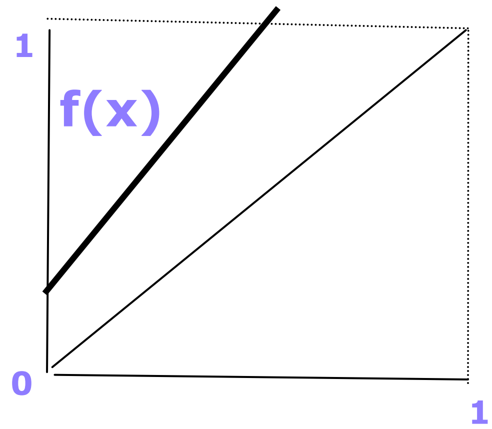
Walrasian equilibrium. Brower's fixed point Theorem. How important are the assumptions?

- 3. *S* not closed



S={x:0<x⩽ 1}
Fixed point x=0,
but x is not in S

4. *S* not bounded



S={x: x≥0}
No fixed point

Walrasian equilibrium. Existence

- The WE existence proof is based on the application of Brower fixed point Theorem to our problem→To look for set S (convex and compact).
- 1. The price set ***P*** (the set of vectors ($p_1, \ldots, p_k$) with non-negative elements) is not compact:
  - Is bounded from below: $p_l \geq 0$, for all l=1,…,k,
    but not from above. It is a closed set. Then ***P*** is not compact.
- 2. ***"Normalize"*** the set of prices ***P*** to make it a compact set: each absolute price p_l' is substituted by a normalized p_l:

$$p_l = \frac{p_l'}{\sum_{j=1}^{k} p_j'}$$

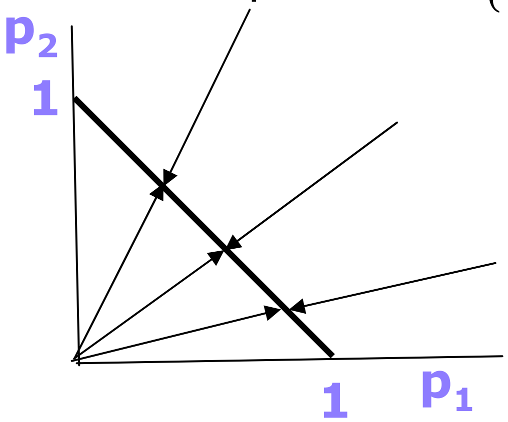
- *with* $\sum_l p_l = 1$. *For instance:* $p_1' = 4$ *and* $p_2' = 6 \rightarrow p_1 = 4/10 = 0.4$ *and* $p_2 = 6/10 = 0.6$ *and* $p_1 + p_2 = 1$. *(**relative prices**)*

Walrasian equilibrium. Existence

- We consider the price vectors belonging to **k-1 dimensional unitary simplex**.

$$S^{k-1} = \left\{ p \in R_+^2 : \sum_{l=1}^{k} p_l = 1 \right\}$$

  - Example: $S^1 = \{(p_1, p_2) \in R_+^2 : p_1 + p_2 = 1\}$



Walrasian equilibrium. Existence

- Set S^{k-1} (the normalized set of prices) is:
  - bounded: $p_l \geq 0$ y $p_l \leqslant 1$, for all $l=1,\ldots,k$
  - closed: *{0,1} belong to* S^{k-1}
  - convex: if p' and p'' are in S^{k-1} (implying that $\sum_l p_l' = 1$ and $\sum_l p_l'' = 1$), then:

    $p = \lambda p' + (1-\lambda) p''$ *is in* S^{k-1}, since

    $\sum_l p_l = \sum_l \lambda p_l' + \sum_l (1-\lambda) p_l'' = \lambda \sum_l p_l' + (1-\lambda) \sum_l p_l'' = 1$

Walrasian equilibrium. Existence

- 3. As z(p) (and the demand functions) is homogeneous of degree zero in prices, prices can be normalized and demands can be expressed as functions of relative prices. $z(p_1,p_2,\ldots,p_k)= z(tp'_1,tp'_2,\ldots,tp'_k)= z(p'_1,p'_2,\ldots,p'_k)$, *with* $t=1/\sum_j p_j'$.
- 4. A potential problem: *z(p)* is continuous whenever prices are strictly positive.
- If some $p_j=0$, by monotonicity of preferences, demands will be infinite→"discontinuity"→z(p) could be not well-defined in the simplex boundaries.
  - Solution to the problem:Modify the non-satiation (monotonicity) assumption: "There exist satiation levels for all goods, but there is always, at least, a good which is bought by the consumer and such that the consumer is never satiated of it".

Walrasian equilibrium. Existence. Walras' Law

- **<u>Walras' Law</u>** ("identity"): For any p *in* S^{k-1}, it is satisfied that *$pz(p)=0$, i.e., the value of the excess demand function is identically equal to zero.*

- *Proof:*
  $pz(p)=p(\sum_i x^i(p,pw^i)-\sum_i w^i)=\sum_i(px^i(p,pw^i)-pw^i)=\sum_i 0=0$.

- <u>Consequences of Walras' Law:</u>
- a) If for all $p \gg 0$, $k-1$ markets clear, then market k will also clear:
  $p_1 z_1+p_2 z_2=0$ by LW, $p_1>0$ y $p_2>0$, if $z_1=0$, LW$\rightarrow z_2=0$

- b) Free goods: if p^* is a WE and $z_j(p)<0$, then $p_j^*=0$,
  in words if in a WE a good has excess supply, then this good will be free:
  $p_1 z_1+p_2 z_2=0$ by LW, if $z_2<0$, LW $\rightarrow p_2=0$.

Walrasian equilibrium. Existence Theorem.

- **Existence Theorem**: If $z: S^{k-1} \rightarrow R^k$ is a continuous function and satisfies that $pz(p)=0$ (LW), then there is a p^* *in* S^{k-1} such that $z(p^*) \leqslant 0$.

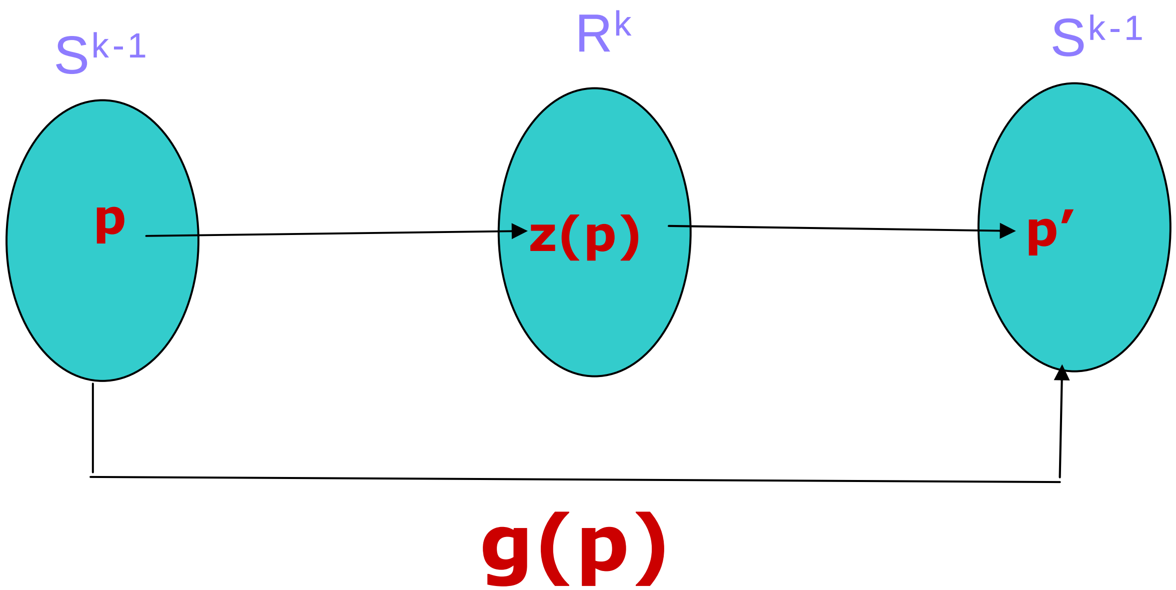
- Proof: 1) to show the existence of a fixed point p^* and 2) to show that this fixed point p^* is a WE.
- **1) *Define the mapping*** $g: S^{k-1} \rightarrow S^{k-1}$ by

$$g_j(p) = \frac{p_j + \max(0, z_j(p))}{1 + \sum_{l=1}^{k} \max(0, z_l(p))}, \quad j = 1, 2, \ldots, k$$

- Rule to revise prices. Note that g is a composed función : for any initial p *in* S^{k-1}, $z(p)$ *in* R^k is obtained and by the rule defining the mapping a new p' in S^{k-1} is obtained.

Walrasian equilibrium. Existence Theorem

- g(p) graphically:



Walrasian equilibrium. Existence Theorem

For instance: Suppose

- $p_1=0.8$ and $z_1(p)=-2$; $p_2=0.2$ and $z_2(p)=8$. Then:
- $g_1(p)=[0.8+0]/[1+0+8]=0.8/9=0.09$ *and* $g_2(p)=[0.2+8]/[1+8]=8.2/9=0.91.$

- ***Notice:***

  a) g is a continuous function, since *z(p)* is a continuous function and each function $max(0,z_j(p))$ is continuous as well.

  b) *g(p) is in* S^{k-1} since

$$\sum_{j=1}^{k} g_j(p) = \sum_{j=1}^{k}\left(\frac{p_j + \max(0, z_j(p))}{1+\sum_{l}\max(0, z_l(p))}\right) = \frac{\sum_{j=1}^{k} p_j + \sum_{j=1}^{k}\max(0, z_j(p))}{1+\sum_{l}\max(0, z_l(p))} = 1$$

The mapping *g* has an economic intuition: if $z_l(p)>0$, then $p_l\uparrow$ (as the walrasian el subastador walrasiano).

Walrasian equilibrium. Existence Theorem

As g is a continuous function mapping S^{k-1} into S^{k-1}, by Brower: there exists a p such that p*=g(p*), that is:*

$$p_j^* = g_j(p^*) = \frac{p_j^* + \max(0, z_j(p^*))}{1 + \sum_{l=1}^{k} \max(0, z_l(p^*))}, \quad j = 1, 2, \ldots k$$

- **2)** ***Now it has to be shown that vector p* is a WE (that $z_j(p^*) \leqslant 0$, j=1,2,…,k).*** From above:

$$p_j^* + \sum_{l=1}^{k} \max(0, z_l(p^*)) = p_j^* + \max(0, z_j(p^*))$$

$$p_j^* \sum_{l=1}^{k} \max(0, z_l(p^*)) = \max(0, z_j(p^*))$$

Walrasian equilibrium. Existence Theorem

- Multiplying by $z_j(p^*)$ and adding the k equations:

$$z_j(p^*)p_j^*\sum_{l=1}^{k}\max(0,z_l(p^*))=z_j(p^*)\max(0,z_j(p^*))$$

$$[\sum_{l=1}^{k}\max(0,z_l(p^*))]\sum_j p_j^* z_j(p^*)=\sum_j z_j(p^*)\max(0,z_j(p^*))$$

- By Walras' Law: $\sum_j p_j^* z_j(p^*)=0$
- and then:

$$\sum_j z_j(p^*)\max(0,z_j(p^*))=0$$

Each term of this sum is either zero or positive (since it is either 0 or $z_j(p^*)^2$). If $z_j(p^*)>0$ then the above equality could not be satisfied, then $z_j(p^*)\leqslant 0$ and p^* is a WE.