Lesson 1

General Equilibrium 1:

Walrasian Equilibrium

Topics to be Discussed

- Introduction
- Walrasian Equilibrium in Exchange Economies
- Existence of Walrasian Equilibrium

General Equilibrium Analysis: Introduction

- General Equilibrium Theory (GET) belongs to Microeconomic Theory (studies the behavior of economic agents, and their interaction in the market)

- Two key analytical devices: *Optimization analysis and equilibrium analysis*

1. Optimization analysis: the economic agent is an optimizer.

2. Equilibrium analysis: What takes place in an economic system when the optimizer behavior of all of its economic agents is compatible.

General Equilibrium Analysis: Introduction

- An agent is in *equilibrium* if she satisfies her rule of behavior: there is no incentive to change.

  Examples: Consumer's equilibrium, firm's equilibrium…(concept from physics).

With several agents:

1. The actions of each agent are in equilibrium
2. The overall behavior is compatible: plans are compatible.

Partial equilibrium model –all prices other than the price of the good being studied are assumed to remain fixed

General Equilibrium Analysis: Introduction

Market interrelationships can be important

- Complements and substitutes
- Increase in firms' input demand can cause market price of the input and product to rise

- To study how markets interrelate, we can use **general equilibrium analysis**
  - Simultaneous determination of the prices and quantities in all relevant markets, taking into account feedback effects
- **General equilibrium model**–all prices are variable and equilibrium requires that all markets clear (all of the interactions between markets are taken into account)

General Equilibrium Analysis: Introduction

- Topics of interest when studying Equilibrium Theory
  - Existence
  - Unicity
  - Stability
- Several GET analysis:
  - Classical models: Marx, Ricardo, etc.
  - Neoclassical: starting with Walras= Market decentralization and developed in the fifties of the last century by Arrow and Debreu…

General Equilibrium Analysis: Introduction. The Invisible Hand of Adam Smith

- Problems studied in the neoclassical GET
  - “The notable degree of *coherence* among a huge number of individuals taking separate decisions about the buying and selling of goods” (Arrow’s Nobel Conference).
  - Problems with **economic coordination**: information is disseminated among agents.

How a decentralized (in information and property rights) system of resource allocation (markets) can solve economic coordination?

(Walras): *Information is transmitted indirectly trough the market prices:* prices act as signals of scarcity and provide the common flow of information to coordinate the economic system.

General Equilibrium Analysis: Introduction

- The neoclassical GET focus mainly on two topics:
- To explain the emerging prices from the economic agents' interactions via market-place. (Existence)
- To explain the role of prices in optimal or efficient states of the economy. (Pareto Efficiency of Walrasian prices)

General Equilibrium Analysis: Introduction

- Traditionalyy: two approaches to GET analysis:
  - Edgeworth: gains from cooperation: To improve upon the allocations through cooperation among agents.
  - Walras: decentralization of chocies through a price-system.

General Equilibrium Analysis: A simple model of Pure exchange: 2 consumers & 2 goods. The gains from trade

- Pure exchange model: the special case of GE models where all of the economic agents are consumers and they exchange their initial endowments.
- Net demander (supplier): the consumer wants to consume more (less) than her initial endowment of that commodity

General Equilibrium Analysis: A simple model of Pure exchange: 2 consumers & 2 goods.

- 2 agents A y B and 2 goods: x_1 y x_2
- No production
- Initial endowments are given by:

$$w^A = (w_1^A, w_2^A) \text{ y } w^B = (w_1^B, w_2^B) \text{ con}$$

$$w_1^A + w_1^B = w_1 \quad \text{y} \quad w_2^A + w_2^B = w_2$$

- Each agent has well-defined preferences over baskets of goods and can consume either her initial endowment or exchange it with the other agents.

General Equilibrium Analysis: A simple model of Pure exchange: 2 consumers & 2 goods.

- Let a consumption basket of A and B be:

$$x^A = (x_1^A, x_2^A) \text{ y } x^B = (x_1^B, x_2^B)$$

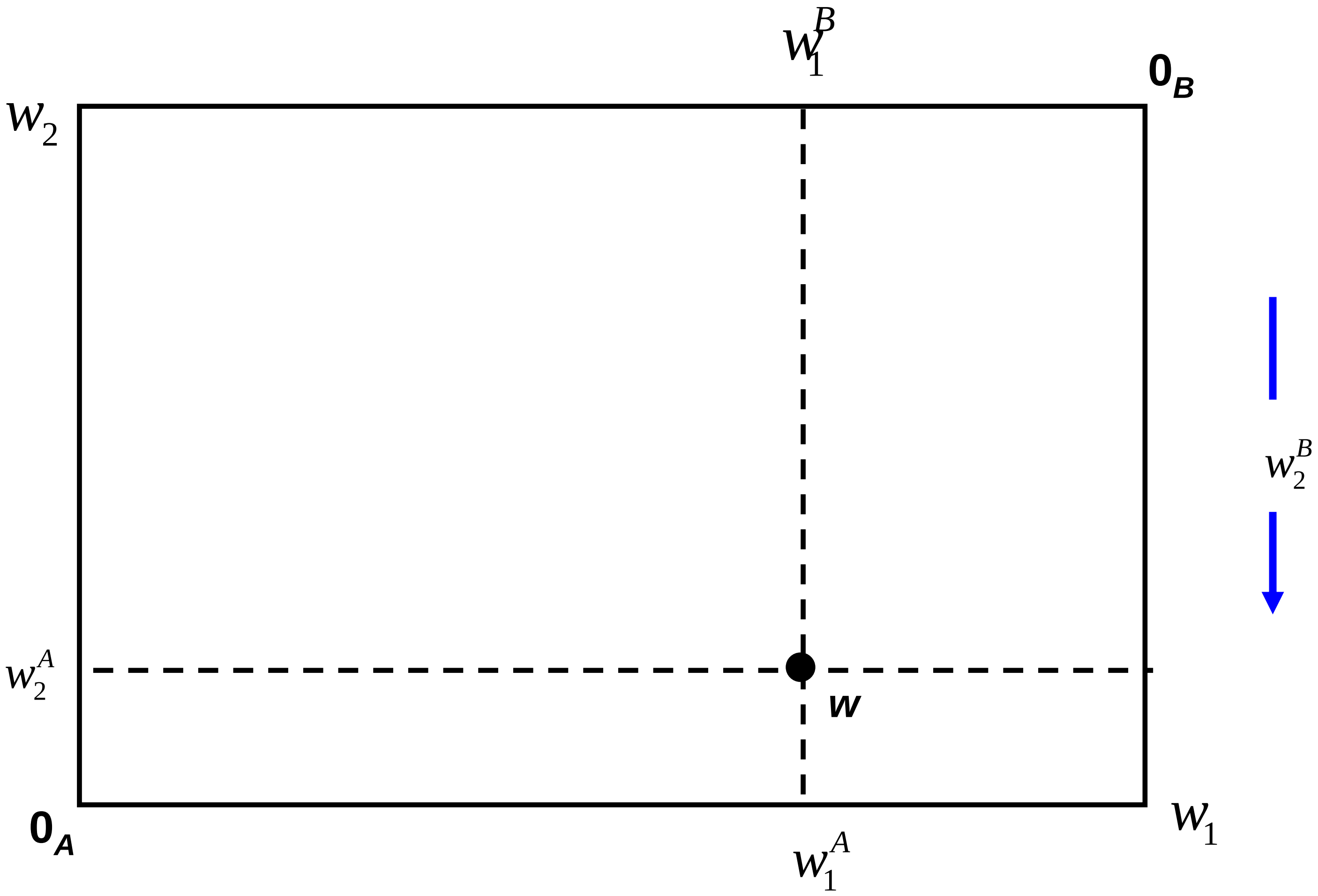
- An *allocation* is a pair of consumption baskets :

$$x = (x^A, x^B)$$

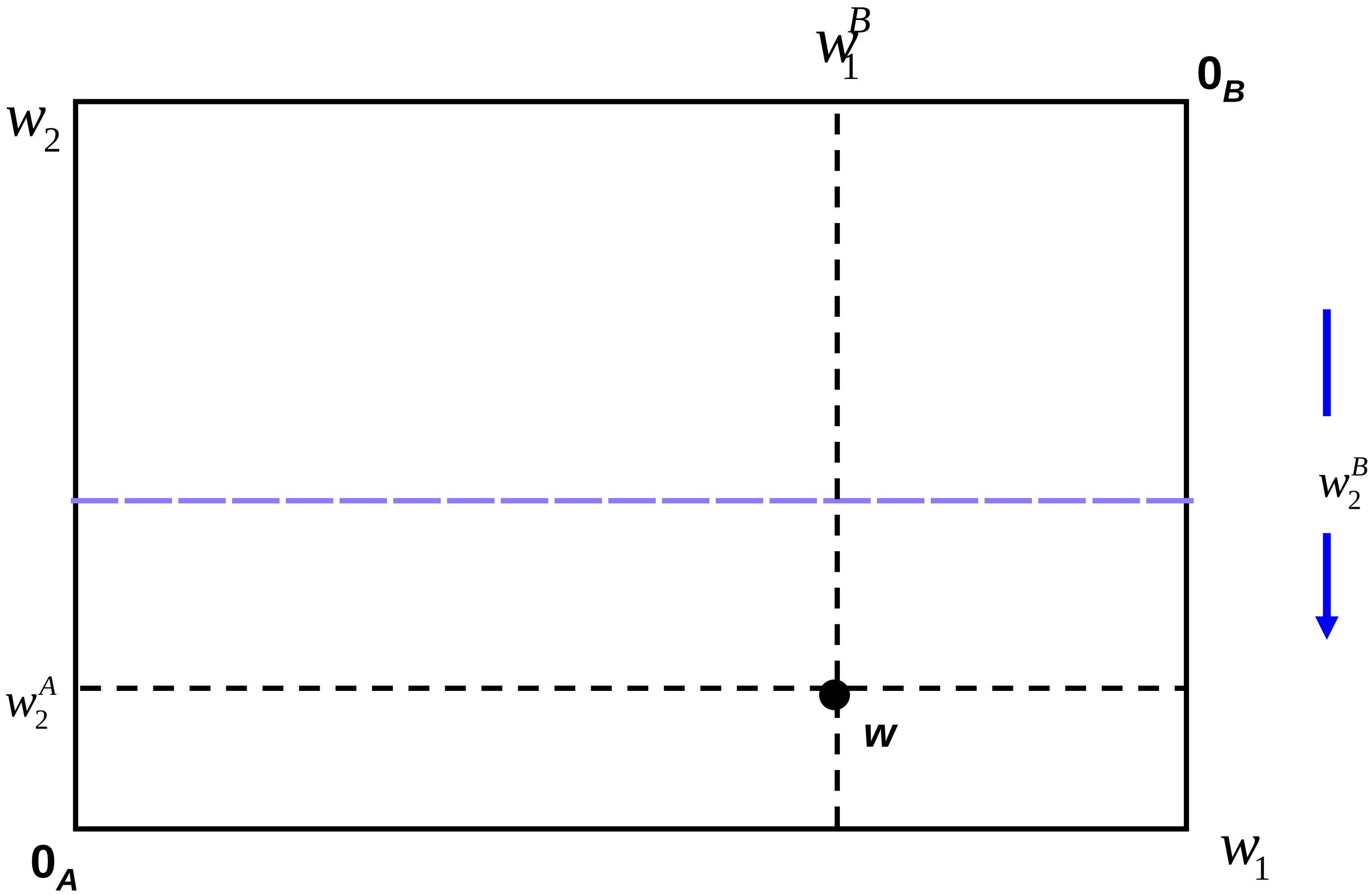
- An allocation is feasible if:

$$x_1^A + x_1^B = w_1^A + w_1^B = w_1 \quad \text{y} \quad x_2^A + x_2^B = w_2^A + w_2^B = w_2$$

A simple model of Pure exchange. The Edgeworth-Bowley box summarizes the set of all feasible allocations.

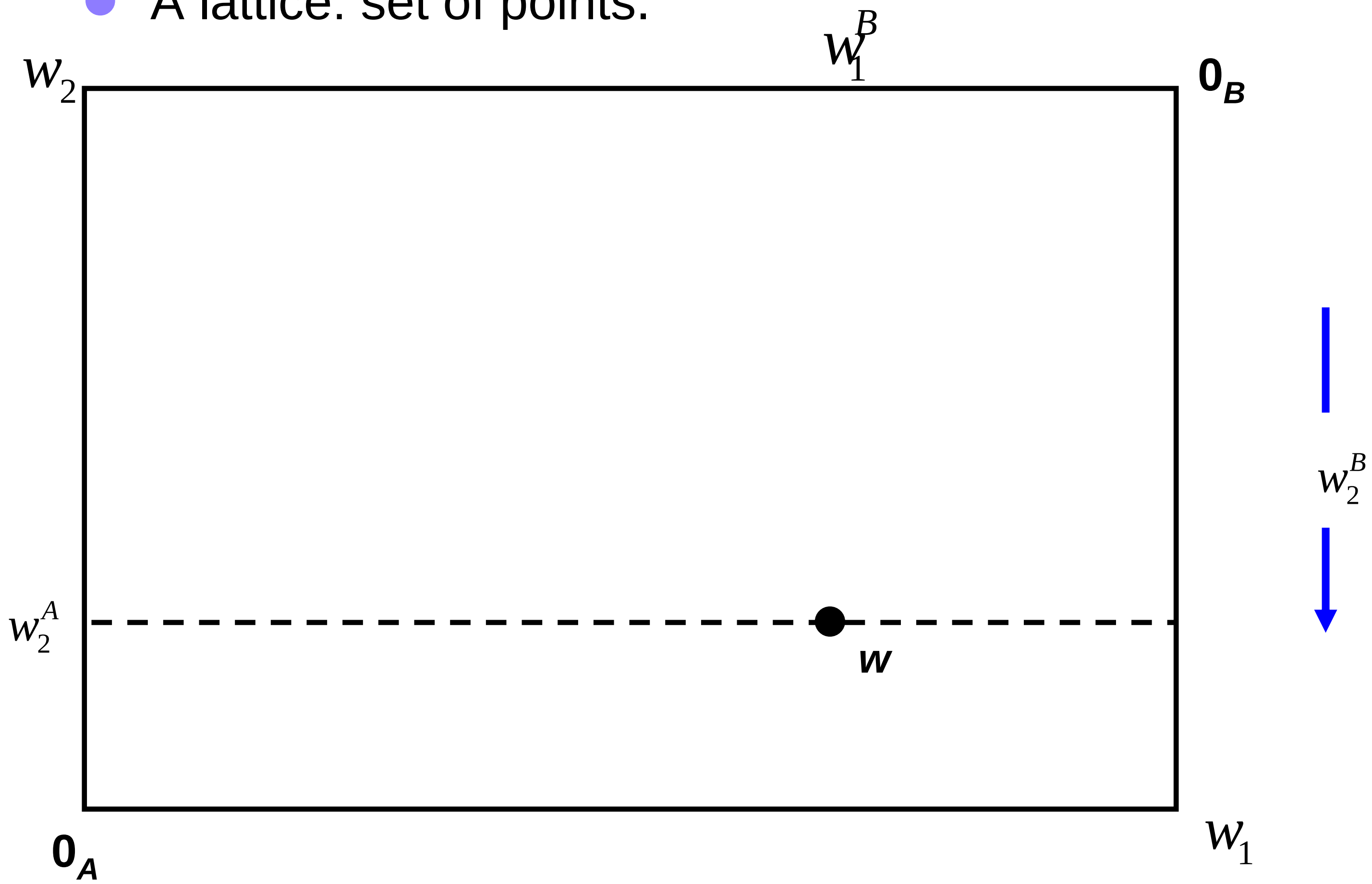


Edgeworth's box when a good is perfectly divisible but the other is not. Set of either horizontal or vertical parallel lines.



Edgeworth's box when no good is perfectly divisible

- A lattice: set of points.



Example: James and Karen are in an economy with 10 units of food and 6 units of clothes.

Individual	Initial Allocation	Trade	Final Allocation
James	7F, 1C	-1F, +1C	6F, 2C
Karen	3F, 5C	+1F, -1C	4F, 4C

- To determine if they are better off, we need to know the preferences for food and clothing

Example: Preferences

- Karen has a lot of clothing and little food. Suppose:
  - MRS of food for clothing is 3
  - To get 1 unit of food, she will give up 3 units of clothing
- James' MRS of food for clothing is only ½
  - He will give up ½ unit if clothing for 1 unit of food

Example: exchange between James and Karen.

- There is room for trade
  - James values clothing more than Karen
  - Karen values food more than James
  - Karen is willing to give up 3 units of clothing to get 1 unit of food, but James is willing to take only ½ unit of clothing for 1 unit of food
- Actual terms of trade are determined through bargaining
  - Trade for 1 unit of food will fall between ½ and 3 units of clothing

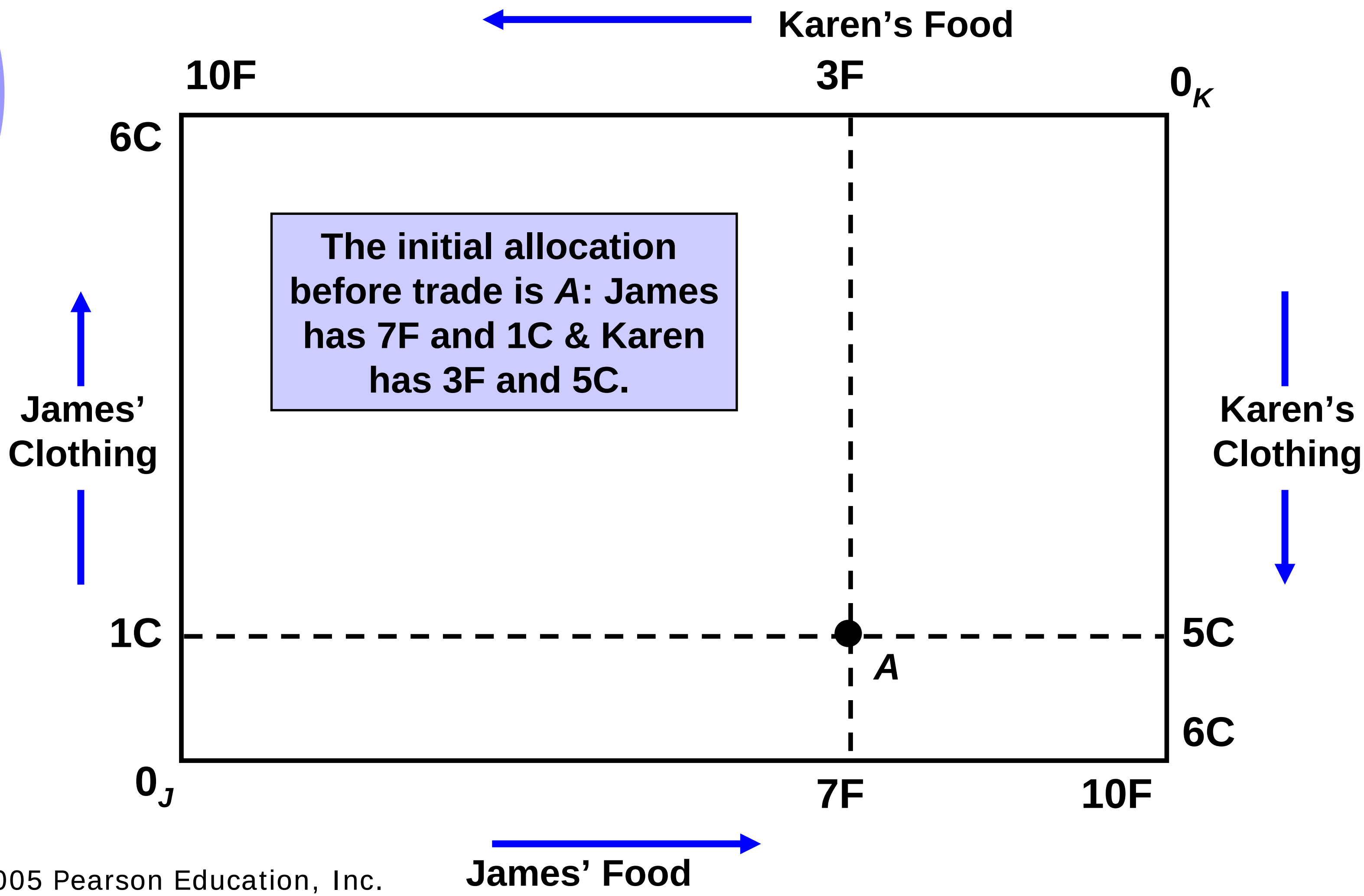
Example: The Advantage of Trade

- Suppose Karen offers James 1 unit of clothing for 1 unit of food
  - James will have more clothing, which he values more than food
  - Karen will have more food, which she values more
- Whenever two consumers’ MRSs are different, there is room for mutually beneficial trade
  - Allocation of resources is inefficient

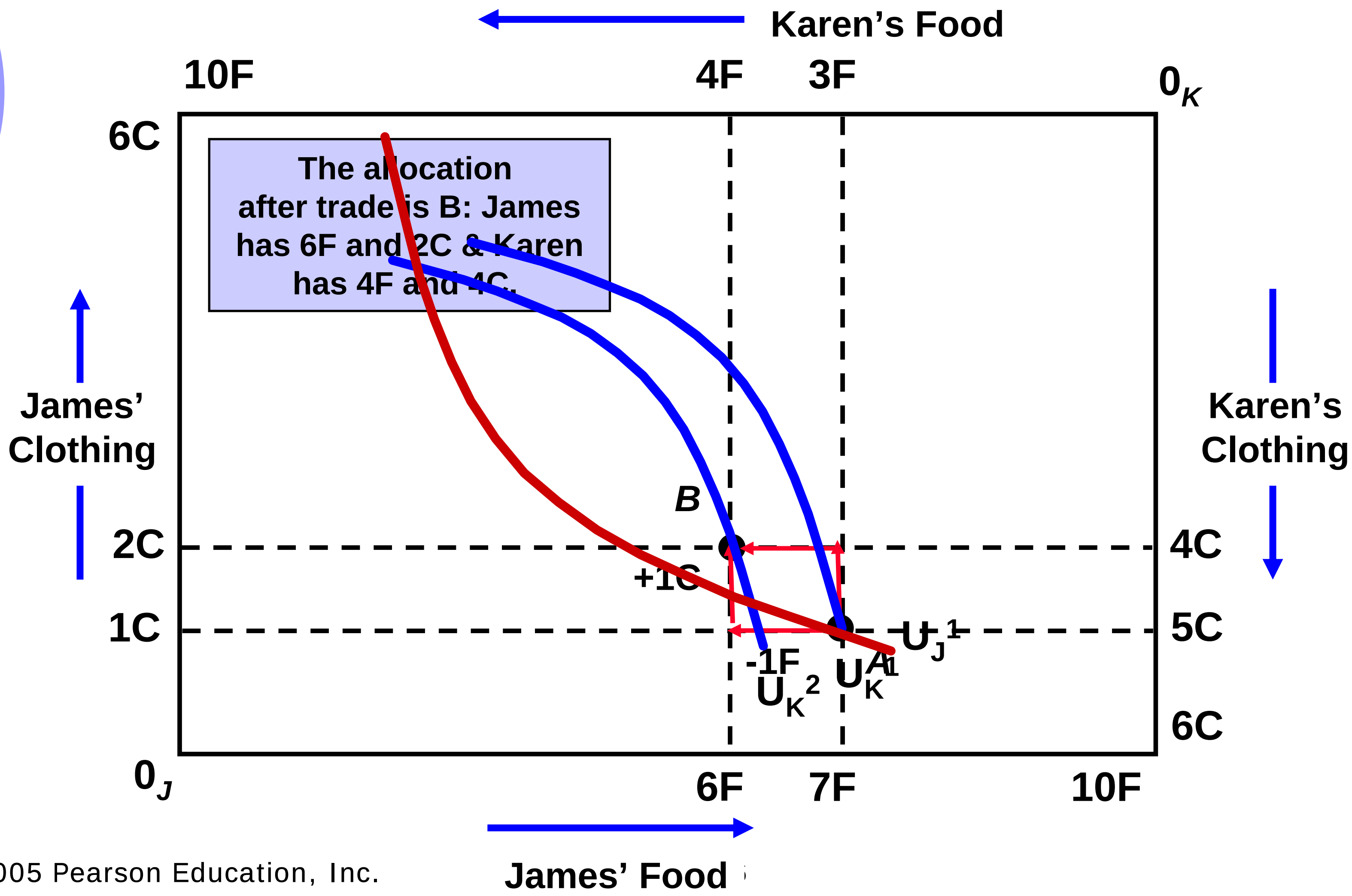
The Edgeworth Box Diagram

- Food is measured across the horizontal axis
- Clothing is measured on the vertical axis
- Length of box is the total amount of food: 10 units
- Height of box is the total amount of clothing: 6 units
- Each point describes the market baskets of *both* consumers
  - James' basket is read from origin O_J
  - Karen's basket is read from origin O_K, in the reverse direction
  - James has 7 units of food and 1 unit of clothing: point A
  - Karen has 3 units of food and 5 units of clothing: point A from different axis

Exchange in an Edgeworth Box



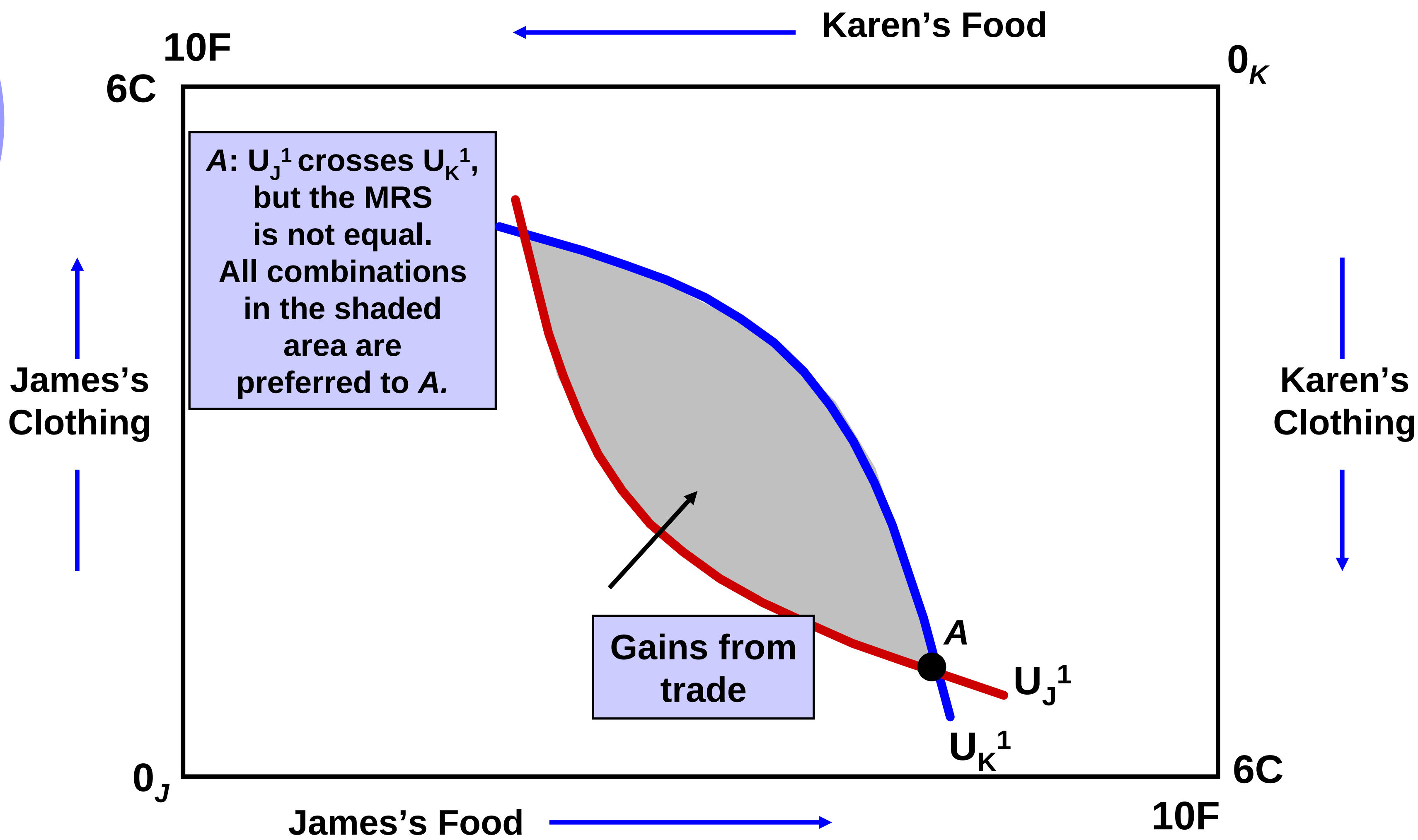
Exchange in an Edgeworth Box



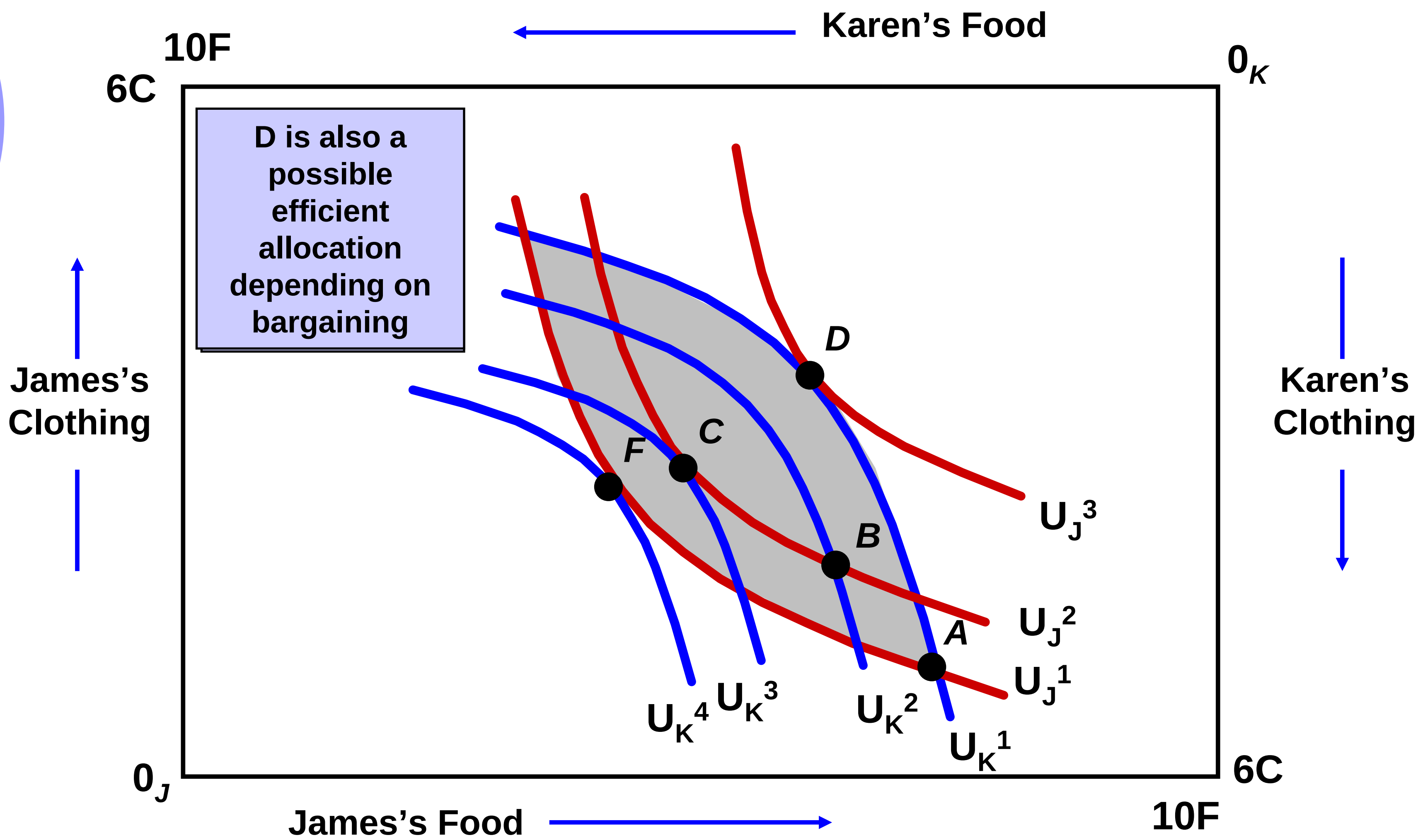
Improvement upon by cooperation

- Is going to be *exchange* in this economy?
- 2 types of allocations which can be improved upon or blocked
- Those which James and Karen would reject just keeping themselves at their initial endowments: INDIVIDUAL RACIONALITY (IR)
- Those which can be improved upon by the joint behavior of both agents: PARETO RATIONALITY.
- An allocation of goods is **Pareto efficient** if no one can be made better off without making someone else worse off

Efficiency in Exchange: Individual Rationality.



IR and Pareto efficiency.



Efficient allocations

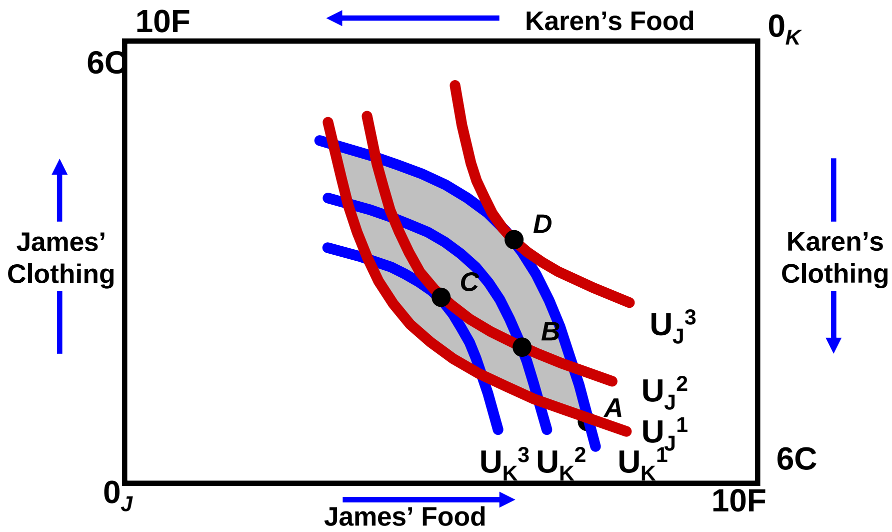
- The shaded area between these two indifference curves represents all the possible allocations of food and clothing that would make both James and Karen better off than A (Describes all mutually beneficial trades)
- We can see both parties are better off at point B since they both end up on a higher indifference curve
  - Not efficient since MRSs are different – indifference curves have different slopes
  - Although a trade might make both parties better off, the new allocation is not necessarily efficient

Efficient Allocations

- How do these parties reach an efficient allocation?
  - When there is no more room for trade
  - When their MRSs are equal
  - They will keep trading, reaching higher indifference curves, until they can no longer do so and still make each better off
  - This is when indifference curves are tangent – they have the same slope and same MRS

Efficiency in Exchange

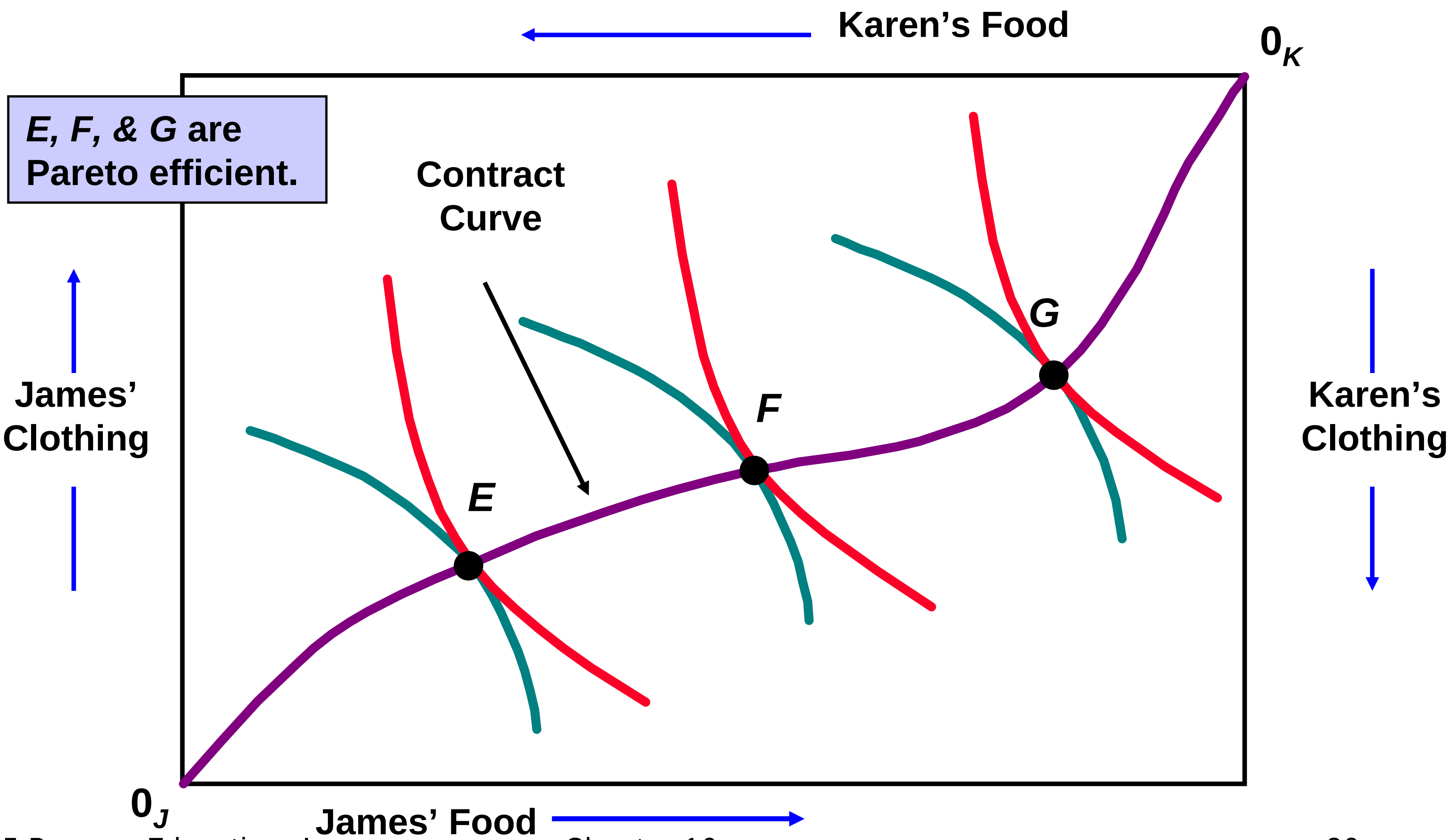
- Any move outside the shaded area will make one person worse off (closer to their origin)
- B is a mutually beneficial trade--higher indifference curve for each person
- Trade may be beneficial but not efficient
- MRS is equal when indifference curves are tangent and the allocation is efficient



Efficiency in Exchange

- The Contract Curve
  - To find all possible efficient allocations of food and clothing between Karen and James, we would look for all points of tangency between each of their indifference curves
  - The **contract curve** shows all the efficient allocations of goods between two consumers.
  - The contract curve is independent of initial endowments.
  - To calculate the contract curve, the utility of an agent is maximized subject to both the feasibility constraint and to the utility level of the other agent´s constraint.

The Contract Curve



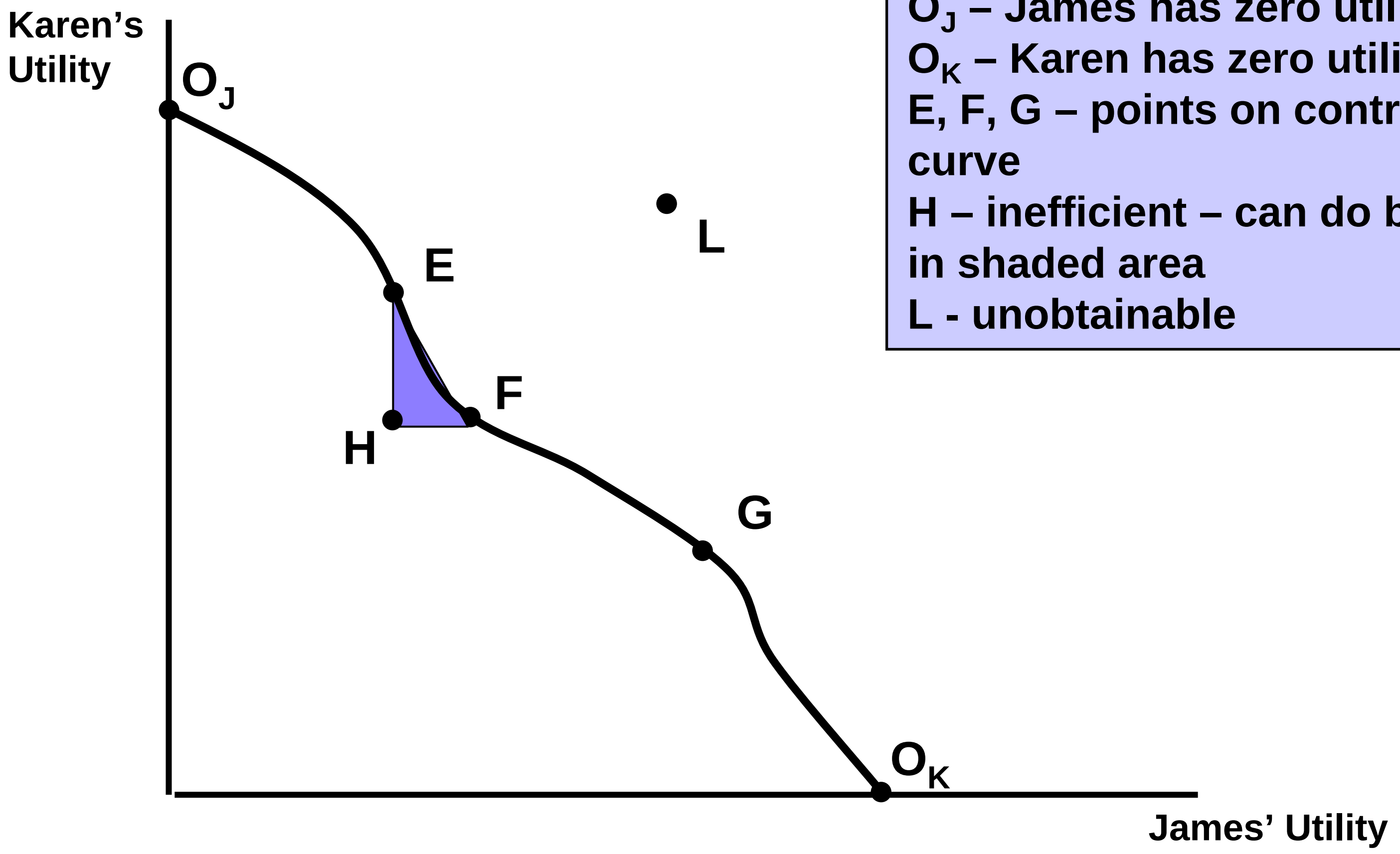
Contract Curve

- All points of tangency between the indifference curves are efficient
  - MRS of individuals is the same
  - No more room for trade
- The contract curve shows all allocations that are Pareto efficient
  - Pareto efficient allocation occurs when further trade will make someone worse off

The Utility Possibilities Frontier

- From the Edgeworth Box, we showed a two person exchange
- The **utility possibilities frontier** represents all allocations that are efficient in terms of the utility levels of the two individuals
  - Shows the levels of satisfaction that are achieved when the two individuals have reached the contract curve

The Utility Possibilities Frontier



O_J – James has zero utility
O_K – Karen has zero utility
E, F, G – points on contract curve
H – inefficient – can do better in shaded area
L - unobtainable

Core of an exchange economy

- The **core** of an exchange economy is *the set of feasible allocations which cannot be improved upon (or blocked) by any coalition of agents*.

- In our 2x2 example: 3 coalitions: {K}, {J} (2 coalitions of one agent) and the grand coalition {K,J}.

- CORE: segment of the contract curve in the shaded area.

- {K}, {J}: Will block no individually rational allocations
- {K,J}: Will block no Pareto rational allocations

Core of an exchange economy with more than two agents.

- Coalition of k agents: Any subset k of agents with mandatory agreements.
- Any k-coalition can block a proposed allocation x whenever the k agents can reallocate their initial endowments among themselves and be better than under x
- Core: RI, Pareto Rationality and rationality of all the remaining coalitions. Example: three agents {A,B,C}
- Coalitions: {A},{B},{C}; {A,B,C,}; {A,B}, {B,C} y
- {A,C}

Core of an exchange economy

Is the core non-empty?

Yes, under convexity of the preferences and perfect divisibility of goods.

BUT:

1. The core is not unique.
2. Huge needs of information.
3. Transaction costs are very high.

Market Exchange: Walras

- Price-decentralization.
- We analyze a process similar to the *competitive mechanism*.
- Agents are price-takers.
- Two caveats:
- This behavior only makes sense in huge economies. When we discuss it for James and Karen, we are assuming that there are many James and many Karens.
- To speak about a “competitive solution” we have to assume that the prices of the goods are known by James and Karen:
- There exits a third person: “***the walrasian auctionier***” who chooses the prices and announces them to the agents, who, in turn, announce the auctionier how much they want to exchange at these prices.

Market exchange: 2x2 Model

- Let us come back to our 2x2 model: James and Karen and food and clothing.
- Let p_A y p_R be the prices of a unit of food and a unit of clothing, respect.
- Given these prices, $p = (p_A, p_R)$, the agents will choose their most preferred exchange that they can afford.
- Are their plans always compatible?
- No, if prices are not equilibrium-prices.

Market exchange: 2x2 Model

- For instance, let

$$p = (p_A, p_R) \;\; \text{y sean } x^J = (x_A^J, x_R^J) \text{ y } x^K = (x_A^K, x_R^K)$$

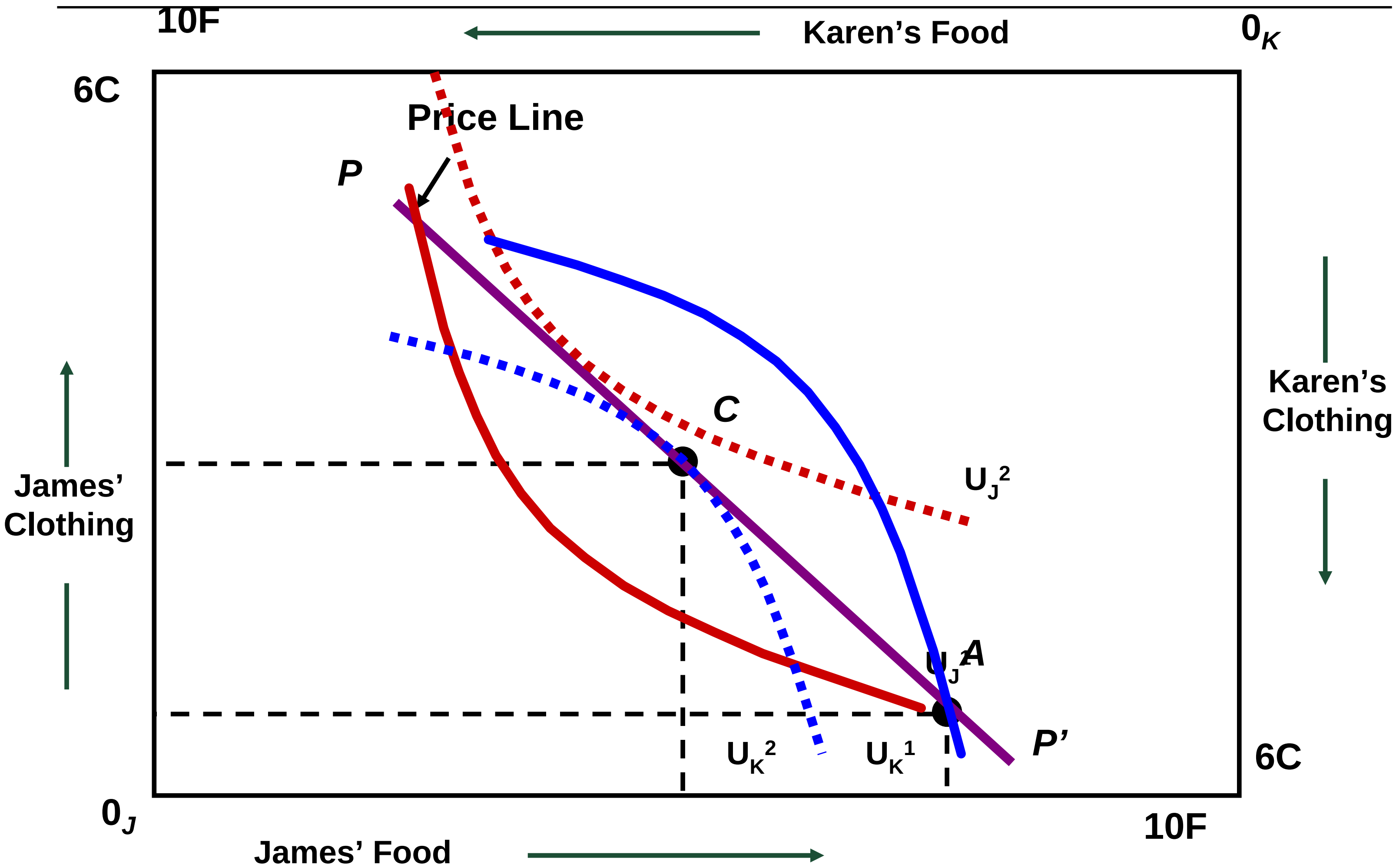
- be James and Karen's demands at these prices, respt. Their initial endowment are:

$$w^J = (w_A^J, w_R^J) \;\; \text{y} \;\; w^K = (w_A^R, w_R^R)$$

- Let the excess demand functions be

$$z_A = x_A^J + x_A^K - (w_A^J + w_A^K) \;\; \text{y} \;\; z_R = x_R^J + x_R^K - (w_R^J + w_R^K)$$

Walrasian Equilibrium in a Pure exchange 2x2 model.



Market Exchange: 2x2 model. There is not market-clearing whenever p is not an equilibrium.

- At *p* markets do not clear.
  - At *p*, James is willing to buy more clothing than the one Karen wants to sell. There is an excess of demand in the clothing market. $z_R > 0$
  - Karen wants to sell mode food than the one James is willing to buy. There an excess of supply in the food market. $z_A < 0$
  - Why? Food is relatively more expensive than clothing: p_A is too high as compared to p_R

Market Exchange: 2x2 model. The Auctionier

- At price vector *p* agents markets' plans are not compatible.
- The auctionier will modify prices according to their excess demand functions:
  - Excess demand will cause price to rise
  - Excess supply will cause price to fall

$$z_A < 0 \rightarrow p_A \downarrow$$
$$z_R > 0 \rightarrow p_R \uparrow$$

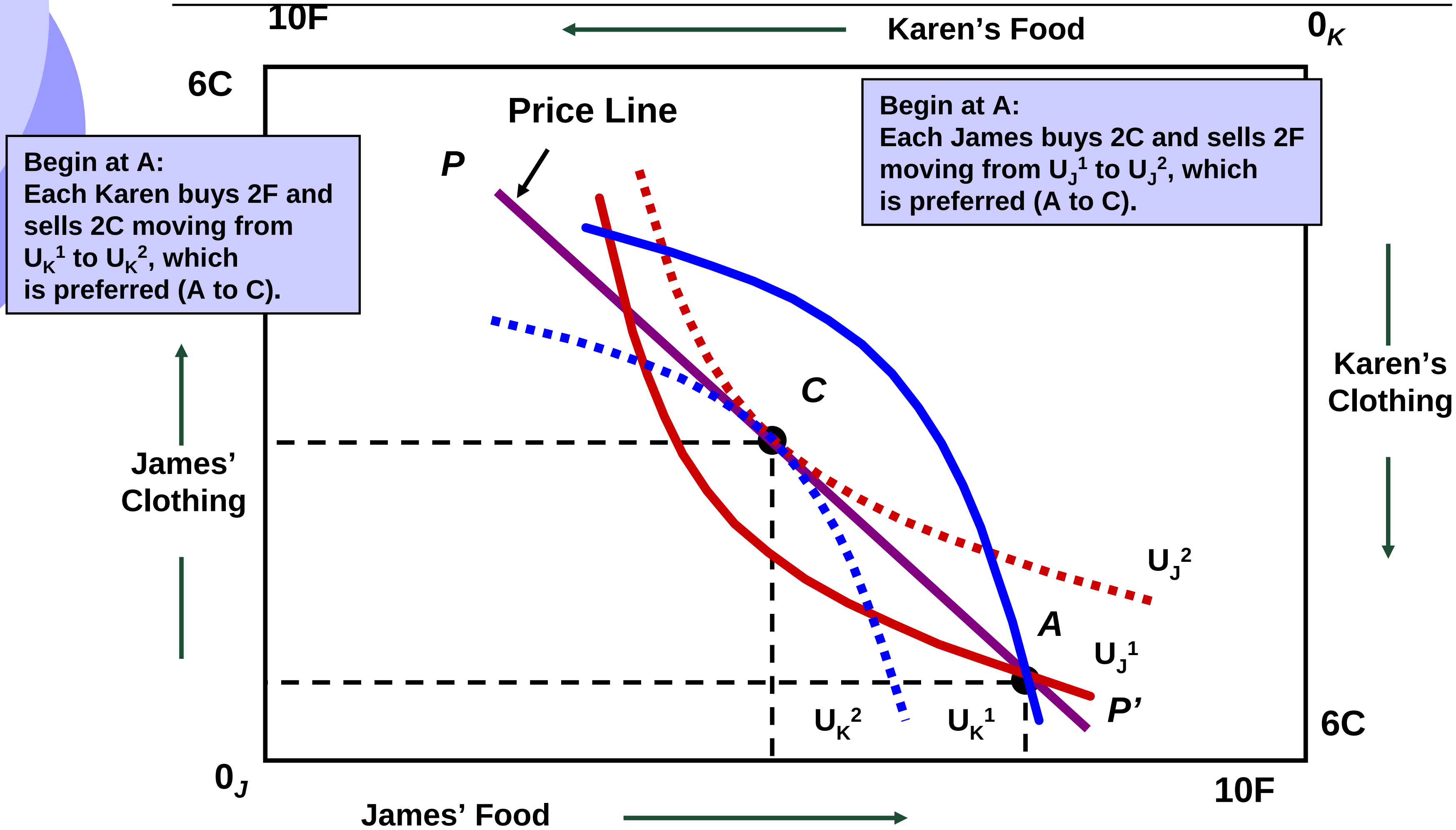
Market Exchange: 2x2 model. The Auctionier

- Disequilibrium is only temporary in a competitive market
  - Excess demand will cause price to rise
  - Excess supply will cause price to fall
- In our example, we have excess supply of clothing and excess demand of food
  - Should expect the price of food to increase relative to price of clothing
  - Prices adjust until equilibrium is reached

Market Exchange: 2x2 model. The Auctionier

- When prices of food and clothing are equal, we can show the price line, PP’ with a slope of –1
    - Shows all possible allocations that exchange can achieve
  - James buys 2 clothing for 2 food: A to C
  - Karen buys 2 food for 2 clothing: A to C
  - Both increase satisfaction.
  - The amount of clothing that Karen wanted to sell is equal to the amount of clothing that James wanted to buy

Market Exchange: 2x2 model. Equilibrium prices= Market Clearing



Market Exchange: 2x2 model. Equilibrium prices= Market Clearing

- At these new prices all the markets clear: $z_A = 0$ y $z_R = 0$
- Agents' market plans are compatible.
- A Walrasian (Competitive) equilibrium is a price vector *p** and a vector of excess demand functions *z**, such that:
- Each agent maximizes her utility at *p**
- There is equilibrium in all the markets

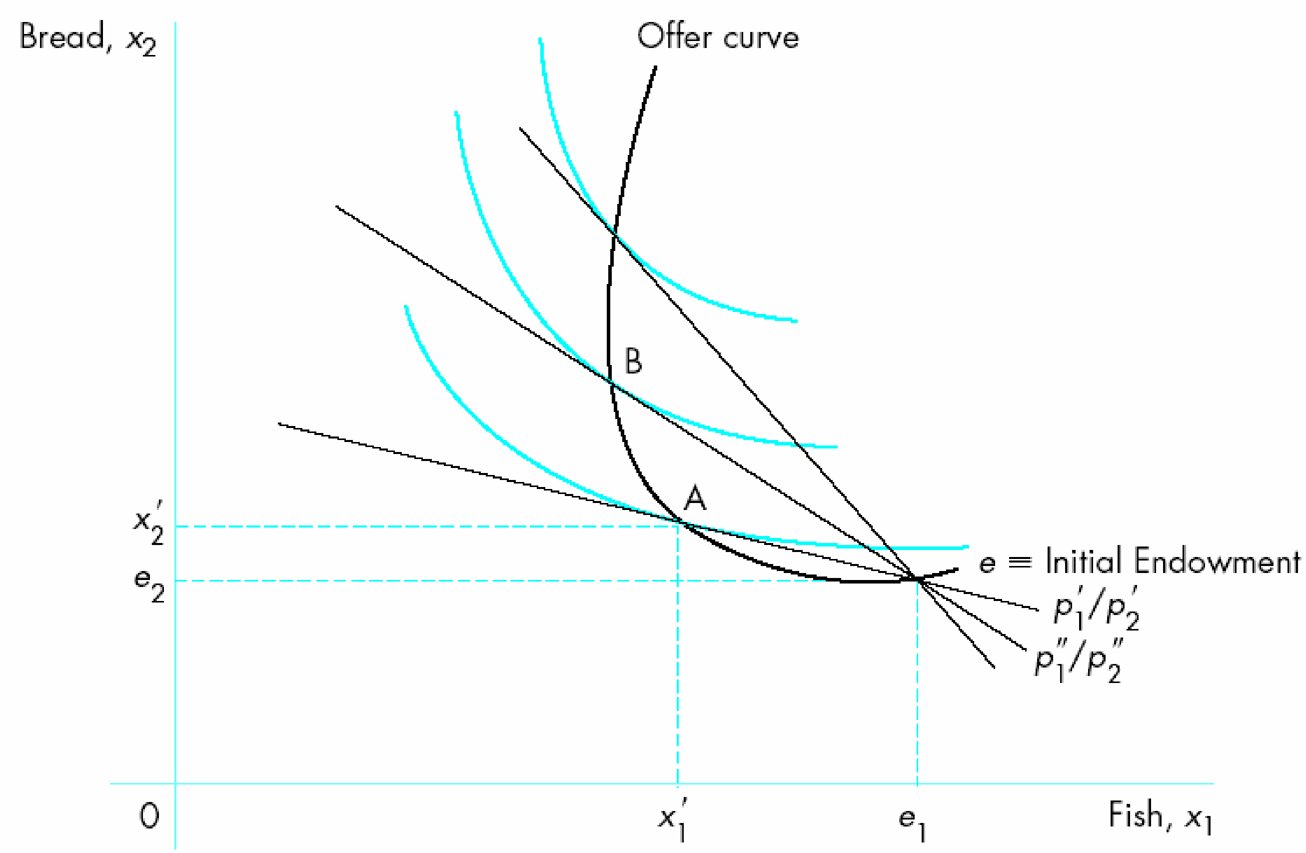
$$z_j^* = 0 \quad \text{si} \quad p_j^* > 0 \qquad \text{(bienes escasos)}$$
$$z_j^* < 0 \quad \text{si} \quad p_j^* = 0 \qquad \text{(bienes libres)}$$

Market Exchange: 2x2 model. Equilibrium prices= Market Clearing

- Walrasian equilibrium
  1. Because the indifference curves are tangent, all MRSs are equal between consumers: *WE=OP* and *EW belongs to the CORE.*
  2. Because each indifference curve is tangent to the price line, each person's MRS is equal to the price ratio of the two goods

$$MRS_{FC}^{J} = P_C / P_F = MRS_{FC}^{K}$$

Offer curves:



Walrasian Equilibrium in a 2x2 model = where the two offer curves cross each other.