

ADVANCED MICROECONOMICS II

Problem set 1 and 2: General Equilibrium.

1. Consider a two-agent exchange economy with utility functions $U_1 = x_{11}x_{12}$, and $U_2 = x_{21}x_{22}$, and with initial endowments $w_1 = (1, 0)$, $w_2 = (0, 1)$. Compute and draw:
 - a) The contract curve and the core of the economy.
 - b) The contract curve and the core of the economy, when the initial endowments are $w_1 = (3/4, 1/4)$, $w_2 = (1/4, 3/4)$. Repeat the analysis for $w_1 = (1/2, 1/2) = w_2$.
 - c) The agents' offer curves and the Walrasian equilibrium. Verify that the Walrasian equilibrium allocation belongs to the core of the economy.
 - d) The Walrasian equilibrium for the initial endowments $w_1 = (3/4, 1/4)$, $w_2 = (1/4, 3/4)$. Repeat the analysis for $w_1 = (1/2, 1/2) = w_2$.

2. Consider a two-agent exchange economy with utility functions $U_1 = x_{11}^\alpha x_{12}^{(1-\alpha)}$, with $\alpha \in (0, 1)$ and $U_2 = \text{Min}\{x_{21}, x_{22}\}$ and with initial endowments $w_1 = (0, 1)$, $w_2 = (1, 0)$. Compute and draw:
 - a) The Walrasian equilibrium
 - b) The contract curve and the core of the economy.

Now suppose that $U_1 = \text{Min}\{x_{11}, x_{12}\}$,

- c) Calculate the contract curve and the core of the economy. Is the Walrasian equilibrium unique?
- d) Compute again the contract curve and the core of the economy, when the initial endowments are $w_1 = (1/4, 3/4)$, $w_2 = (3/4, 1/4)$.

3. Consider each one of the following exchange economies with two agents and two goods:
 - a) $U_1 = x_1 + y_1$; $w_1 = (3, 3)$ and $U_2 = \text{Min}\{x_2, y_2\}$; $w_2 = (3, 0)$
 - b) $U_1 = x_1 + y_1$; $w_1 = (3, 3)$ and $U_3 = \text{Min}\{x_3, 5y_3\}$; $w_2 = (0, 3)$
 - c) $U_2 = \text{Min}\{x_2, y_2\}$; $w_2 = (3, 0)$ and $U_3 = \text{Min}\{x_3, 5y_3\}$; $w_2 = (0, 3)$

Calculate the contract curve, the core and the Walrasian equilibrium of each economy.

- e) Consider the economy of the above three agents: $U_1 = x_1 + y_1$, with $w_1 = (3, 3)$, $U_2 = \text{Min}\{x_2, y_2\}$ with $w_2 = (3, 0)$, and $U_3 = \text{Min}\{x_3, 5y_3\}$ with $w_3 = (0, 3)$, and the following allocations:

$$a_1 = [(11/4, 15/4), (3, 3), (5/4, 1/4)]$$

$$a_2 = [(7/4, 11/4), (3, 3), (5/4, 1/4)]$$

$$a_3 = [(9/4, 15/4), (3, 2), (3/4, 1/4)]$$

$$a_4 = [(11/4, 15/4), (2, 2), (5/4, 1/4)]$$

Do they belong to the core of the economy?. Justify your answer.

4. The excess demand functions of goods 1 and 2, in a three-good economy are given by:

$$z_1(p_1, p_2, p_3) = -p_2 / (p_1 + p_2) + p_3 / (p_1 + p_3)$$

$$z_2(p_1, p_2, p_3) = -p_3 / (p_2 + p_3) + p_1 / (p_1 + p_2)$$

Calculate the excess demand function of good 3.