Filter example: We consider the three-stage filter shown in Fig. 2, which directly implements our approach. To illustrate the detection and diagnosis capability of the approach, we will consider a parametric fault; namely, a deviation in R_1 of 20%. Fig. 3 shows the simulation results for each stage in the test mode. Signals labelled '*_NF*' correspond to the fault-free circuit and those labelled '*_FR1*' correspond to the faulty circuit. The input (V_{in}) used is a square signal between 0 and 1V. As can be seen, the third (Fig. 3*a*) and second (Fig. 3*b*) stages operate as expected. The obtained responses are close to the corresponding fault-free output (both signals are indistinguishable). However, in the test of the first stage (Fig. 3*c*), a disparity with the fault-free response is observed; namely, a fault in the DC gain. Using this measurement and interpreting the first-order transfer function corresponding to the faulty stage, we can predict the cause of the fault. R_1, R_2 and R_7 are responsible for the DC gain, but R_2, R_7 and C_2 also determine the time constant of the stage response. Since this time constant is as expected (as shown in Fig. 3), the error is in R_1. Hence, the fault is detected and its diagnosis coincides with the real fault.

Conclusions: A DfT methodology has been reported for analogue fault diagnosis in active analogue filters. This methodology improves that reported in [8, 9], and its impact on the filter performance is negligible. In terms of hardware and software, the cost is low, requiring only opamps with a duplicated input stage and a previous simulation to obtain the nonfaulty output signals for each stage in the test mode. The applicability has been demonstrated with a simple example using HSPICE simulations. Since each stage of the filter is tested separately, single-faults and multiple-faults can be diagnosed. The methodology can be extended to other types of filters and circuits using opamps.

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Bit allocation algorithm for codebook design in vector quantisation fully based on human visual system nonlinearities for suprathreshold contrasts

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Indexing terms: Image processing, Vector quantisation, Human factors

A fully perceptual oriented approach for codebook design in the vector quantisation of images for compression purposes is presented. The proposed algorithm not only exploits the usually considered dependence on frequency of the human eye, but also makes use of the nonlinear behaviour of the visual system for suprathreshold contrasts. From experimental data of human contrast incremental thresholds of sinusoidal grids, we present a perceptually optimal information allocation function. Simulations show that by introducing perceptual contrast nonuniformities, a more efficient bit allocation is attained than that for the JPEG-like contrast uniform algorithm.

Introduction: The central aim of lossy image compression techniques is to adapt redundancy reduction algorithms to the limitations of the human visual system (HVS), to make the variations unnoticeable. The more recent and successful lossy compression methods are those that perform a vector quantisation (VQ) of the coefficients of a given image transform [1]. The main problem to overcome with these VQ algorithms is the design of a codebook that optimises the subjective quality of the decoded images. A variety of VQ compression algorithms base their codebook design on statistical properties of the images [2]. However, the effect of quantisation on the human eye may not always correspond exactly with mathematical differences in error levels. The HVS capabilities have been introduced as a coefficient-dependent weighting function [3]. In these perceptually weighted algorithms, the number of quantisation levels per coefficient is determined by the human threshold sensitivity whereas the amplitude nonuniformities are still given by the statistical properties of the training set [2], or are not considered [4].

We present an information allocation algorithm for codebook design that can give rise to a general nonuniform partition of the coefficient-amplitude transform domain and can be easily related to the perceptual capabilities of the HVS in suprathreshold conditions. This algorithm improves the subjective quality of the decoded images.

Codebook design controlled by information allocation function (IAF): VQ of a transformed signal involves partitioning of the coefficient-amplitude space. in the proposed scheme, the whole codebook design process is fixed by the information allocation function (IAF). The IAF is a function defined in the coefficient-amplitude space giving the amount of information assigned to encode each zone of that space, given by

$$IAF(f,C) = \frac{d^2 I}{df\,dC} \tag{1}$$

where I is the information (in bits), and f and C indicate the zone of the coefficient-amplitude (frequency-contrast) space.

The central idea is that the nonuniform distribution of codewords in the vector space must be derived from the IAF in such a manner that we have a peak density of codewords (and minimum values of quantisation step size) where the IAF reaches its maximum. The number of bits assigned to encode each coefficient f can be immediatly obtained by integrating the IAF for amplitudes. We call this integral the cumulative information allocation function (CIAF), where

$$CIAF(f) = \frac{dI}{df} = \int_{C_m}^{C_M} IAF(f,C)dC \tag{2}$$

where C_m and C_M are the boundaries of the amplitude range.

The CIAF gives us a criterion to make a coefficient-selective bit allocation, but gives us no intormation about the distribution of the quantisation levels over the amplitude range of a given coeffi-

cient. This question is solved by using a set of look-up tables (one per coefficient) called nonuniform quantisation functions (NUQFs) obtained from the IAF by the following equation:

$$NUQF(f,C) = \frac{C_M - C_T(f)}{\int_{C_m}^{C_M} \frac{1}{IAF(f,C)}dC} \int_{C_m}^{C} \frac{1}{IAF(f,C)}dC + C_T(f) \quad (3)$$

where $C_T(f)$ is the threshold amplitude for the coefficient f. The NUQF are monotonically increasing functions representing a nonlinear mapping of the amplitude range $[C_m, C_M]$ into the range $[C_T(f), C_M]$. The slope of the NUQF varies with the amplitude nonuniformities of the IAF, having a minimum slope when the IAF reaches a maximum. To summarise: the IAF represents an intuitive distribution of information in the space to be partitioned, the CIAF gives the number of quantisation levels assigned to each coefficient and each NUQF gives the definitive nonuniform spacing of those quantisation levels. Fig. 1 summarises (with a 1-D example) the codebook design process controlled by the IAF.

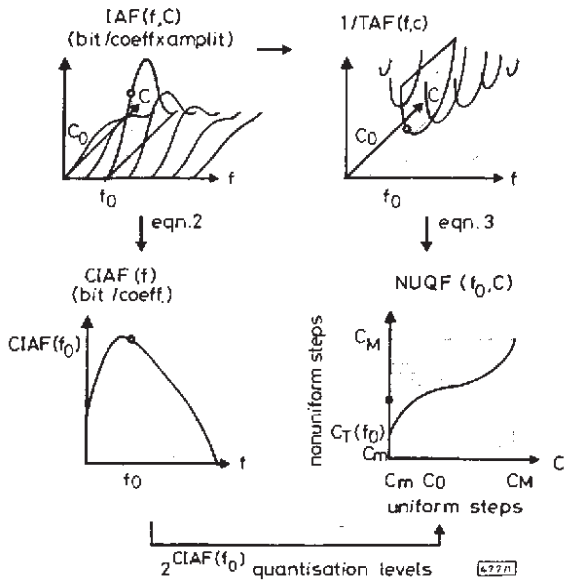

Fig. 1 *1-D example of codebook design process controlled by IAF*

Perceptually optimal information allocation function: In applications where the image is going to be judged by a human observer, VQ must benefit from the visual system limitations, reducing information where the system is not sensitive. In the most general case, the sensitivity peaks of the system are not overlapped with the probability density peaks. Therefore the subjective rule can be stated as follows: the bigger the sensitivity is, the narrower the quantisation step must be. The implementation will be optimal if the quantisation step widths equal (or are proportional) to the HVS tolerances to changes in the coefficient or amplitude, for every point in the coefficient-amplitude space.

In the psychophysical perception literature, those limitations of the ability of discrimination are referred to as incremental thresholds of frequency (Δf) and contrast (ΔC) [5, 6]. If the variation of the coefficients (or amplitude) in the encoding process is lower than the corresponding incremental threshold Δf (or ΔC), the system cannot perceive the variation.

The IAF codebook design algorithm can be easily adapted to obtain quantisation step sizes proportional to the system tolerances as required for optimal subjective performance. A perceptually optimal bit allocation for spatial frequency encoding can be obtained by defining the IAF as the inverse of the area of psychovisual tolerance regions. That is

$$IAF(f,C) = \frac{K}{\Delta f \Delta C} \quad (4)$$

where K is a normalisation constant given by the total amount of information employed when encoding the image. Eqn. 4 states that more information is assigned to the narrower tolerance regions. Recent exhaustive experimental measurements have shown that tolerance of the system to changes in the coefficient or in the amplitude of each basic stimulus depends on the point of the coefficient-amplitude space [5, 6]:

$$\Delta f(f,C) = af + b \quad (5a)$$

$$\Delta C(f,C) = C_T(f) + \frac{C}{\left(\frac{C_T(f)}{k(F)}\right)^{\frac{1}{n(f)}} + C}\left(k(f)C^{n(f)} - C_T(f)\right) \quad (5b)$$

where a and b are constants [5], $n(f)$ and $k(f)$ are experimentally measured parameters [6] and $C_T(f)$ is the threshold contrast of a sinusoid of frequency f to be just perceived. The inverse of $C_T(f)$ is known as the contrast sensitivity function (CSF) [7] and is usually taken as a reference for frequency-selective functions in perceptually weigthed algorithms.

Taking into account the slow variation of the frequency tolerance ($a = 0.025$ for f in cycles per degree[5]) and considering that discrete implementation of the transforms in the encoding process implies a fixed uniform frequency sampling and avoids frequency variations, we will consider Δf = constant. Therefore we propose a perceptually optimal IAF given by the inverse of eqn. 5*b*.

The suprathreshold term in the IAF (the C dependent term) represents a significant qualitative improvement compared with previous perceptually weighted algorithms [4]. If such a suprathreshold dependence were removed, the bit allocation would be controlled by the CSF (the human frequency selective function) as in the standard perceptual oriented VQ methods. Therefore, for example, the JPEG compression algorithm can be derived from our approach for the particular case of contrast independent IAF. The suprathreshold nonuniformities give a physiological basis to empirical changes needed in the quantisation matrix of the JPEG standard when exclusively supported by the threshold sensitivity (filter function CSF).

Fig. 2 *Image processed by uniform JPEG-like method (1.6 bit/pixel) and perceptual IAF nonuniform method (1.6 bit/pixel)*

a Uniform JPEG-like method
b Perceptual IAF nonuniform method

Results: We compare our perceptual contrast nonuniform bit allocation algorithm with the JPEG-like algorithm (uniform step size over the entire contrast range). The same number of quantisation levels per coefficient are used in both schemes. Fig. 2*a* and *b* show decoded *Lena* images obtained by applying each algorithm. Imposing the same compression ratio, simulations clearly show a better subjective quality for the contrast nonuniform scheme.

Despite the relatively low compression ratio of the proposed example, the results derived from simulations demonstrate the importance of considering the HVS nonlinearities for suprathreshold contrast in future research.

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Counting lattice points on ellipsoids: Application to image coding

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Indexing terms: Vector quantisation, Image coding

The enumeration problem of lattice vectors in a lattice codebook shaped for elliptical source statistics is solved. The counting of codebook vectors is crucial for the computation of the bit rate, i.e. for the determining the codeword lengths to be transmitted to the decoder.

Introduction: In lattice-based vector quantisation, it is known that the shape of truncation must fit, as closely as possible, the probability density function of the source to minimise the overload noise of the quantiser. This is strongly linked to the shaping advantage of vector quantisation over scalar quantisation, pointed out by Lookabaugh *et al.* in [6]. This gain is obtained by taking into account the spatial distribution of the source vectors in the design of the codebook. Thus, uniform, Gaussian, and Laplacian memoryless sources lead to hypercubic, spherical, and pyramidal codebooks, respectively.

Gaussian sources with memory are also of great interest in signal processing, particularly in image or speech coding. For example, wavelet coefficients [1] of robot vision or satellite images are strongly correlated. Thus, vectors constituted by neighbouring coefficients are elliptically distributed around zero in a privileged direction parallel to $(1, \ldots, 1)$. Lattice vector quantisation applied to these data will be more efficient when applying an elliptical truncation [2, 5].

We address the case of Gaussian sources with memory, coded by elliptical lattice-based codebooks. In a prefix code encoding scheme, such codebooks need to solve the problem of counting lattice points lying on elllpsoids for codeword length computation [7]. Indeed, knowledge of the total number of codebook vectors [2] is not sufficient in such an encoding scheme. We propose a solution based on the generating theta series designed by Conway and Sloane [3] for the main lattices $\mathbf{Z}^n$, D_n, E_8 which are widely used in data compression applications.

Lattices and elliptical surfaces: The problem of enumeration can be solved by defining concentric surfaces of constant radius on which lattice points can be counted. Thus, the generating theta series proposed by Conway and Sloane enable the counting of lattice points lying on spheres and therefore within spherical codebooks. Furthermore, Fischer proposed a solution for pyramidal codebooks designed in the $\mathbf{Z}^n$ lattice [4] and Solé proposed a solution for pyramidal codebooks designed in the main lattices [8]. We propose to solve the problem for elliptical codebooks designed in the main lattices. First, we can define a centred n-dimensional ellipsoid with axis lengths equal to $a_i r$ as the set

$$\{X \in \mathbf{R}^n \mid (\|X\|_2^2)_W = r^2\} \tag{1}$$

where $(\|x\|_2^2)_W = XWX^t$, r is a nonnegative constant and W is a weighted diagonal matrix such that $w_{ii} = 1/a_i^2$, $a_i > 0 \ \forall \ i$ and $w_{ij} = 0 \ \forall \ i \neq j$. Thus, the counting of lattice vectors lying within an elliptical codebook such as $r = l$ can be seen as the counting of lattice vectors lying on concentric ellipsoids for r varying from 0 to l.

Modified theta series for counting lattice points on ellipsoids: For a given lattice Λ, the number N_r of points at distance r from the origin (metric in the weighted L_2 norm) is given by

$$\Theta_\Lambda^W(q) = \sum_{X \in \Lambda} q^{(\|X\|_2^2)w} = \sum_{r=0}^{+\infty} q^{r^2} \left|\{X \in \Lambda \mid (\|X\|_2^2)_W = r^2\}\right| \tag{2}$$

The coefficient of q^{r^2} is the number of lattice vectors lying on a centred ellipsoid of radius r (metric in the weighted L_2 norm). Conway and Sloane solved the problem for $W = I$ where I is the identity matrix and propose a method of counting lattice points lying on spheres (using theta series). For more details see [3]. We propose to solve the problem for $W \neq I$ and thus to count lattice points lying on ellipsoids. We first recall the Jacobi theta functions

$$\theta_2(q) = \sum_{m=-\infty}^{+\infty} q^{(m+1/2)^2} \qquad \theta_3(q) = \sum_{m=-\infty}^{+\infty} q^{m^2}$$

$$\theta_4(q) = \sum_{m=-\infty}^{+\infty} (-q)^{m^2} \tag{3}$$

where $q = e^{i\pi z}$. We note that

$$\Theta_{\mathbf{Z}}(q) = \theta_3(q) \qquad \theta_4(q) = \theta_3(q^4) - \theta_2(q^4)$$
$$\text{and } \theta_3(q) = \theta_3(q^4) + \theta_2(q^4) \tag{4}$$

It is useful for computing theta series to consider lattices constructed from algebraic codes. Conway and Sloane give two constructions based on binary linear codes to design main lattices: construction A and construction B [3]. Furthermore, to each binary linear code C is attached a weight enumerator $W_C(x, y)$, where x and y denote the number of zeros and ones in a codeword, respectively. The weight enumerators of codes associated with the main lattices are well known. Furthermore, two important theorems (Theorems 3 and 15, Chapter 7 of [3]) enable us to write

$$\Theta_\Lambda(q) = W_C(\theta_3(q^4), \theta_2(q^4)) \tag{5a}$$
for construction A

$$\Theta_\Lambda(q) = \frac{1}{2} W_C(\theta_3(q^4), \theta_2(q^4)) + \frac{1}{2}[\theta_4(q^4)]^n \tag{5b}$$
for construction B

From the Jacobi functions given in eqn. 3, we can define

$$\theta_j^{a_i}(q) = \theta_j(q^{1/a_i^2}) \tag{6}$$

where $q = e^{i\pi z}$ and $a_i \neq 0$. Then, using construction A for $\mathbf{Z}^n$ and D_n lattices, and construction B normalised by 1/2 for the E_8 lattice, the corresponding weight enumerators are given by

$$W_{C(\mathbf{Z}^n)}(x, y) = \prod_{i=1}^{n}(x_i + y_i)$$

$$W_{C(D_n)}(x, y) = \prod_{i=1}^{n}[(x_i + y_i)] + \prod_{i=1}^{n}[(x_i - y_i)] \tag{7}$$

$$W_{C(2E_8)}(x, y) = \prod_{i=1}^{8} x_i + \prod_{i=1}^{8} y_i$$

From eqns. 5, 7 and 4, we can define modified theta series in terms of functions given by eqn. 6 as

$$\Theta_{\mathbf{Z}^n}(q) = \prod_{i=1}^{n}[\theta_3^{a_i}(q^4) + \theta_2^{a_i}(q^4)] = \prod_{i=1}^{n}[\theta_3^{a_i}(q)]$$

$$\Theta_{D_n}^W(q) = \frac{1}{2}\left[\prod_{i=1}^{n}[\theta_3^{a_i}(q^4) + \theta_2^{a_i}(q^4)] + \prod_{i=1}^{n}[\theta_3^{a_i}(q^4) - \theta_2^{a_i}(q^4)]\right]$$
$$= \frac{1}{2}\left[\prod_{i=1}^{n}\theta_3^{a_i}(q) + \prod_{i=1}^{n}\theta_4^{a_i}(q)\right]$$

$$\Theta_{2E_8}^W(q) = \frac{1}{2}\left[\prod_{i=1}^{8}\theta_2^{a_i}(q^4) + \prod_{i=1}^{8}\theta_3^{a_i}(q^4) + \prod_{i=1}^{8}\theta_4^{a_i}(q^4)\right] \Rightarrow$$

$$\Theta_{E_8}^W(q) = \frac{1}{2}\left[\prod_{i=1}^{8}\theta_2^{a_i}(q) + \prod_{i=1}^{8}\theta_3^{a_i}(q) + \prod_{i=1}^{8}\theta_4^{a_i}(q)\right] \tag{8}$$