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ON THE INTRINSIC TWO-DIMENSIONALITY OF THE CSF AND ITS MEASUREMENT

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Two-dimensional Point Spread Function

MOTS CLÉS :

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Filtrage des Fréquences Spatiales
Fonction de Sensibilité au Contraste Bidimensionnelle
Fonction Réponse Impulsionnelle Bidimensionnelle

SUMMARY : Classically the visual system sensitivity has been studied separately as a function of spatial frequency and orientation, without explicitly pointing out the relation between both parameters.

As the visual system processes two-dimensional signals, its Contrast Sensitivity Function (CSF) is intrinsically two-dimensional too, and therefore, the dependence of the 1-D CSF on orientation is obtained by considering different radial profiles of the 2-D CSF.

In the present work, we study the simplest and most straightforward method to obtain the maximum amount of information about the 2-D CSF with a minimum of measurements. It follows from the results that, for emmetropic observers, it is enough to measure two perpendicular radial profiles of the CSF to obtain enough information about the whole 2-D frequency domain, where as for the most common astigmatism cases, it's necessary to include another intermediate measure.

Assuming that the Modulation Transfer Function (MTF) is proportional to the CSF, it's possible to compute the visual system Point Spread Function (PSF), optical and neural parts, from the function experimentally obtained. To estimate the accuracy of the method, the 2-D PSF is calculated and, we simulate, by computer, the perception of a test using the 2-D CSF measured for emmetropic and astigmatic observers.

Au sujet de la bidimensionalité intrinsèque de la CSF et sa mesure

RÉSUMÉ : Classiquement, on a étudié séparément la sensibilité visuelle comme une fonction de la fréquence et de l'orientation, sans que l'importance de la relation entre les deux n'ait été remarquée.

Puisque le système visuel traite des signaux bidimensionnels, sa Fonction de Sensibilité au Contraste (CSF) est aussi intrinsèquement bidimensionnelle. La dépendance de la CSF 1-D étant fonction de l'orientation, elle peut s'obtenir en considérant différentes sections de la CSF 2-D.

Dans ce travail, on étudie, la méthode la plus simple et la plus directe de manière à obtenir le plus d'informations sur la CSF 2-D, en effectuant le moins de mesures possibles. Grâce à cette procédure, il suffira à des observateurs emmétropes, de ne mesurer que deux sections perpendiculaires de la CSF pour obtenir assez d'informations dans tout le domaine fréquenciel bidimensionnel ; alors que pour des astigmates les plus fréquents une mesure supplémentaire sera nécessaire.

En considérant que la Fonction de Transfert de Modulation (MTF) est proportionnelle à la CSF, on peut simuler à l'aide d'un ordinateur, la fonction réponse impulsionnelle (PSF) tant de la partie optique que de la partie neurale du système visuel, d'après la fonction obtenue expérimentalement. Pour estimer la qualité de la procédure, on calcule la PSF 2-D et on simule la perception, par des observateurs emmétropes et astigmates, d'un test en utilisant les CSF 2-D mesurées.

INTRODUCTION

INTRINSIC TWO-DIMENSIONALITY OF THE CSF

It is well known that the response of the human visual system depends in a non-trivial way on the spatial frequency of the input stimulus. As it was shown by Shade [1], Campbell & Green [2], and Campbell & Robson [3], among others, the system behaves (for threshold contrasts) as a passband filter.

Davidson [4] specified the conditions under which it is possible to accomplish such a characterization. In the low contrast range, the system response is approximately linear and, in practice, its global performance is homogeneous.

To characterize the system by means of a transfer function, which represents the attenuation experimented by each sinusoidal component of the input image, offers two very important advantages compared with the classical methods based in the measurement of the visual acuity. The first, a qualitative one, because it defines the system's performance before any kind of stimulus, whereas the visual acuity represents only the small-detail detection limit. The second, a quantitative one, because, once the transfer function is known, the result of the information preprocessing by the visual system can be quantitatively reproduced [5]. To do this, it suffices to apply the filter function on the spectrum of the input intensity distribution, as follows :

$$\mathbf{i}'(\mathbf{x}) = \mathrm{FT}^{-1}[\mathrm{CSF}(\mathbf{k}) \cdot \mathrm{FT}[\mathbf{i}(\mathbf{x})]] \qquad (1)$$

where $\mathbf{i}(\mathbf{x})$ represents the input image, $\mathbf{i}'(\mathbf{x})$ the perceived image, CSF (k) the contrast sensitivity function defined in the 2-D spatial-frequency domain, and FT a Fourier transform.

Traditionally the visual system sensitivity has been calculated as a function of the frequency of a test grating of a fixed orientation. On the other hand, the dependence of the function on grating orientation has been studied without emphasizing the existing link between frequency and orientation.

Either if we want to obtain a general characterization that includes the effects due to orientation, or if we must use Eq. (1) in simulation applications (like those in which visual acuity is obtained reproducing the perception of Snellen letters [5]), it is necessary to have at one's disposal values of the CSF in enough points $\mathbf{k}$ of its entire 2-D domain.

For these reasons, the development of a simple method to obtain the 2-D CSF is interesting both from a theoretical viewpoint (as it would enable us wholly to characterize the system) and from a practical one (as it would permit us to perform numerical simulations of perception).

In this work, we show that to obtain an adequate characterization that would take into account the asymmetries of sensitivity with orientation, it is enough to have two radial profiles of the 2-D CSF in perpendicular directions and to perform a linear interpolation for angles between these two directions.

MEASUREMENT OF THE CSF ON ITS TWO-DIMENSIONAL DOMAIN

It is known [7] that the spectrum of a sinusoidal grating of spatial frequency k and orientation θ consists of two Dirac's delta functions in the 2-D frequency domain, located at $\mathbf{k} = \pm (k_x, k_y)$ where :

$$\begin{aligned} k_x &= k \cos(\theta) \\ k_y &= k \sin(\theta) . \end{aligned} \qquad (2)$$

From this, when we determine the threshold detection contrast for a sinusoidal grating, we also determine the value of the CSF in the corresponding point $\mathbf{k}$.

By a suitable choice of the test grating it is possible to minimize the experimental process and yet to obtain sufficient information about the CSF to characterize the system correctly.

It is aparent that the detection process is strongly dependent on frequency whereas it depends in a lesser degree on orientation (except in cases of strong astigmatism). This suggest that the two-dimensional CSF varies strongly with the radius and is radially symmetric to some extent, thus, to obtain sufficient information it could be enough to use a reasonable sample of frequencies in a few radial directions.

Any other value can be calculated by linear interpolation between the measured values at one frequency k and different orientations θ_1, θ_2 by means of the following equation :

$$\mathrm{CSF}(k, \theta) = \mathrm{CSF}(k, \theta_1) + \frac{\mathrm{CSF}(k, \theta_2) - \mathrm{CSF}(k, \theta_1)}{\theta_2 - \theta_1} (\theta - \theta_1) . \qquad (3)$$

Clearly, the quickest procedure is to measure only one radial profile of the CSF (as it is usually done when the threshold contrast for gratings of fixed orientation is measured) and extrapolate the results until we have generated the complete function taking advantage of its radial symmetry, but, in this way, the information about possible orientation asymmetries is lost, and the characterization so obtained is deficient.

To gain some insight about the function asymmetries, it is necessary to measure more radial profiles. The main difficulty consists in reaching a compromise between the fiability of the results (the measurement of many planes being necessary to achieve a good result) and the manageability of the method.

In order to estimate the number of CSF radial profiles that is needed to carry out an adequate characterization of the visual system, we have measured different CSF radial profiles (at 0°, 45° and 90°, respectively). These measurements were made by two observers, and their 2-D CSF and 2-D PSF was computed with the usual assumption that the MTF and the CSF are proportional [8]. Using one, two or three radial profiles to calculate the CSF, we study to which extent a greater number of measurements gives additional information.

EXPERIMENTAL METHOD

The CSF of each observer was calculated using gratings with variable frequency and orientation

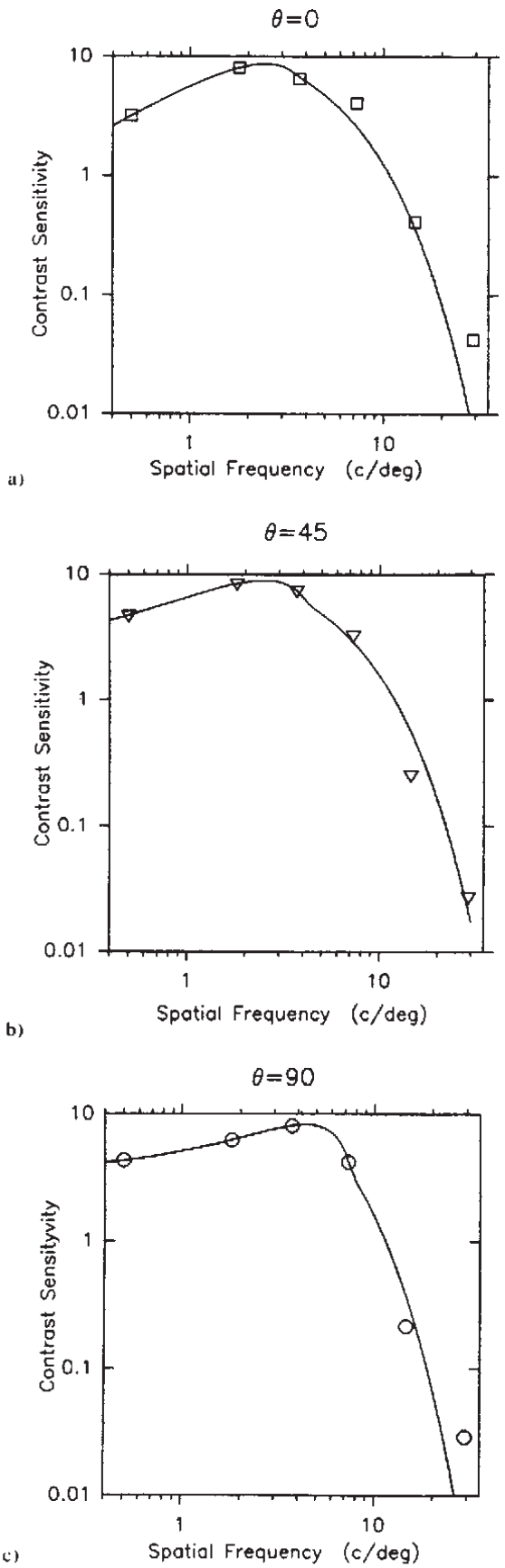


FIG. 1. — *Experimental measurements of three radial profiles of the* CSF *for observer JM* (EMMETROPIC), *and the correspondent fit curves. The results were obtained determining the sensitivity to square gratings with orientation* θ : (a) $\theta = 0°$; (b) $\theta = 45°$; (c) $\theta = 90°$. *Contrast sensitivity is expressed in* 1/*(threshold contrast) units, and contrast is expressed in %.*

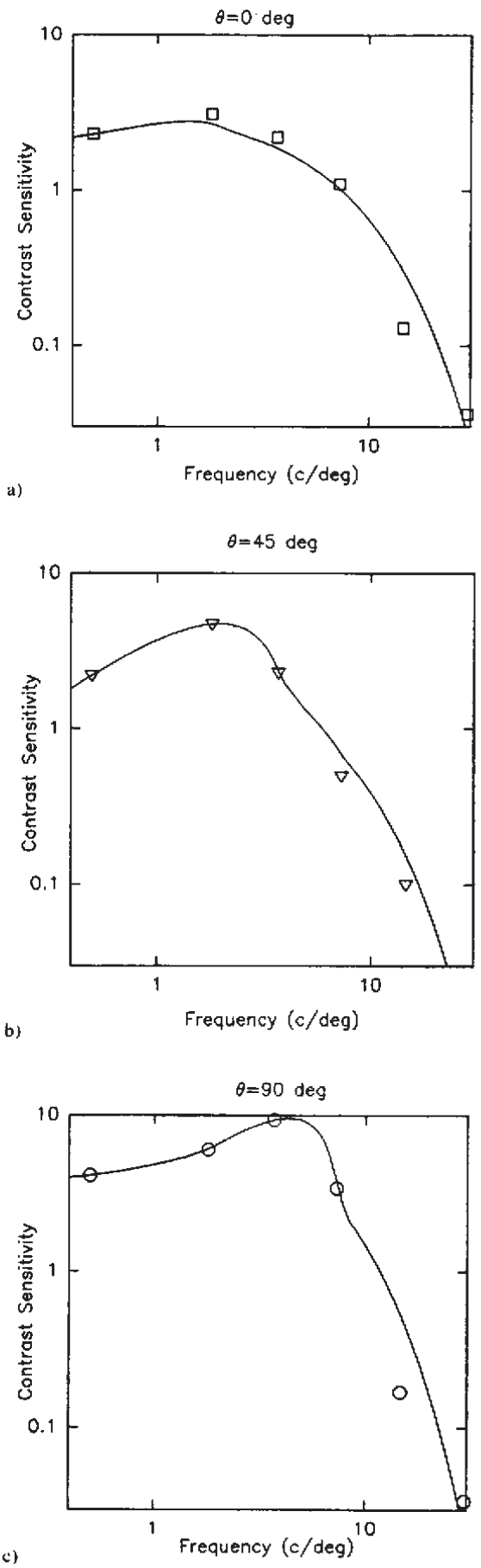


FIG. 2. — *Experimental measurements of three radial profiles of the* CSF *for observer* RZ (ASTIGMATIC-3 dp, O°), *and the correspondent fit curves.*

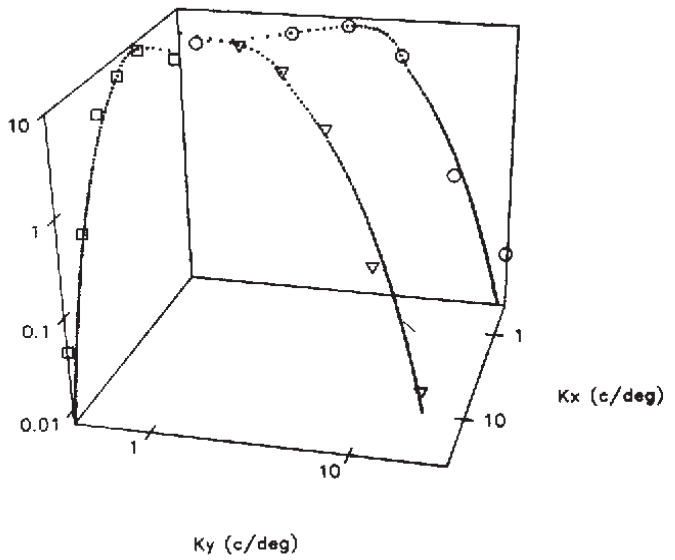


FIG. 3. — *Results of figure* 1 (*Obs. JM* EMMETROPIC) *plotted in two-dimensional spatial frequency domain. The location in this domain is given by equation* (2). *These are the basic data for the interpolation. The interpolation process will generate intermediate radial profiles to achieve a smooth (linear) transition between the measured profiles.*

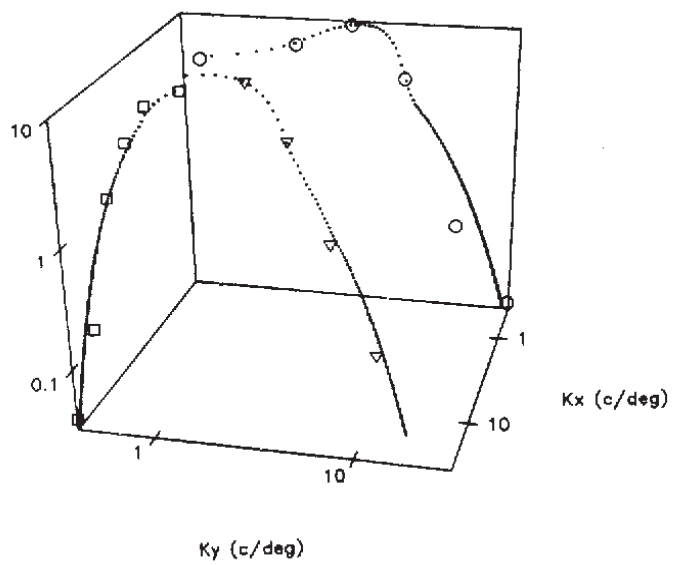


FIG. 4. — *Results of figure* 2 (*Obs.* RZ, ASTIGMATIC) *plotted in the two-dimensional spatial frequency domain.*

electronically generated on the face of a DM2 Joyce Display Oscilloscope with phospor WA-white and a resolution of 1 024 × 1 024 pixels and 8 bits/pix. Patterns were under computer control (IBM AT) via a VSG2 waveform generator based on a TMS320C25 digital signal processor operating at a clock speed of 40 MHz which gives a basic instruction timing of 100 μsec (10 MIPS). Once a given orientation has been fixed, the program presents in a random way gratings with different frequencies, and the threshold

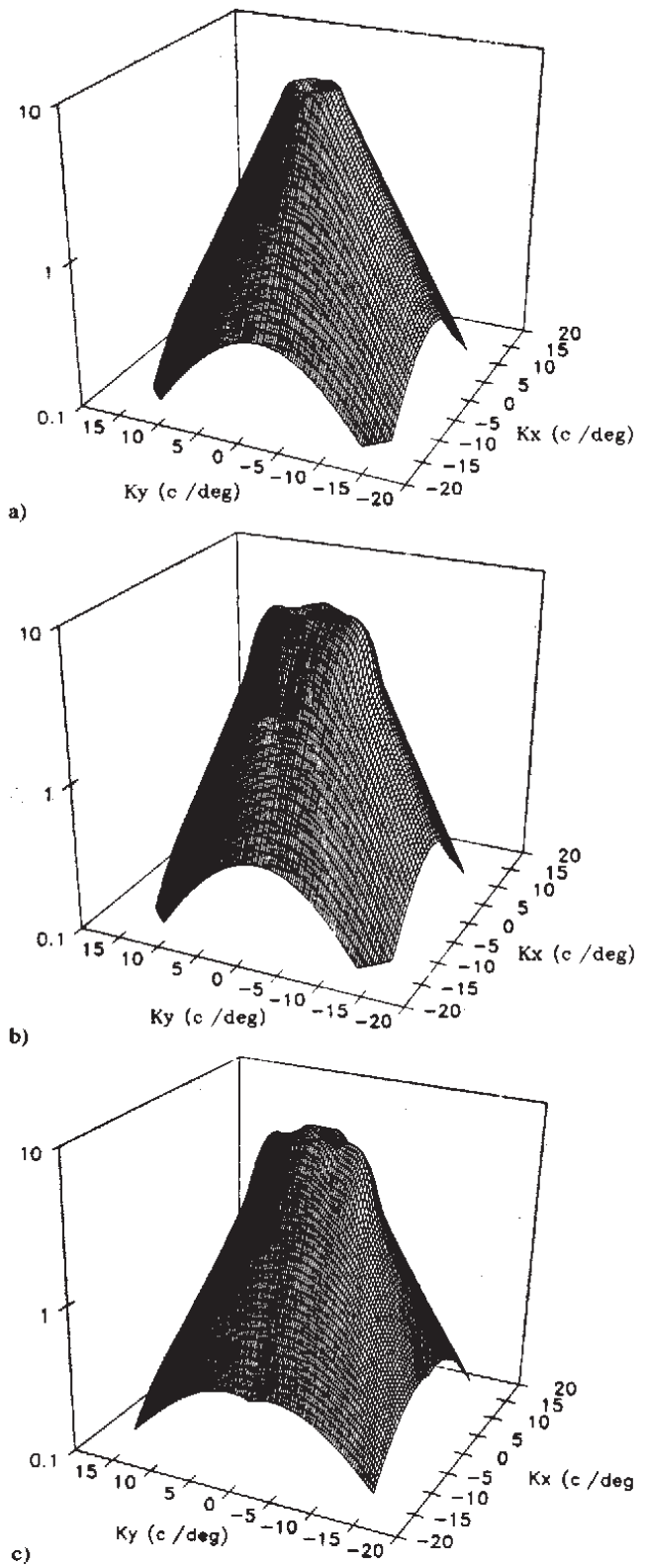


FIG. 5. — *Two-dimensional* CSFs *for observer* JM (EMMETROPIC) *obtained by the interpolation (see text, Eq.* 3) *considering respectively (from fig. 3) :* (a) *an unique radial profile of the* CSF (0°) ; (b) *two radial profiles* (0° *and* 90°) ; (c) *three radial profiles (at* 0, 45, *and* 90°). *The shape of these surfaces appears to be somewhat different to the figure 3 due to change in the scale. In this figure it can't be applied the log scale in the frequency domain as usual in* 1D *representations (fig. 1) because of the presence of negative frequency values. We haven't represented all the* 256*256 *calculated values, but only the values pertaining to the most interesting interval of frequencies. Outside this interval the filters decrease with the constant slope given by the fitting.*

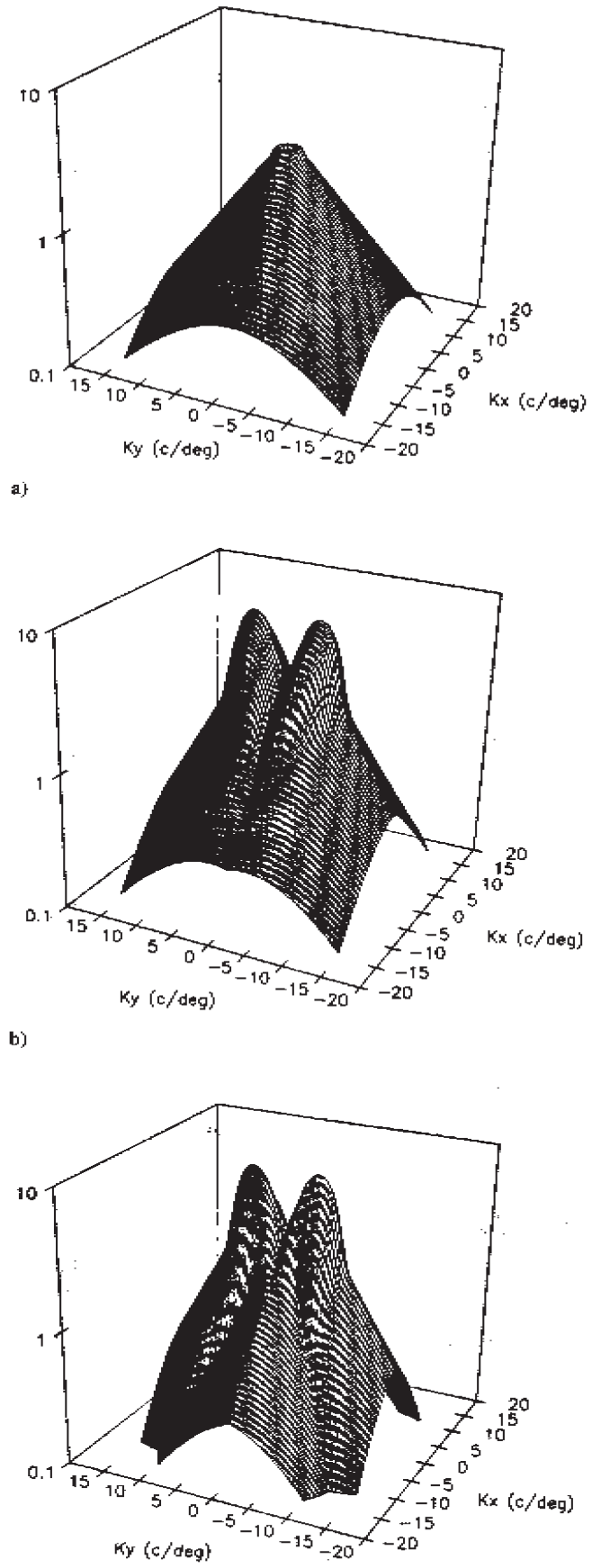


FIG. 6. — *Two-dimensional* CSFs *for observer* RZ (ASTIGMATIC) *obtained by interpolation (see text, Eq.) considering respectively (from fig. 4)* : (a) *an unique radial profile of the* CSF (0°) ; (b) *two radial profiles* (0° *and* 90°) ; (c) *three radial profiles (at* 0, 45, *and* 90°).

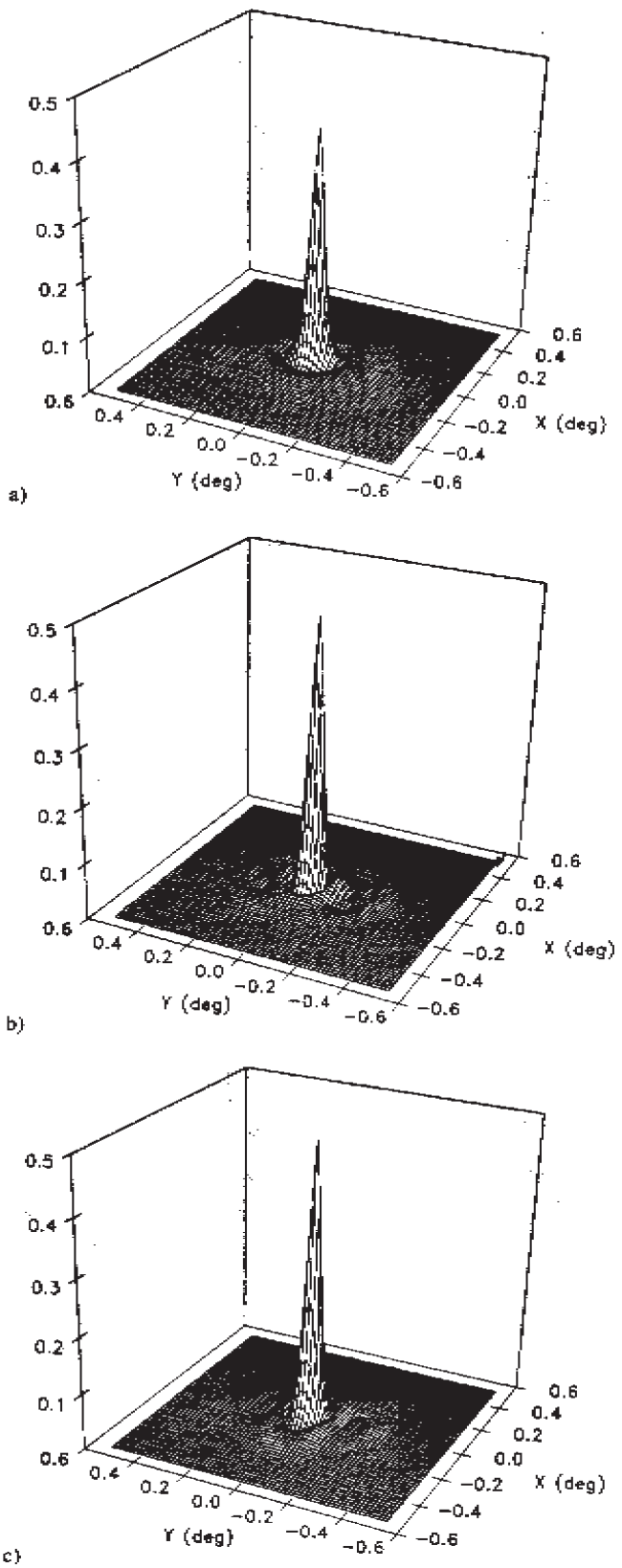
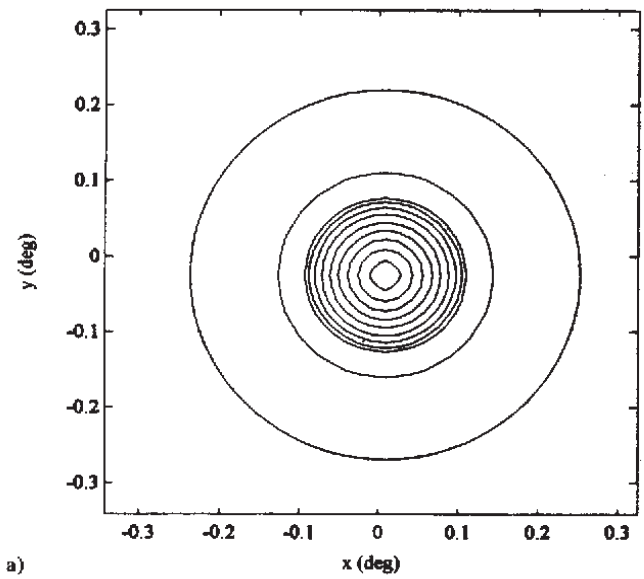
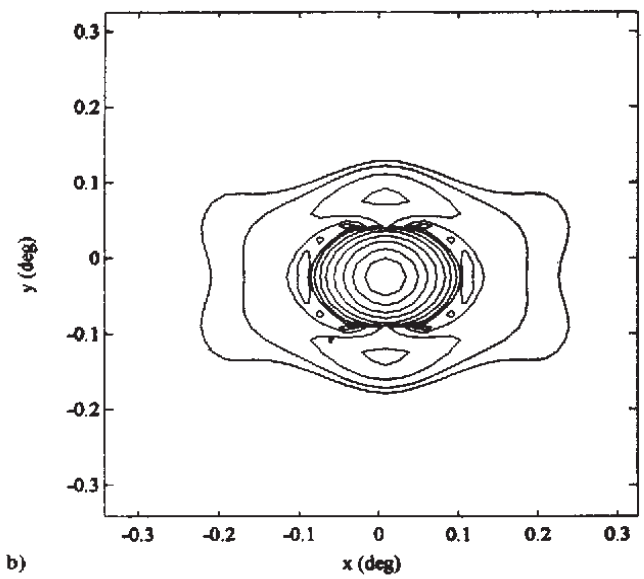


FIG. 7. — *Module of the* PSFs *for observer* IM (EMMETROPIC) *derivated, by means of equation* (4), *from :* (a) *the* CSF *of figure 5(a) considering an unique radial profile ;* (b) *the* CSF *of figure 5(b), that includes the* 0 *and* 90° *radial profiles ;* (c) *the* CSF *of figure 5(c), that includes all the measurements. The scale of the ordinate axis is in arbitrary units. It depends on the particular normalization of the filter. We have represented the spatial interval in which the* PSF *has a non-zero value. The module representation makes the inhibitory (negative) lobes appear to be positive.*



a)



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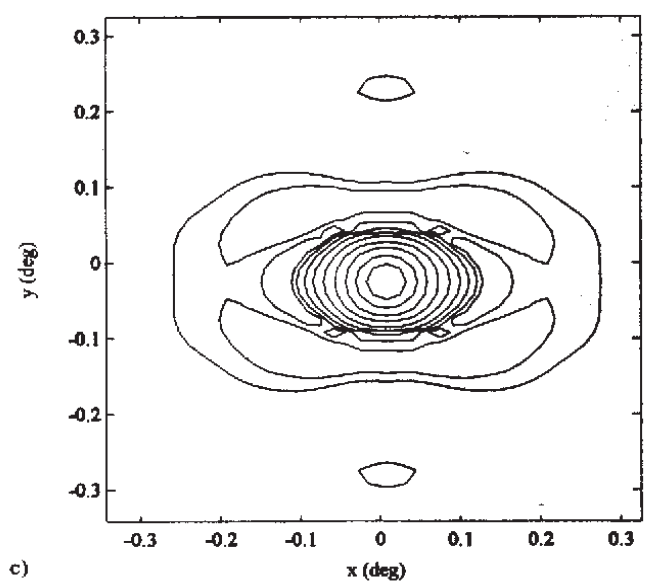
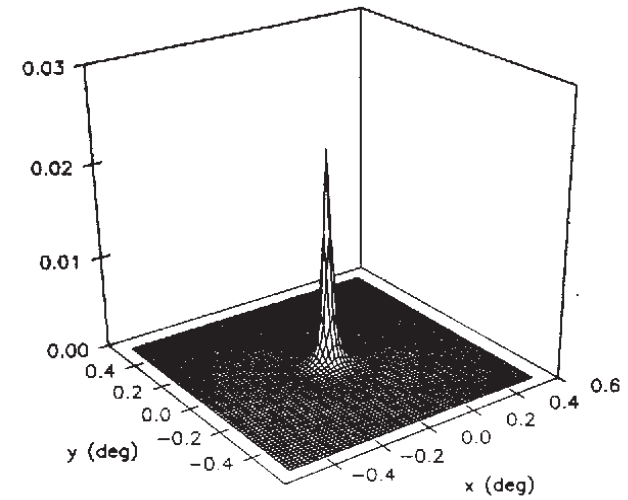
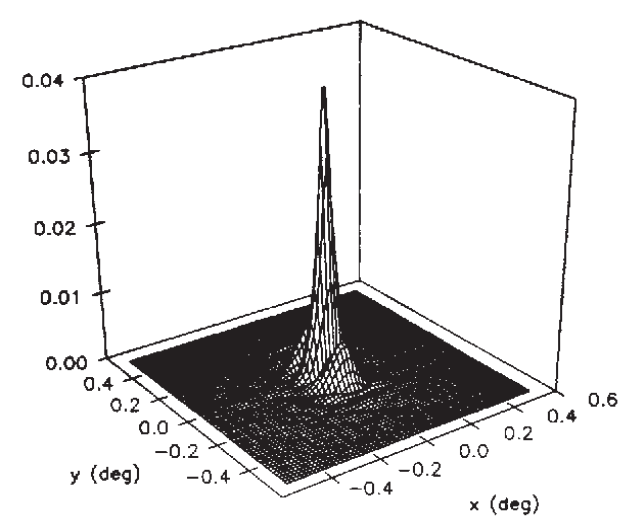


c)

FIG. 8. — *Contour curves (logaritmically spaced to enhance the small details) of the* PSFs *for observer* JM (EMMETROPIC). *Figures 8(a), 8(b) and 8(c), correspond to contour plots of figures 7(a), 7(b) and 7(c).*



a)



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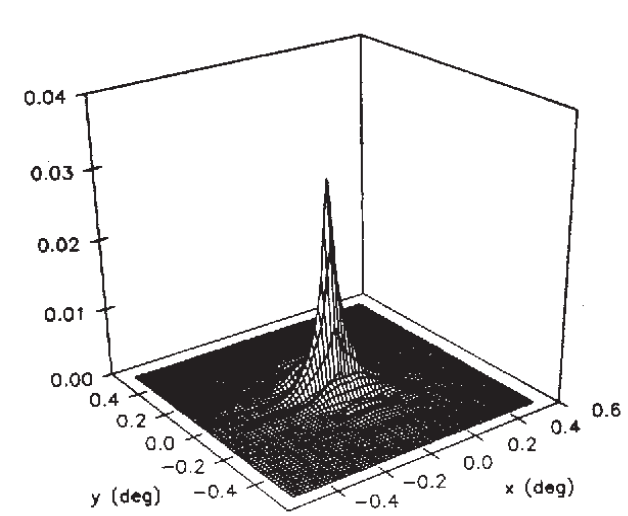

c)

FIG. 9. — *Module of the* PSFs *for observer* RZ (ASTIGMATIC) *derivated, by means of equation (4) from :* (a) *the* CSF *of figure 6(a), considering an unique radial profile ; (b) the* CSF *of figure 6(b), that includes the* 0 *and* 90° *radial profiles ;* (c) *the* CSF *of figure 6(c), that includes all the measurements.*

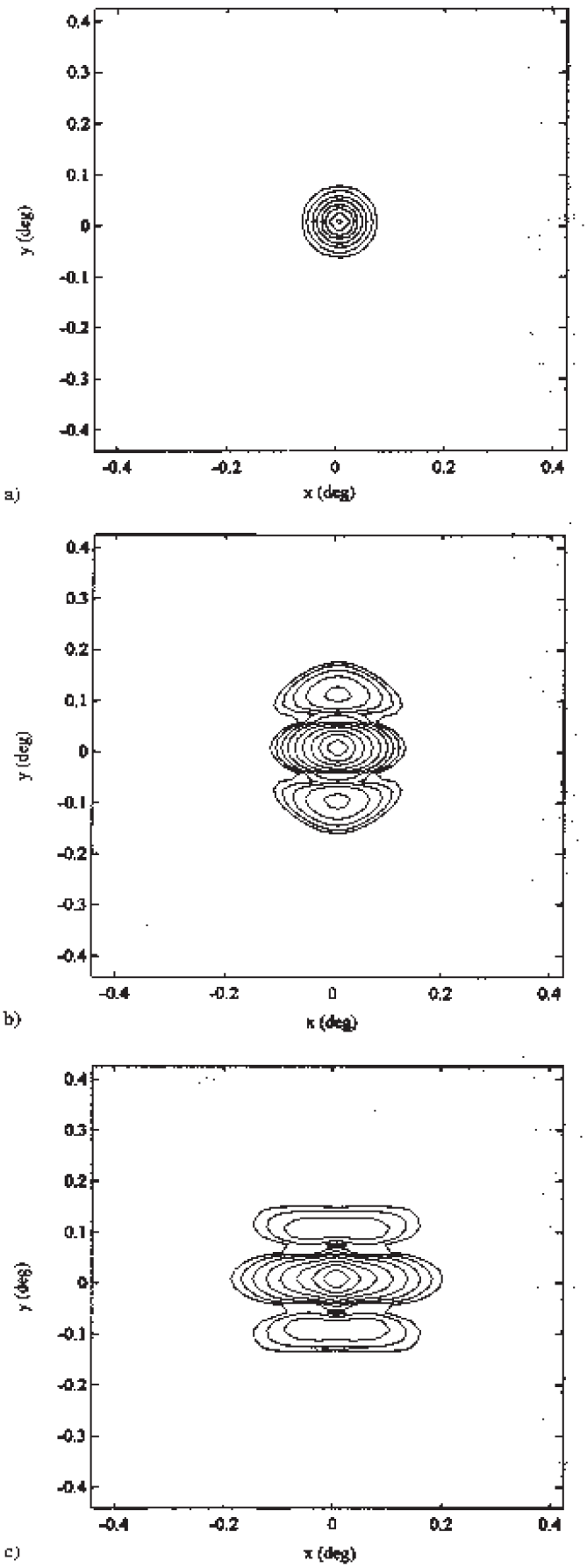

FIG. 10. — *Contour curves (logarithmically spaced to enhance the lower details) of the* PSFs *for observer* RZ (ASTIGMATIC). *Figures 10(a), 10(b) and 10(c), correspond to contour plots of figures 9(a), 9(b) and 9(c).*

contrast is determined by means of a two-alternative forced-choice method used in previous works [9].

Square gratings of six different spatial frequencies were used (0.5, 1.8, 3.7, 7.3, 14.6 and 29.2 c/deg), at angles 0°, 45° and 90°, with an average luminance of the order of 300 cd/m^2. The observation distance was fixed at 1 m, so that the test subtended an angle of 8°. The choice of these particular frequencies is due to the fact that the program automatically corrects the chosen frequencies taking into account the observation distance so as to avoid aliasing with the screen phospors.

RESULTS AND DISCUSSION

A general characterization by means of the CSF or the PSF requires a sampling of the frequency domain precise enough to get rid of errors associated with the computation of the Discrete Fourier Transform. Let's bear in mind that, disregarding the effect due to the phase and assuming that the MTF is proportional to the CSF, we can write the PSF as follow :

$$\mathrm{PSF}\ (\mathbf{x}) = \mathrm{TF}^{-1}[\mathrm{CSF}\ (\mathbf{k})]\ . \qquad (4)$$

As we need to increment the number of available points, we fit the sensitivity decrease in the high frequency range with an exponential function, and with a polynomial of the necessary degree in the low frequencies range. In this way, with the equations obtained, we'll be able to compute the value of the radials profiles of the CSF for the planes measured, in the frequency range of [0-30 c/deg], covered by the experiment.

Figures 1, 2, 3 and *4* show the experimental results and the fitted curves for observer JM (emmetropic) and RZ (astigmatic, −3 dp, 0°).

From these values, and using Eq. (3) we can generate a 256 × 256 matrix (an integer power of 2 is needed to apply the FFT algorithm) uniformly distributed in the region of the frequential domain where the CSF has an appreciable value. *Figures 5* and *6* show the results obtained for both observers, using one, two or three radial profiles to generate the surfaces.

With these values and by means of Eq. (4) we obtain the values of the PSF shown in *figures 7, 8, 9* and *10*.

To compute the scale of the spatial domain where the PSF is defined, we used the known fact [10] that the spectrum $F(\mathbf{k})$ of a discrete sampled signal $f(\mathbf{x})$, repeats itself periodically in the frequency domain at intervals $\mathbf{k}_s$ (where $\mathbf{k}_s$ is the sampling frequency) so, in absence of aliasing, all the information about $f(\mathbf{x})$ is contained in the rectangular frequency interval determined by the limit $\pm k_s/2$. In our case, the PSF Fourier Transform is defined between ± 30 c/deg, and therefore, the sampling

frequency used in the discrete spatial domain must be 60 c/deg. Thus if we take into account that k_s is defined as $k_s = n/X_M$, n being the number of intervals contained in the spatial domain of width X_M, ($n = 255$ in our case), we can solve the equation for X_M. With this, the side of the spatial domain subtends 4.25 deg (it covers the range between ± 2.125 deg).

To perform a perception simulation, we use Eq. (1) with the mentioned assumptions. In the simulation, the test contrast was 0.1 in all cases, near enough to the threshold value as to assume that the system behaves linearly. The values of the employed filters were normalized so that the maximum value was equal to one. The small value of the CSF in the origin, causes a strong attenuation of the average luminance of the output image. To avoid this, we set ad hoc that CSF (0) = 1 to preserve the average luminance. This doesn't make our result less general as it just implies the use of a given origin of luminances. The size of the field used in the simulation is such that with our sampling, its spectrum domain equals that of the measured 2D-CSF. As we have said before the field must be a square whose side subtends 4.250 deg. The intensity values for each point of the object (and the image) vary from 0 to 1, being displayed with a gray level proportional to their value (0 = black, 1 = white).

The input test is shown in *figure 11*. The results of the simulation for both the emmetropic and the astigmatic observers, are shown in *figures 12* and *13* respectively.

The results for the emmetropic observer both in the PSF and the simulation *(figs. 7, 8* and *12)* confirm that the use of just one radial profile of the CSF leads to a too simplified characterization, that doesn't take

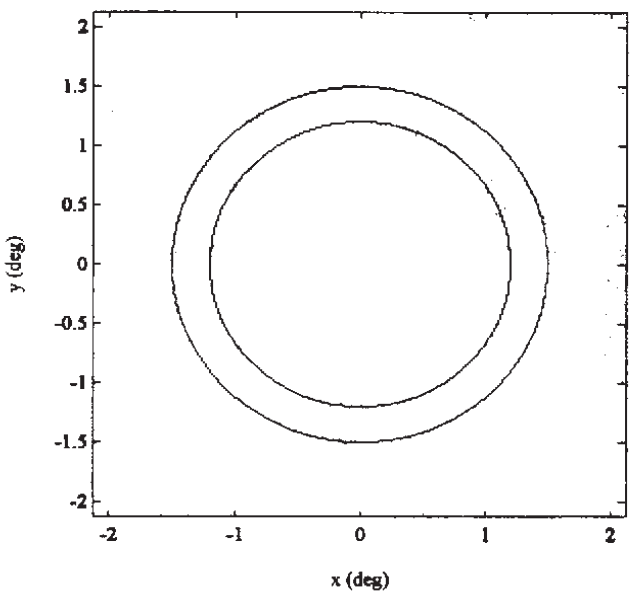

FIG. 11. — *Input test used in the simulations : white circle of radius* 1.5° *and* 0.3° *width, and normalized luminance of* 1, *on a gray background with a luminance of* 0.9. *The contrast of the test is* 0.1.

into account the system response asymmetries. In *figures 7(a)* and *8(a)* it can be seen a perfect radially symmetric PSF, and simulation *figure 12(a)* shows that the system response is the same in all directions.

When we introduce a new radial profile in a perpendicular direction, we correct in a substantial way the former result as we introduce asymmetries in the 2D-CSF. The passband to vertical frequencies is wider *(fig. 5(b))* and the PSF in the y axis is correspondingly narrower *(fig. 7(b)* and *8(b))* ; in consequence the resolution for horizontal lines is higher *(fig. 12(b))*.

Introducing another radial profile with an intermediate orientation, we obtain a new correction to our previous results (the inhibitory lateral lobes in the PSF are modified ; compare *figs. 8(b)* and *8(c))*. Nevertheless, the basic asymmetry of the function showed in the previous step is not changed, as can be seen from comparing the *figures 7(b)* and *7(c)* and the simulations *12(b)* and *12(c)*. The PSF is still wider in the x direction and therefore the resolution in the for the horizontal details is better than for the vertical ones.

The differences between *figures 5(b)* and *5(c)* are of the same order that the standard deviation of our experimental measures, and the general behaviour obtained from both characterizations is similar. For all this, we may conclude that for emmetropic observers, we don't get substantial information measuring intermediate orientations ; a simple linear interpolation for the angles between two perpendicular directions shows the system asymmetries.

The obtained width at half height (0.065 deg. approximately) is of the same order than the one obtained by other authors [11, 12]. Our value is slightly higher than that of the mentioned works [11, 12] ; this is due to our having included the neural contribution besides the optical one with the corresponding widening in the PSF.

For the astigmatic observer, the insuficience of the modelization based on an unique radial profile is evident too. The CSF so obtained has a perfect radial symmetry *(fig. 6(a))*, so it involves an isotrope PSF *(figs. 9(a), 10(a))*, and a direction independent global behaviour *(fig. 13(a))*. Of course, this results have no value as a characterization of an astigmatic observer.

If we introduce a new radial profile in the perpendicular direction (90 deg., the best performance direction of the observer RZ), the asymmetry of the CSF is pointed out *(fig. 6(b))*. The introduction of the new profile gives basic information about the great differences between the contrast sensitivity to perpendicullary oriented stimuli. According with the better sensitivity results on the *Ky* direction (horizontally oriented gratings), the corresponding widening of the PSF in the x direction is achieved *(figs. 9(b)* and *10(b))*, and this induces a direction dependent behaviour of the system. Note *(fig. 13(b))* that the

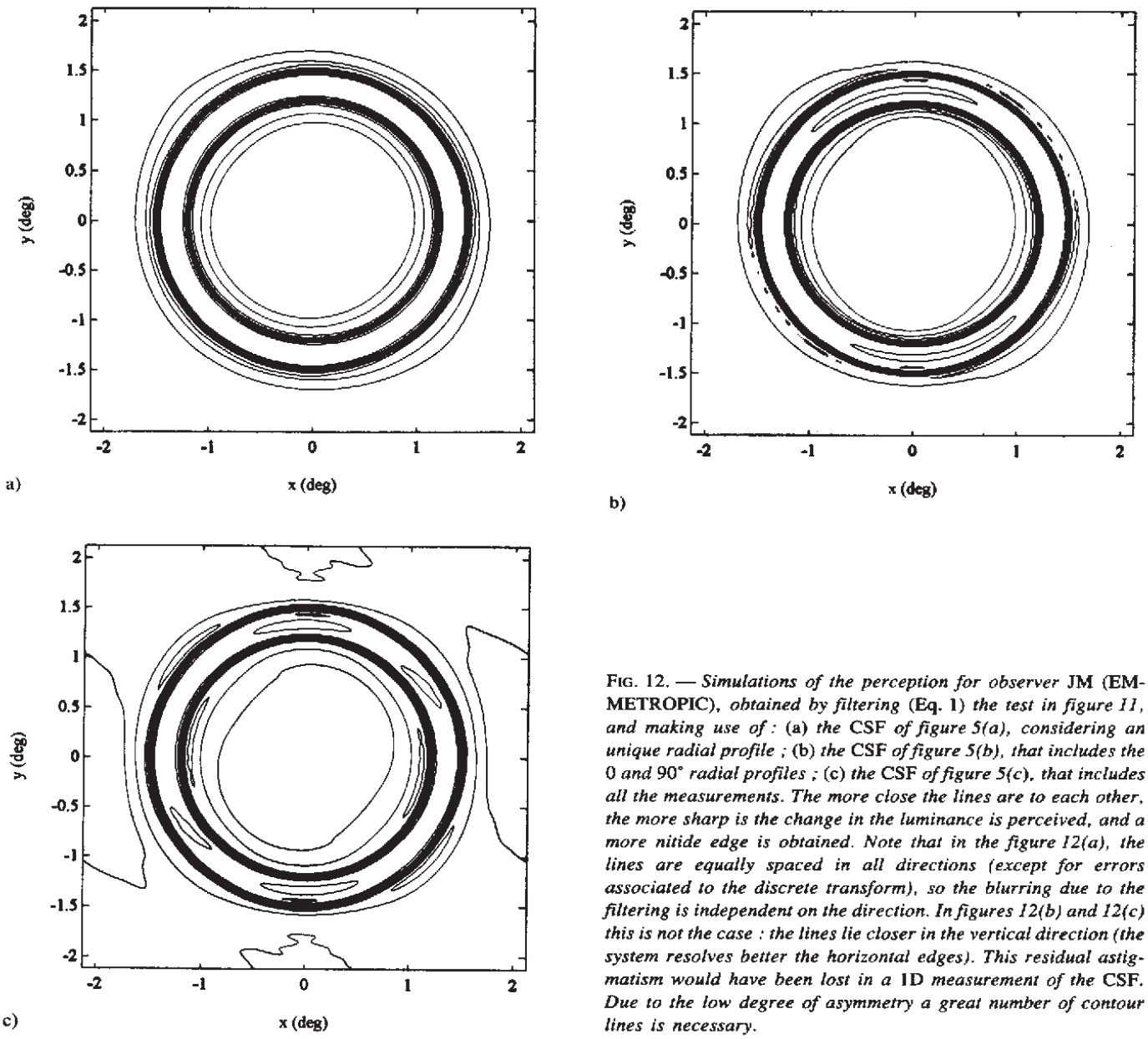

FIG. 12. — *Simulations of the perception for observer* JM (EMMETROPIC), *obtained by filtering* (Eq. 1) *the test in figure 11, and making use of :* (a) *the* CSF *of figure 5(a), considering an unique radial profile ;* (b) *the* CSF *of figure 5(b), that includes the* 0 *and* 90° *radial profiles ;* (c) *the* CSF *of figure 5(c), that includes all the measurements. The more close the lines are to each other, the more sharp is the change in the luminance is perceived, and a more nitide edge is obtained. Note that in the figure 12(a), the lines are equally spaced in all directions (except for errors associated to the discrete transform), so the blurring due to the filtering is independent on the direction. In figures 12(b) and 12(c) this is not the case : the lines lie closer in the vertical direction (the system resolves better the horizontal edges). This residual astigmatism would have been lost in a* 1D *measurement of the* CSF. *Due to the low degree of asymmetry a great number of contour lines is necessary.*

progressive blurring of the test edges is smooth between the best (90 deg.) and the worst (0 deg.) direction.

The use of another radial profile (45 deg.) substantially modifies the previous results, showing that the sensitivity of the astigmatic observer to obiquely oriented gratings is analog to that of the vertical oriented ones. This implies that the transition between the 0 deg. profile and the 90 deg. profile isn't linear. Compare the *figures 6(b)* and *6(c)*, and note that the introduction of the 45 deg. profile breaks the smooth transition shown in *figure 6(b)*, impossing a low sensitivity in a greater angular range. The result is a new widening of the PSF in the x direction *(figs. 9(c)* and *10(c))*. In the case of the astigmatic observer, the correction in the PSF's width due to the 45 deg. profile is quantitatively important : the width at half heigth in the x direction of 0.07 deg. in the two radial profiles PSF is corrected to 0.1 deg. in the three radial profiles PSF, while the width in the y direction remains unchanged at about 0.055 deg. This involves a more sharp decrease of the sensitivity of the system as one goes from the vertical to the horizontal direction. As can be seen in *figure 13(c)*, the correction affects the simulation too. The angular range in which the test is nitide is reduced respect the previous case *(fig. 13(b))*, and the blur has a non linear increase with the orientation.

In the case of RZ, including another intermediate measure (67°) doesn't introduce additional information.

The results for the astigmatic observer suggest that the transition in the behaviour between the best and the worst directions is, in general, non linear. We can

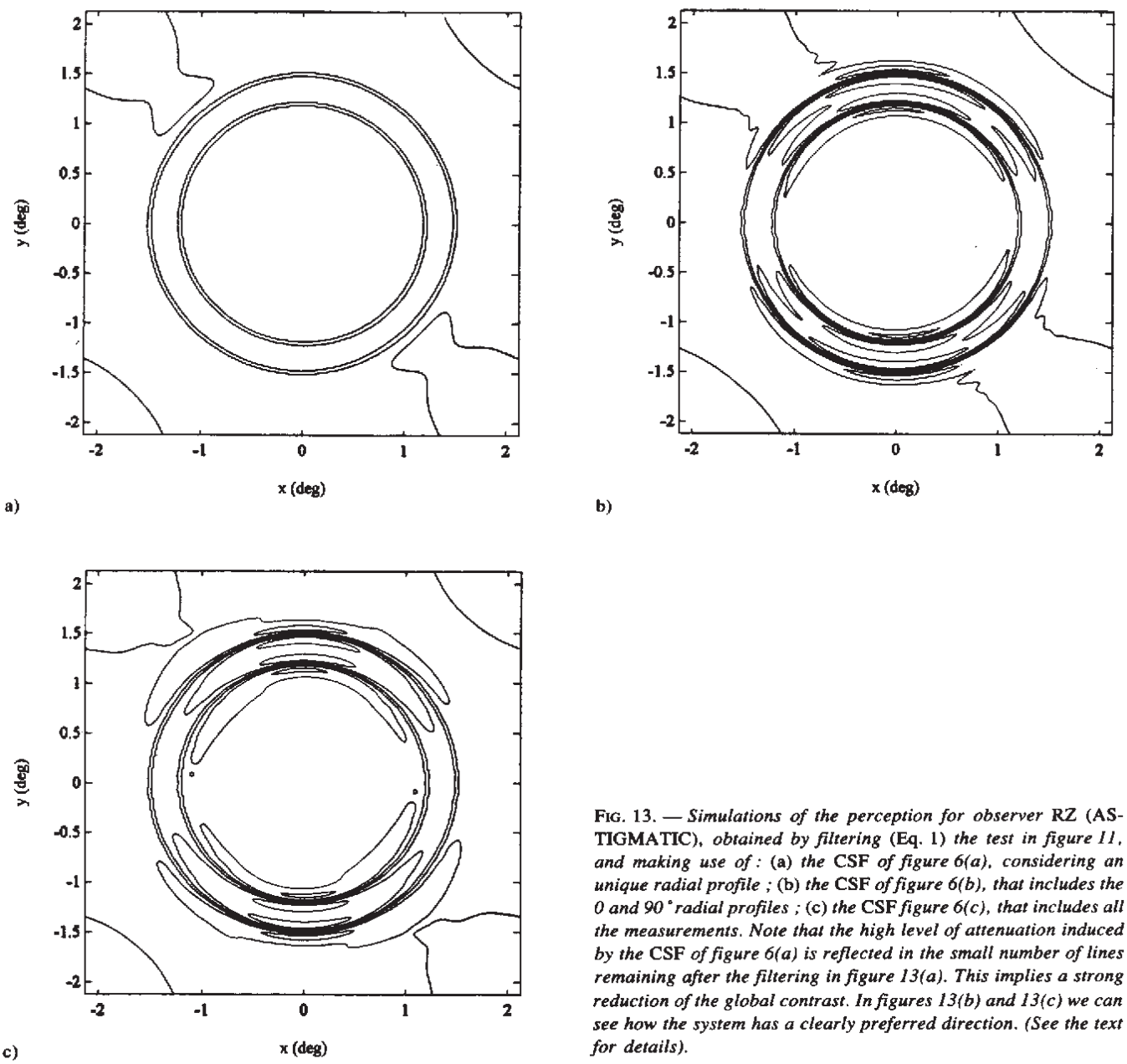

FIG. 13. — *Simulations of the perception for observer* RZ (ASTIGMATIC), *obtained by filtering* (Eq. 1) *the test in figure 11, and making use of :* (a) *the* CSF *of figure 6(a), considering an unique radial profile ;* (b) *the* CSF *of figure 6(b), that includes the 0 and 90° radial profiles ;* (c) *the* CSF *figure 6(c), that includes all the measurements. Note that the high level of attenuation induced by the* CSF *of figure 6(a) is reflected in the small number of lines remaining after the filtering in figure 13(a). This implies a strong reduction of the global contrast. In figures 13(b) and 13(c) we can see how the system has a clearly preferred direction. (See the text for details).*

conclude that for a complete characterization of an astigmatic observer, it's necessary, at least, one intermediate measure between the best and its normal direction, to take into acocunt this non linearity.

To summarize, special emphasis must be put on the two-dimensionality of the CSF if a general characterization that includes the system asymmetries is desired. Our results show that this general and predictive characterization can be achieved by means of the measurement of two perpendicular radial profiles of the CSF in the case of emmetropic observers, where as for astigmatic observers, is necessary at least a third measure in a intermediate direction. The 2D filters so obtained allow us to perform a numerical simulation of the perception of any complex test at threshold contrast level.

Our method, as above described, complements other methods [12] for computing the 2D PSF of the visual system optical part. Combining both, we can obtain separately the optical and neural part of the 2D transfer function, in such a way that the global behaviour would be simulated associating in cascade the filters obtained.

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