Perturbation Analysis of the Changes in V1 Receptive Fields due to Context

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1. Introduction

Background:

V1 represents low contrast stimuli in a wavelet-like basis [1, 2, 3].
However, V1 non-linearities imply changes in the axes of the representation:
The local receptive fields *are not* **the basis functions at threshold level!**

Our Contribution:

We study the local behavior of the image representation in V1 using a perturbation analysis.
The proposed analysis explicitly shows:
- The changes of the axes of the representation.
- The changes of the size and orientation of the discrimination regions.

Applications:

- **Stimulus design** in vision research experimentation [2].
- Measuring **perceptual distances** between images [4, 5].

2. Principal Axes and Discrimination Regions in Non-Linear Systems

Notation:

$$a \xrightarrow{T^{-1}} c \xrightarrow{F} c' \xrightarrow{R} r$$

Spatial Domain — Linear Transform Domain — Weighted Transform Domain — Response Domain

Local Principal Axes of The System: Incremental directions in the spatial domain (distortion images or incremental stimuli) that excite a single sensor.

Discrimination Regions: Set of images that depart one JND from a given image.

3. Perturbation Analysis of Non-linear Systems

Principal Axes:

Expansion of the non-linear response, $r = R(F \cdot T^{-1} \cdot a)$

$$r + \Delta r = R(F \cdot T^{-1} \cdot (a + \Delta a)) = R(F \cdot T^{-1} \cdot a) + \nabla R(c') \cdot F \cdot T^{-1} \cdot \Delta a$$

Condition for the k-th axis,

$$\Delta r_i = \sum_j \left(\nabla R(F \cdot T^{-1} \cdot a) \cdot F \cdot T^{-1} \right)_{ij} \Delta a_j^{(k)} = \delta_{(i-k)}$$

k-th axis:

$$\Delta a^{(k)} = T \cdot F^{-1} \cdot \nabla R(F \cdot T^{-1} \cdot a)^{-1} \cdot \delta^{(k)} \qquad (1)$$

Discrimination Regions:

Assuming constancy of perceptual distance and using Riemannian formulation

$$d_a(a, a + \Delta a)^2 = d_r(r, r + \Delta r)^2 = \tau$$

$$\Delta a^T \cdot W_a(a) \cdot \Delta a = \Delta r^T \cdot I \cdot \Delta r$$

Perceptual metric in the spatial domain:

$$W_a(a) = T^{T^{-1}} \cdot F \cdot \nabla R(F \cdot T^{-1} \cdot a)^T \cdot \nabla R(F \cdot T^{-1} \cdot a) \cdot F \cdot T^{-1} \qquad (2)$$

4. Perturbation Analysis of Gain Control in V1

V1 Model:

ICA linear transform [6] $\quad c = T^{-1} . a$

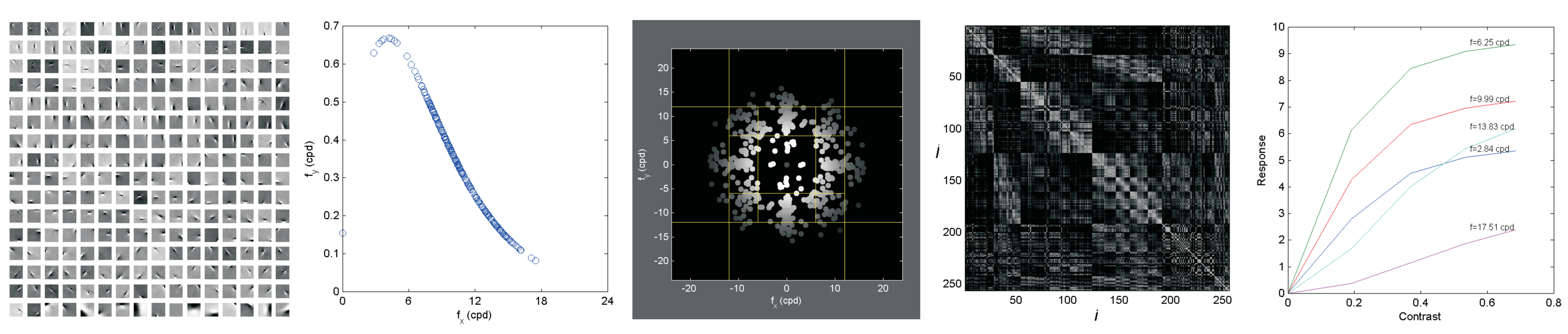
CSF-like frequency weighing [7] $\quad c' = F . c$

Divisive Normalization [8]

$$r_i = \frac{|c'_i|^2}{\beta_i + \sum_j h_{ij} \cdot |c'_j|^2}$$

(input-dependent) Jacobian,

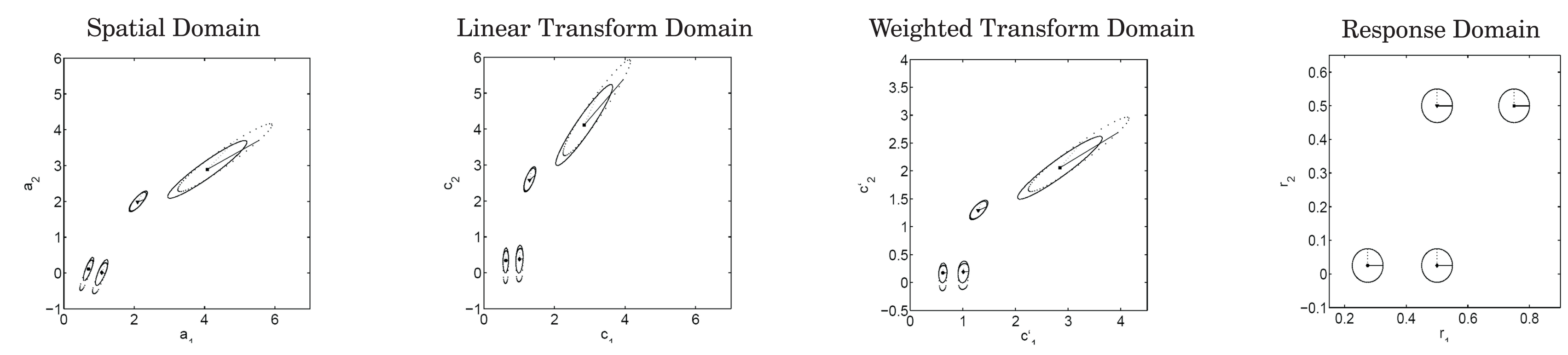
$$\nabla R(F \cdot T^{-1} \cdot a)_{ij} = \frac{\partial R(c')_i}{\partial c'_j} = 2 \left(\frac{|c'_i|}{\beta_i + \sum_j h_{ij} |c'_j|^2} \delta_{ij} - \frac{|c'_i|^2 |c'_j|}{(\beta_i + \sum_j h_{ij} |c'_j|^2)^2} h_{ij} \right)$$



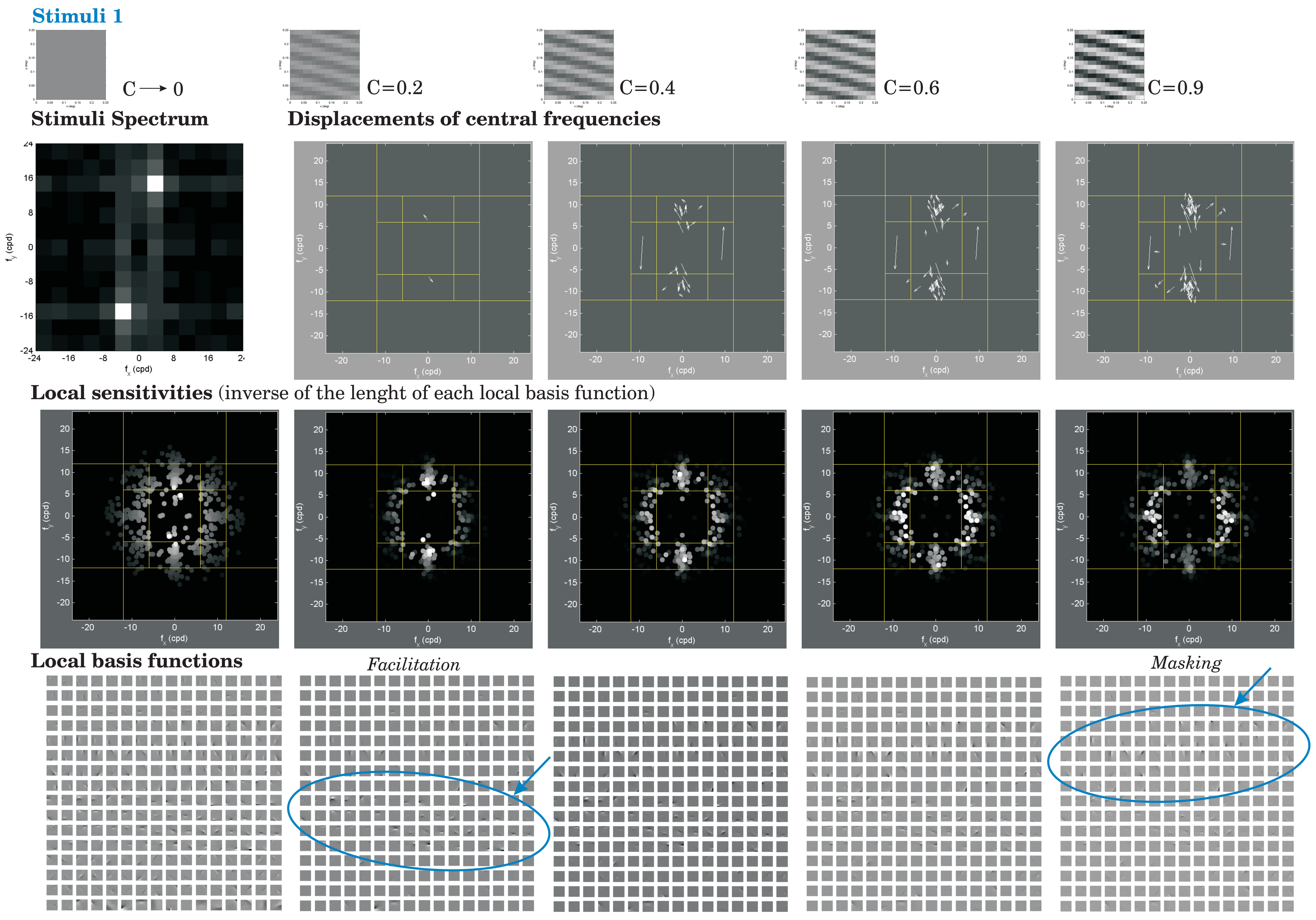
ICA Axes — T | CSF-like weighting (1D view) — F | CSF-like weighting (2D view) — F | Masking interaction matrix — h | Response — r

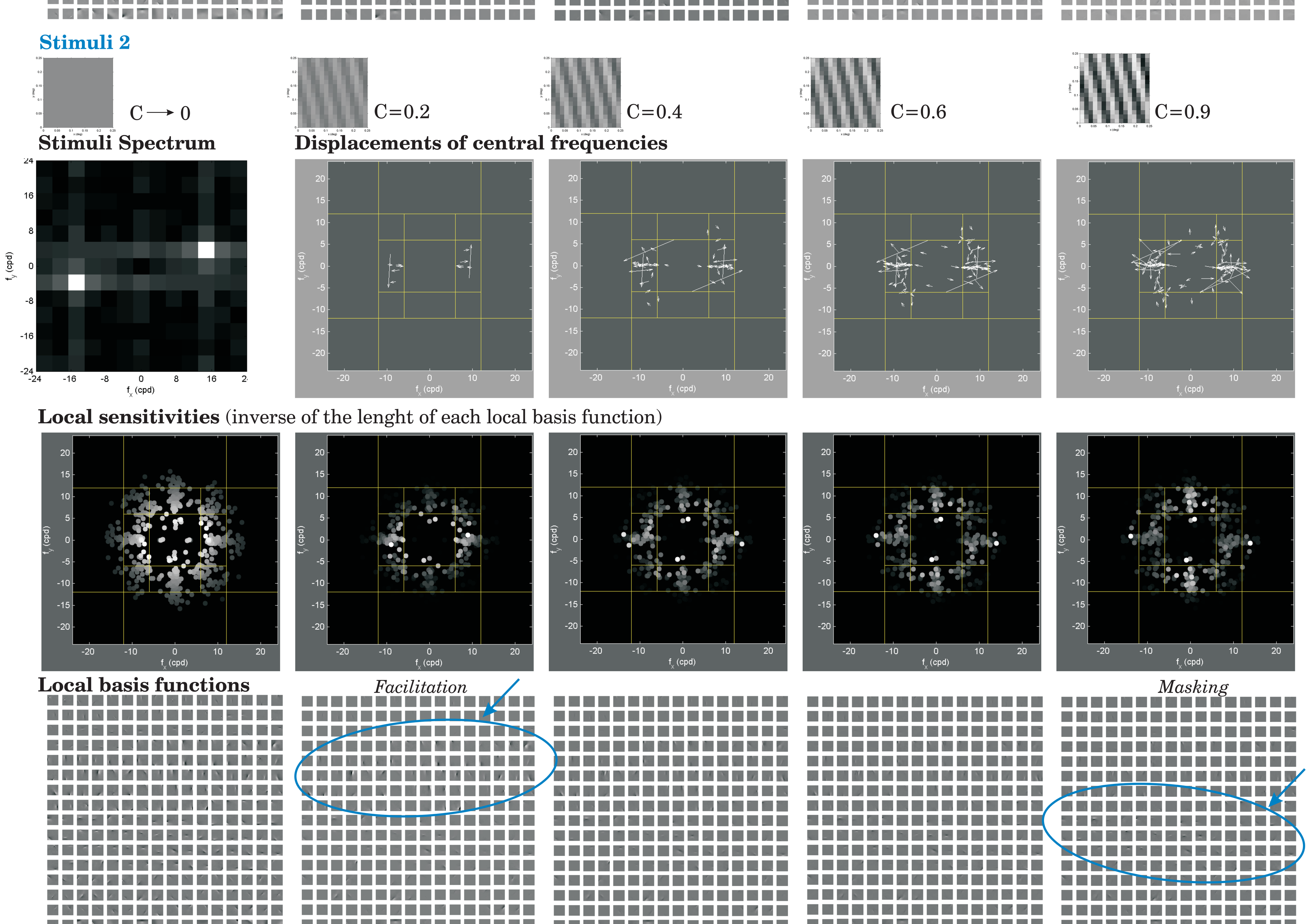
2D Example:

$$T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \quad F = \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix} \quad \beta = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad h = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$$



5. Results





6. Conclusions

The perturbation analysis provides closed expressions to compute the local principal axes of the system (eq.1) and the local metric matrix that describes the size and orientation of the local discrimination ellipsoid (eq. 2).

The local receptive fields given a particular stimulus are a linear combination of the receptive fields at threshold level (uniform background). This linear combination is given by the Jacobian of the non-linearity at that point (eq. 2), i.e. it depends on the input stimuli and the adaptation state.

The proposed analysis (together with standard parameters in the divisive normalization model) shows that the optimum frequency of the sensors moves towards/backwards the frequency content of the stimulus. This modification depends on the frequency content and the contrast of the stimulus.

7. References

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