Geometric Properties of the Disk Algebra.

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(Joint Work with D. Garcia, S.K. Kim and M. Maestre)

POSTECH

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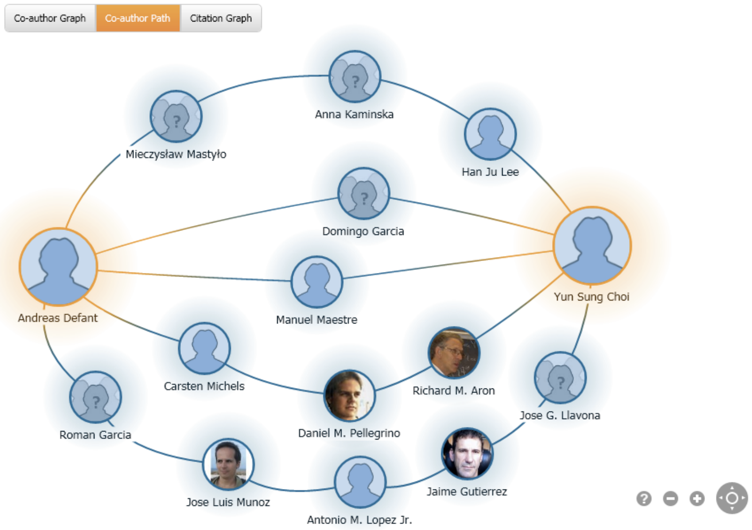
Andreas Defant



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Urysohn Lemma

Theorem (Urysohn Lemma)

A Hausdorff topological space is normal if and only if given two disjoint closed subsets can be separated by a continuous function with values on $[0,1]$ *and taking the value 1 on one closed set an 0 on the other.*

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A Hausdorff topological space is normal if and only if given two disjoint closed subsets can be separated by a continuous function with values on $[0,1]$ *and taking the value 1 on one closed set an 0 on the other.*

Related to the numerical index, Daugavet property, Bishop-Phelps-Bollobás property, etc.
several geometrical properties of the space $C(K)$ (resp. $C(K,X)$) on a Hausdorff compact set K (resp. with values in a Banach space X) have been obtained,

Most of the results at one point or another use the classical Urysohn Lemma.

Why a complex version of Urysohn Lemma

Many of that results could not be extended to a uniform algebra
(i,e. to a closed subalgebra of a complex $C(K)$ that separates points and contains the constant function $\mathbf{1}$)
since the Urysohn Lemma cannot be true, in general, if we ask the function to be in a given uniform algebra.

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For example consider the most representative case, the disk algebra $A(\mathbb{D})$ of functions continuous on the closed unit complex disk $\overline{\mathbb{D}}$ and holomorphic in the open disk $\mathbb{D}$ of $\mathbb{C}$.

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Why Complex Version ? (Stone-Weierstrass Theorem) The only uniform algebra in the real case is $C(K)$

Complex Version of Urysohn Lemma

Here, $C(K)$ stands for the space of complex-valued continuous functions defined on a compact Hausdorff space K equipped with the supremum norm $\|\cdot\|_\infty$.

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Given $x \in K$, we denote by $\delta_x : A \to \mathbb{C}$ the evaluation functional at x given by $\delta_x(f) = f(x)$, for $f \in A$.

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The natural injection $i : K \to A^*$ defined by $i(t) = \delta_t$ for $t \in K$ is a homeomorphism from K onto $(i(K), w^*)$.

Complex Version of Urysohn Lemma

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For a uniform algebra A of $C(K)$, if

$$S = \{x^* \in A^* : \|x^*\| = 1, x^*(\mathbf{1}) = 1\},$$

then the set $\Gamma_0(A)$ of all $t \in K$ such that δ_t is an extreme point of S is a boundary for A that is called the Choquet boundary of A.

Complex Version of Urysohn Lemma - Uniform Algebra

Theorem (Cascales, Guirao, and Kadets, 2012)

Let $A \subset C(K)$ *be a uniform algebra for some compact Hausdorff space* K *and* $\Gamma_0 = \Gamma_0(A)$.

Then, for every open set $U \subset K$ *with* $U \cap \Gamma_0 \neq \phi$ *and* $0 < \epsilon < 1$, *there exist* $f \in A$ *and* $t_0 \in U \cap \Gamma_0$ *such that* $f(t_0) = \|f\|_\infty = 1$, $|f(t)| < \epsilon$ *for every* $t \in K \setminus U$ *and*

$$f(K) \subset R_\epsilon = \{z \in \mathbb{C} : |\ Re(z) - 1/2| + (1/\sqrt{\epsilon})\ |Im(z)| \leq 1/2\}.$$

In particular,

$$|f(t)| + (1 - \epsilon)|1 - f(t)| \leq 1, \text{ for all } t \in K.$$

Complex Version of Urysohn Lemma - Uniform Algebra

We apply this Urysohn type Lemma to extend results on $C(K)$ or $C(K, X)$ to A^X concerning the numerical index, Daugavet equation, lushness and the approximate hyperplane series property (in short $AHSP$).

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If A is a uniform algebra, then A^X is defined to be a subspace of $C(K, X)$ such that

$$A^X = \{f \in C(K, X) \ : \ x^* \circ f \in A \ \text{ for all } \ x^* \in X^*\}.$$

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Given $f \in A$ and $x \in X$, we define $f \otimes x \in C(K, X)$ by $(f \otimes x)(t) = f(t)x$ for $t \in K$. We write

$$A \otimes X = \{f \otimes x \ ; \ f \in A, \ x \in X\}.$$

From the definition of A^X we note that $A \otimes X \subset A^X$.

Main Results

we study some geometrical properties of certain classes of uniform algebras, in particular
the disk algebras $A_u(B_X)$ of all uniformly continuous functions on the closed unit ball and holomorphic on the open unit ball of a complex Banach space X.

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Main Results

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the disk algebras $A_u(B_X)$ of all uniformly continuous functions on the closed unit ball and holomorphic on the open unit ball of a complex Banach space X.

(1) $A_u(B_X)$ has k-numerical index 1 for every k, has the lush property and has the AHSP property.

(2) The algebra of the disk $A(\mathbb{D})$, and more in general any uniform algebra such that its Choquet boundary has no isolated points, has the polynomial Daugavet property.

Numerical Index

For a Banach space X, we write $\Pi(X)$ to denote the subset of $X \times X^*$ given by

$$\Pi(X) := \{(x, x^*) \ : \ x \in S_X, \ x^* \in S_{X^*}, \ x^*(x) = 1\}.$$

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Given a bounded function $\Phi : S_X \to X$, its *numerical range* is defined by

$$V(\Phi) := \{x^*(\Phi(x)) \ : \ (x, x^*) \in \Pi(X)\}$$

and its *numerical radius* is defined by

$$v(\Phi) := \sup\{|\lambda| \ : \ \lambda \in V(\Phi)\}.$$

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Let us comment that for a bounded function $\Phi : \Omega \to X$, where $S_X \subset \Omega \subset X$, the above definitions are applied by just considering $V(\Phi) := V(\Phi|_{S_X})$.

Numerical Index

For $k \in \mathbb{N}$, we define

$$n^{(k)}(X) = \inf\{v(P) \ : \ P \in \mathcal{P}(^k X; X), \ \|P\| = 1\},$$

where $\mathcal{P}(^k X; X)$ is the space of all continuous k-homogeneous polynomials from X into X, and call it the *polynomial numerical index of order k of X*.

When $k = 1$, it is the numerical index of X.

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When $k = 1$, it is the numerical index of X.

If we consider elements of $\mathcal{A}_u(B_X, X)$ instead of continuous k-homogeneous polynomials, we can define the *analytic numerical index of X* by

$$n_a(X) = \inf\{v(f) \ : \ f \in \mathcal{A}_u(B_X, X), \ \|f\| = 1\}.$$

Numerical Index

Since the space $\mathcal{P}(X;X)$ of all continuous polynomials from X into X is dense in $\mathcal{A}_u(B_X, X)$ we have that

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i.e. $n_a(X)$ can be called the "*non-homogeneous polynomial numerical index of X*".

Clearly,

$$n_a(X) \le n^{(k)}(X)$$

for every $k \in \mathbb{N}$.

We also denote by $\mathcal{P}(X)$ the space of scalar-valued polynomials on X.

Numerical Index

Theorem

Suppose that A be a uniform algebra. Then $n^{(k)}(A^X) \geq n^{(k)}(X)$ *for every* $k \geq 1$ *and* $n_a(A^X) \geq n_a(X)$.

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Sketch of Proof ($n^{(k)}(X) \leq n^{(k)}(A^X)$ for every $k \geq 1$) Let $P \in S_{\mathcal{P}(^k A^X, A^X)}$ and $0 < \epsilon < 1$ be given. Choose $f_0 \in S_{A^X}$ so that $\|P(f_0)\| > 1 - \frac{\epsilon}{6}$. Find $t_1 \in \Gamma_0$ such that $\left\|P(f_0)(t_1)\right\| > 1 - \frac{\epsilon}{6}$. Since P is continuous at f_0, there exists $0 < \delta < 1$ such that $\|P(f_0) - P(g)\| < \frac{\epsilon}{6}$ for every $g \in A^X$ with $\|f_0 - g\| < \delta$.

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Let

$$W = \{t \in K \ : \ \|f_0(t) - f_0(t_1)\| < \delta/6, \ \|P(f_0)(t) - P(f_0)(t_1)\| < \epsilon/3\}.$$

This set is open in K and $t_1 \in W \cap \Gamma_0$.

Sketch of Proof

By the complex version of Urysohn Lemma
there exist a function $\phi : K \to \overline{\mathbb{D}}$ and $t_0 \in W \cap \Gamma_0$
such that $\phi \in A$, $\phi(t_0) = 1$, $|\phi(w)| < \frac{\delta}{6}$ for every $w \in K \backslash W$, and

$$|\phi(t)| + \left(1 - \frac{\delta}{6}\right)|1 - \phi(t)| \leq 1$$

for every $t \in K$.

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Define $\Phi(x) = x_1^*(x)\big(1 - \frac{\delta}{6}\big)(1 - \phi)f_0 + \phi x \in A^X$ for $x \in X$.

Sketch of Proof

By the complex version of Urysohn Lemma
there exist a function $\phi : K \to \overline{\mathbb{D}}$ and $t_0 \in W \cap \Gamma_0$
such that $\phi \in A$, $\phi(t_0) = 1$, $|\phi(w)| < \frac{\delta}{6}$ for every $w \in K \backslash W$, and

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Define $Q \in \mathcal{P}(^k X; X)$ by

$$Q(x) = P\big(\Phi(x)\big)(t_0), \ (x \in X). \tag{1}$$

Numerical Index

Theorem

Let A be a uniform algebra and X be a Banach space. Assume that A^X has the following property: For every $P \in \mathcal{P}({}^k X; X)$ and $t \in K$, $Q : A^X \longrightarrow C(K, X)$ where $Q(f)(t) = P(f(t))$ satisfies that $Q(f) \in A^X$ for every $f \in A^X$. Then $n^{(k)}(A^X) = n^{(k)}(X)$.

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Corollary

For any Banach space X, and $k \geq 1$, the following hold.

1. *For a uniform algebra A we have $n^{(k)}(A) = 1$ for every $k \geq 1$ and $n_a(A) = 1$.*
2. $n^{(k)}(\mathcal{A}(\mathbb{D}^n, X)) = n^{(k)}(X)$ *and* $n_a(\mathcal{A}(\mathbb{D}^n, X)) = n_a(X)$ *for every* $n \in \mathbb{N}$.
3. $n^{(k)}(\mathcal{A}_u(B_X)) = 1$ *for every* $k \geq 1$ *and* $n_a(\mathcal{A}_u(B_X)) = 1$.

Polynomial Daugavet Property

A Banach space X is said to have the Daugavet property if the norm identity, so called the Daugavet equation,

$$\|Id + T\| = 1 + \|T\|$$

holds for every rank-one operator (and hence for every weakly compact operator) $T \in L(X)$.

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If this happens for every weakly compact polynomial, we say that X has the polynomial Daugavet property.

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Theorem

Suppose that A is a uniform algebra whose Choquet boundary has no isolated points. For every $P \in S_{\mathcal{P}(A^X)}$, $f_0 \in S_{A^X}$ and $\epsilon > 0$, there exist some $\omega \in S_{\mathbb{C}}$ and $g \in B_{A^X}$ such that $Re \omega P(g) > 1 - \epsilon$ and $\|f_0 + \omega g\| > 2 - \epsilon$.

Polynomial Daugavet Property

Corollary

If A is a uniform algebra whose Choquet boundary has no isolated points, then every weakly compact $P \in \mathcal{P}(A^X; A^X)$ satisfies the Daugavet equation which means that A^X has the polynomial Daugavet property.

Polynomial Daugavet Property

Corollary

If A is a uniform algebra whose Choquet boundary has no isolated points, then every weakly compact $P \in \mathcal{P}(A^X; A^X)$ satisfies the Daugavet equation which means that A^X has the polynomial Daugavet property.

Since given a ball U in $\mathbb{C}^n$ or the polydisk $\mathbb{D}^n$ the Choquet boundary of $\mathcal{A}(U)$ does not have isolated points,

Corollary

If U is a ball in $\mathbb{C}^n$ or the polydisk $\mathbb{D}^n$, then $\mathcal{A}(U, X)$ has the polynomial Daugavet property for every complex Banach space X. In particular, $\mathcal{A}(U)$ has the polynomial Daugavet property.

Lushness

The concept of lushness was introduced to characterize an infinite dimensional Banach space with the numerical index 1.

The lushness has been known to be the weakest among quite a few isometric properties in the literature which are sufficient conditions for a Banach space to have the numerical index 1.

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The lushness has been known to be the weakest among quite a few isometric properties in the literature which are sufficient conditions for a Banach space to have the numerical index 1.

A Banach space X is said to be *lush* if for every $x, y \in S_X$ and for every $\epsilon > 0$ there is a slice

$$S = S(B_X, x^*, \epsilon) = \{x \in B_X \ : \ Rex^*(x) > 1 - \epsilon\}, \ x^* \in S_{X^*}$$

such that $x \in S$ and $dist(y, aconv(S)) < \epsilon$.

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Corollary

If X is lush, then $\mathcal{A}_u(B_Y)$ is lush for any Banach space Y.

AHSP

A Banach space X is said to have the *AHSP* if for every $\epsilon > 0$ there exist $\gamma(\epsilon) > 0$ and $\eta(\epsilon) > 0$ with $\lim_{\epsilon \to 0^+} \gamma(\epsilon) = 0$ such that for every sequence $(x_k)_{k=1}^{\infty} \subset B_X$ and for every convex series $\sum_{k=1}^{\infty} \alpha_k$ satisfying

$$\left\| \sum_{k=1}^{\infty} \alpha_k x_k \right\| > 1 - \eta(\epsilon)$$

there exist a subset $A \subset \mathbb{N}$, a subset $\{z_k : k \in A\} \subset S_X$ and $x^* \in S_{X^*}$ such that
(i) $\sum_{k \in A} \alpha_k > 1 - \gamma(\epsilon)$
(ii) $\|z_k - x_k\| < \epsilon$ for all $k \in A$, and
(iii) $x^*(z_k) = 1$ for all $k \in A$.

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(iii) $x^*(z_k) = 1$ for all $k \in A$.

This property was introduced to characterize a Banach space X such that the pair (ℓ_1, X) has the Bishop-Phelps-Bollobás property for operators.

AHSP

Since every lush space has the *AHSP*, We have the following.

Corollary

Suppose that A is a uniform algebra.

1. *If X is lush, then A^X has the AHSP. In particular, A has the AHSP.*
2. *If A^X is lush, then X has the AHSP.*

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Suppose that A is a uniform algebra.
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Corollary

If X has the AHSP, then $\mathcal{A}_u(B_Y)$ has the AHSP for any Banach space Y.


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