Interaction induced edge channel equilibration

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WS10-ETOLDs, Valencia, Spain
Monday the 31st of May 2010

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Markus Büttiker

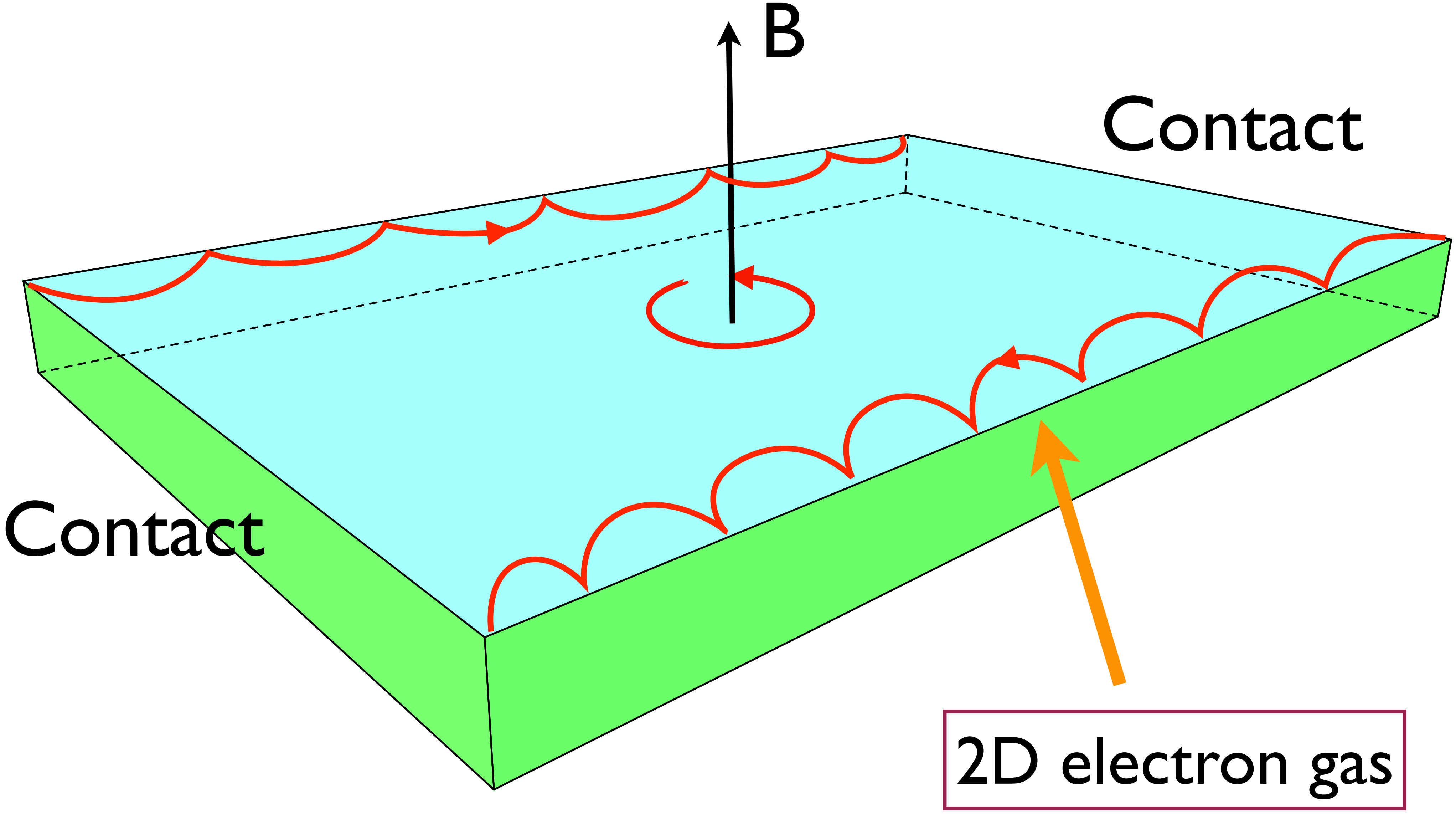
Ref.: Phys. Rev. B Rapid Comm. **81**, 041311 (2010)

Outline

- Background:
  Edge states in the integer quantum hall effect
- Main idea:
  Measuring the non-equilibrium electronic distribution in an edge state *and* it´s relaxation
- The experiments by F. Pierre et al!
- The theory:
  Perturbative and non-perturbative results for the distribution function.
- Summary

What is an Edge state?

Classical: Cyclotron orbits and Skipping orbits

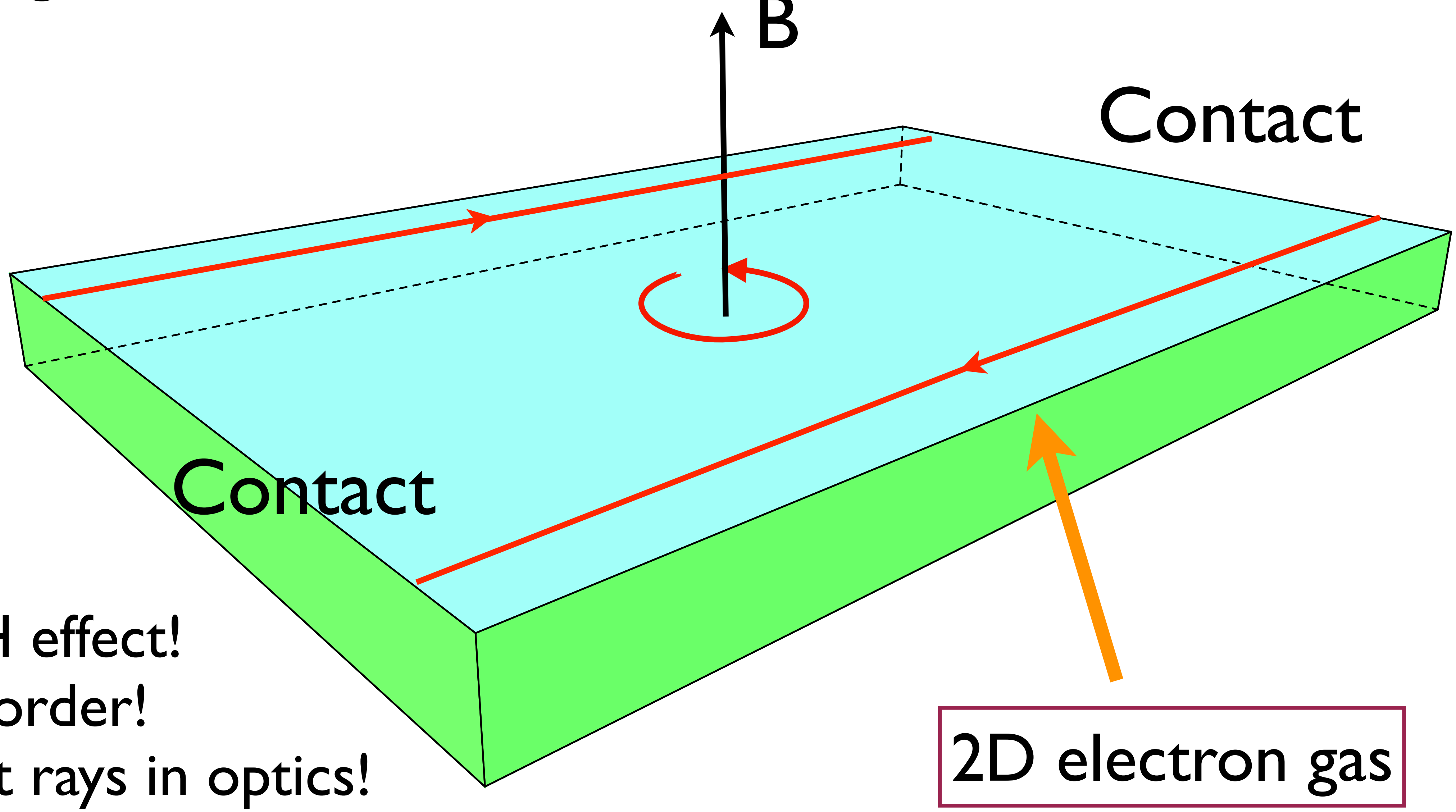


Halperin, Phys. Rev. B **25**, 2185 (1982)

What is an Edge state?

Current flows along the edge of the sample!

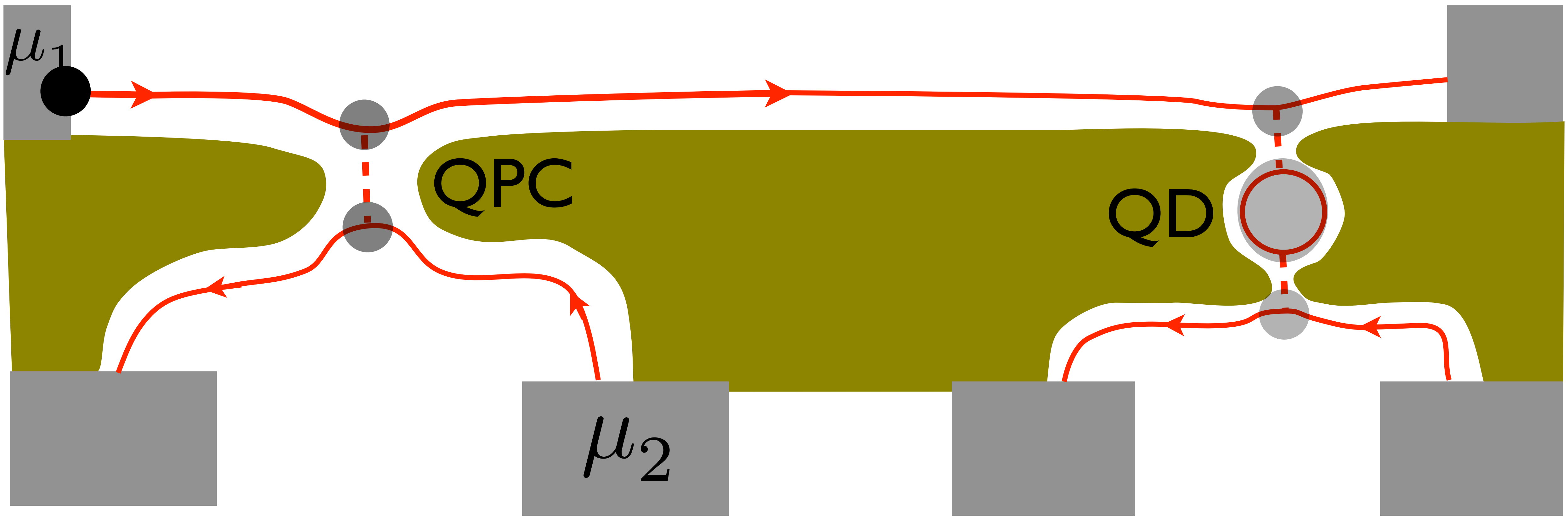
Classical: Cyclotron orbits and Skipping orbits
Quantum: Edge states!



- Explains the IQH effect!
- Robust again disorder!
- Analogue to light rays in optics!
- Allows for nanostructure engineering!!

Halperin, Phys. Rev. B **25**, 2185 (1982)

Measuring the electronic distribution in an edge state



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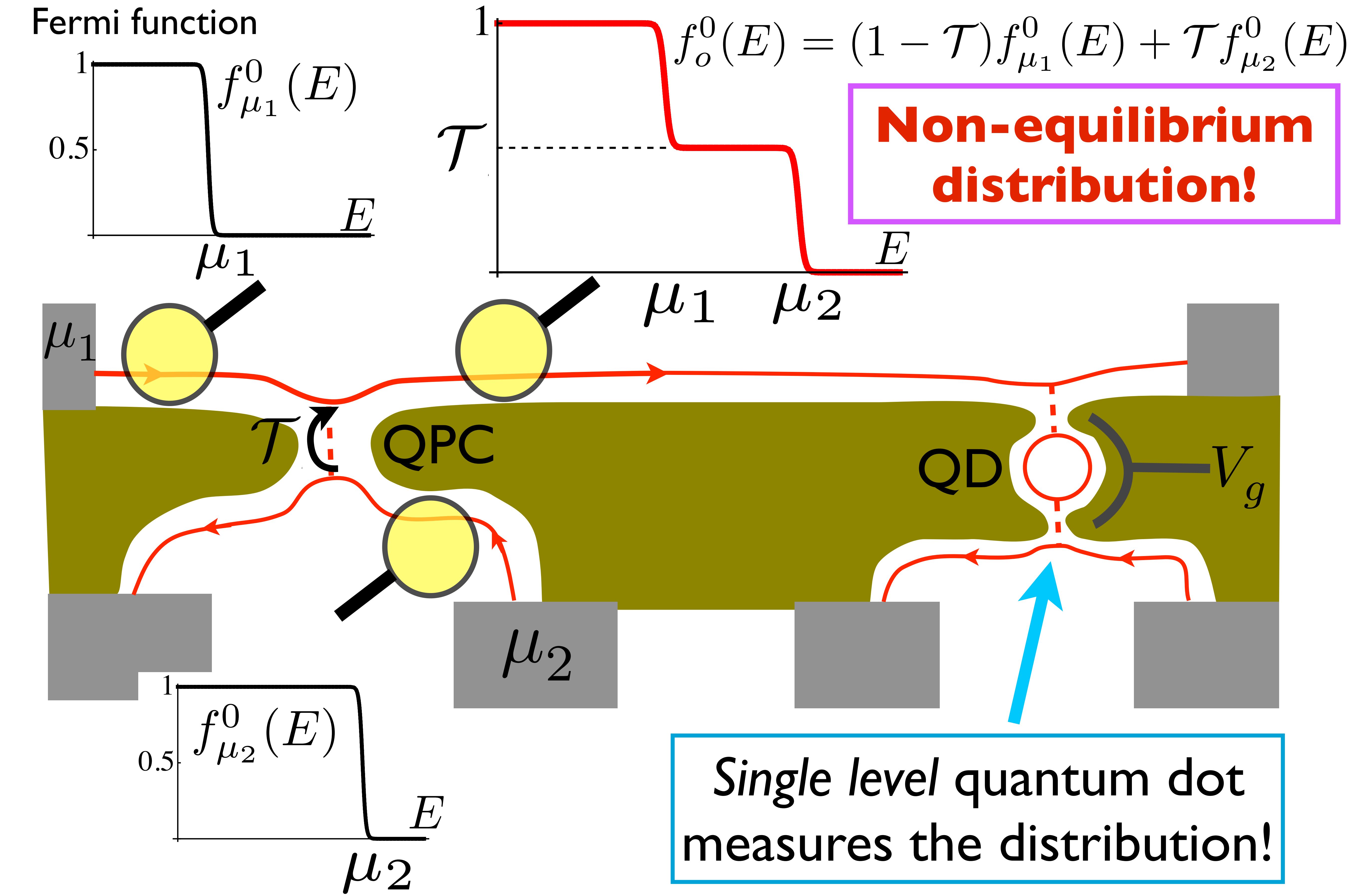
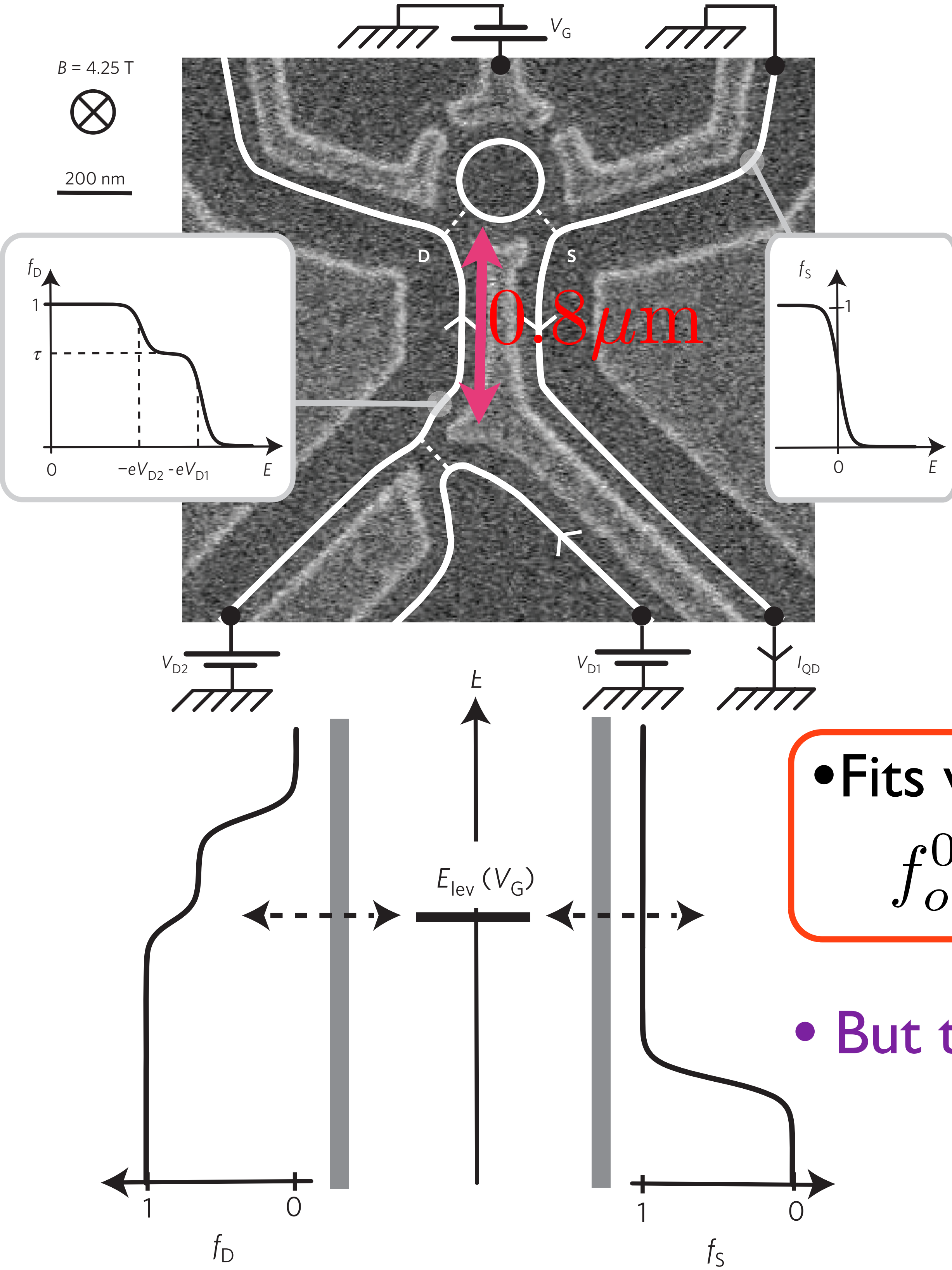
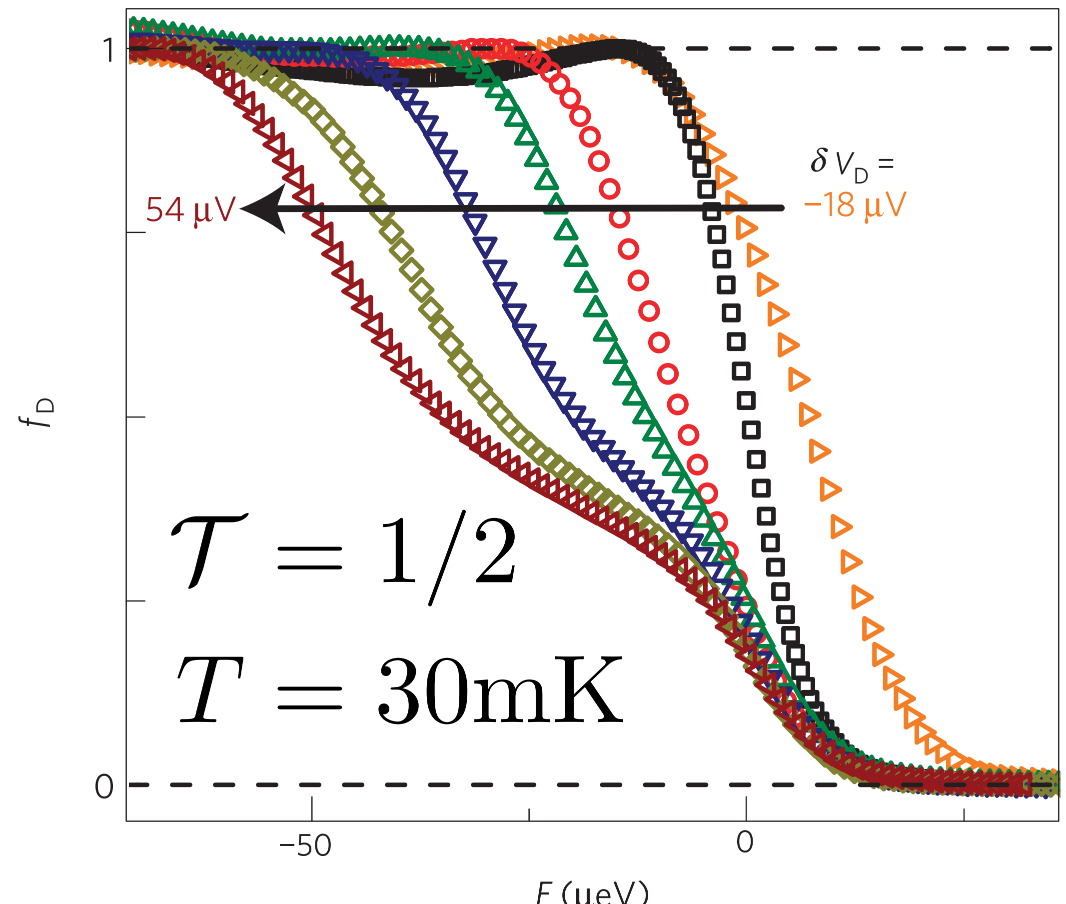


First Experiment: Proof of principal

Nature Physics, **6**, 34 - 39 (2009):

C. Altimiras, H. le Sueur, U. Gennser, Cavanna, D. Mailly and F. Pierre





- Fits well with non-interacting scattering theory:

$$f_o^0(E) = (1-\mathcal{T}) f_{\mu_1}^0(E) + \mathcal{T} f_{\mu_2}^0(E)$$

- But the length of propagation is 0.8 micro meter...

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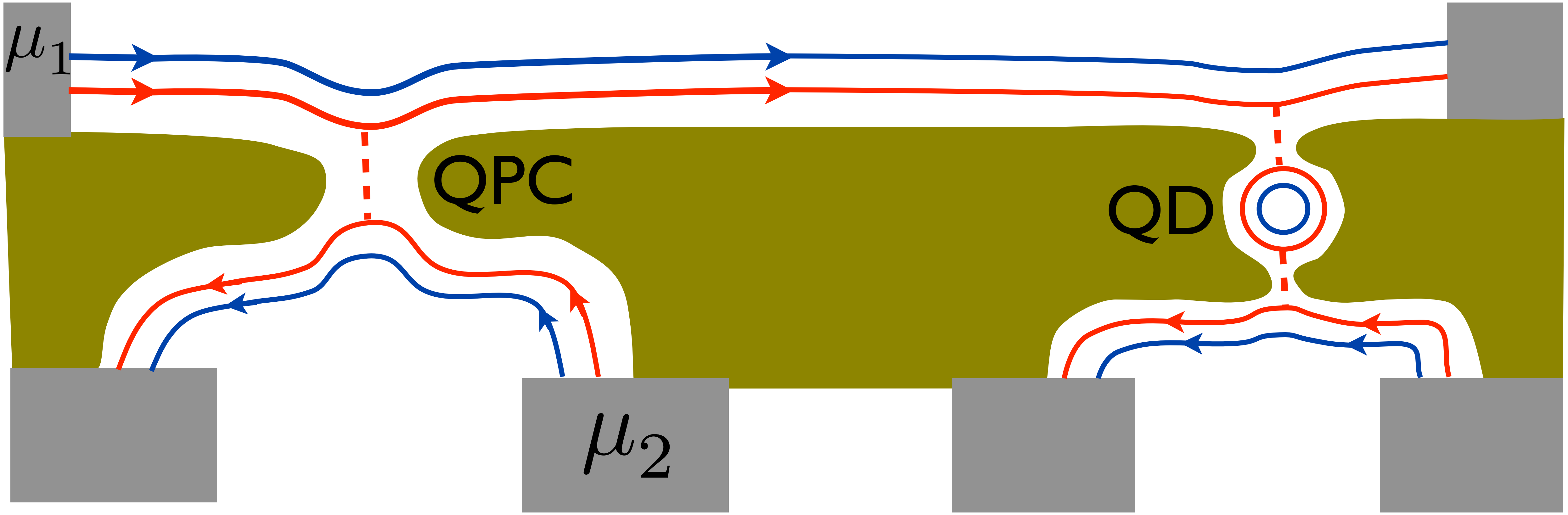
Note: Experiments have **two** edge states (filling factor 2)

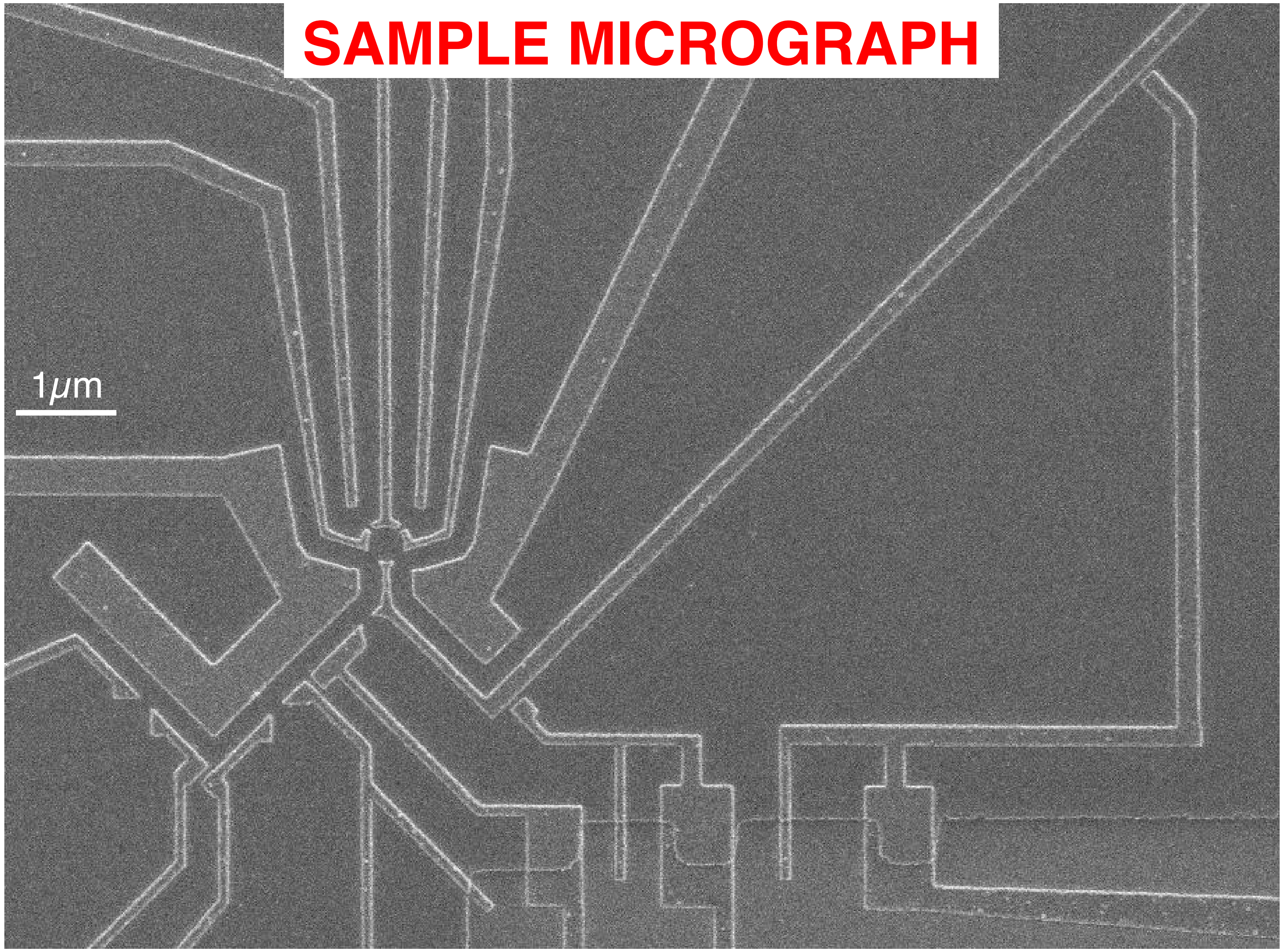


For only **one** edge state the experiments are more difficult....

Second Experiment: Relaxation in edge states

H. le Sueur, C. Altimiras, U. Gennser, Cavanna, D. Mailly and F. Pierre
(unpublished, arXiv:1003.4962)

Goal: Vary **length** of propagation between QPC and QD



courtesy of F. Pierre *et al*

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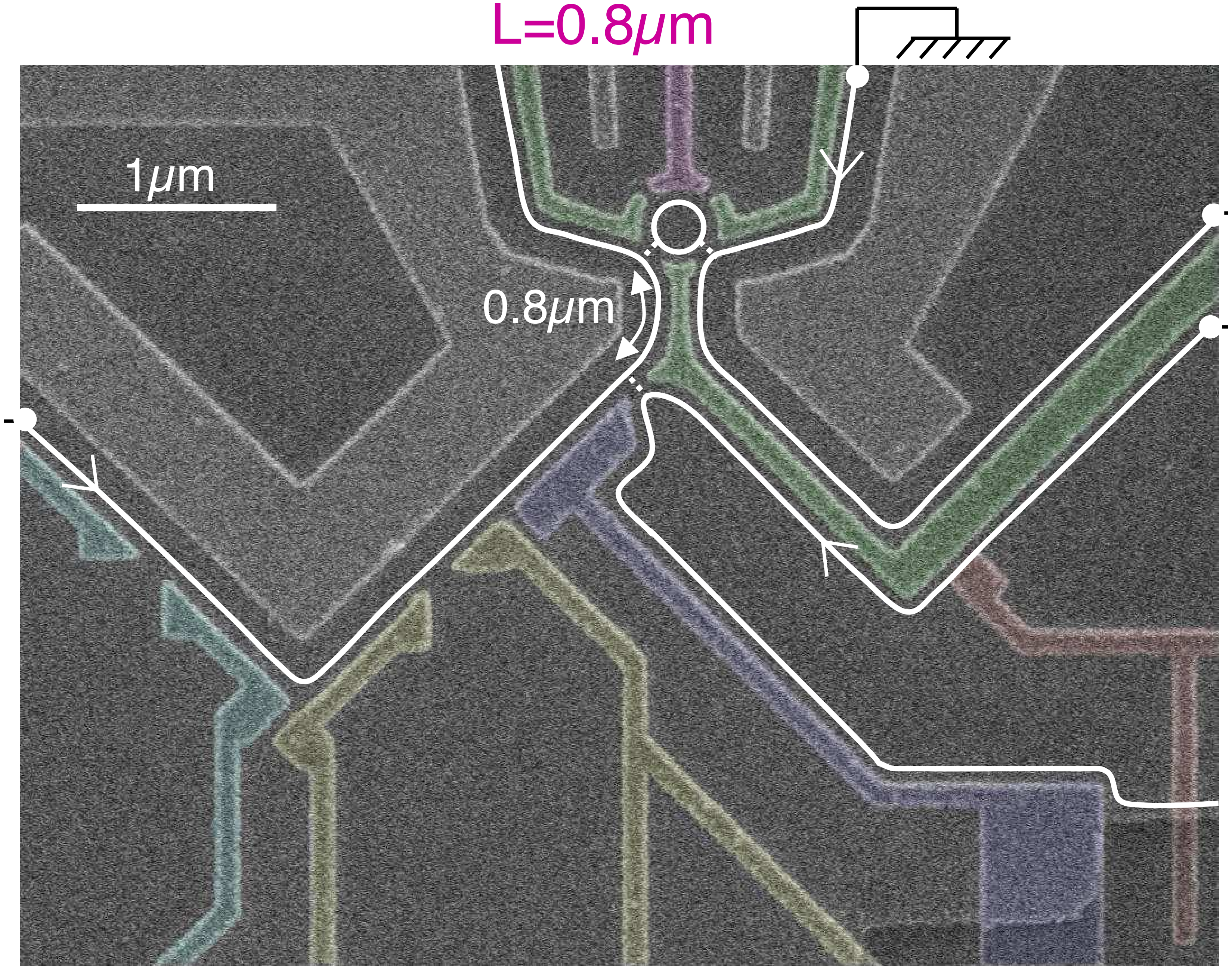

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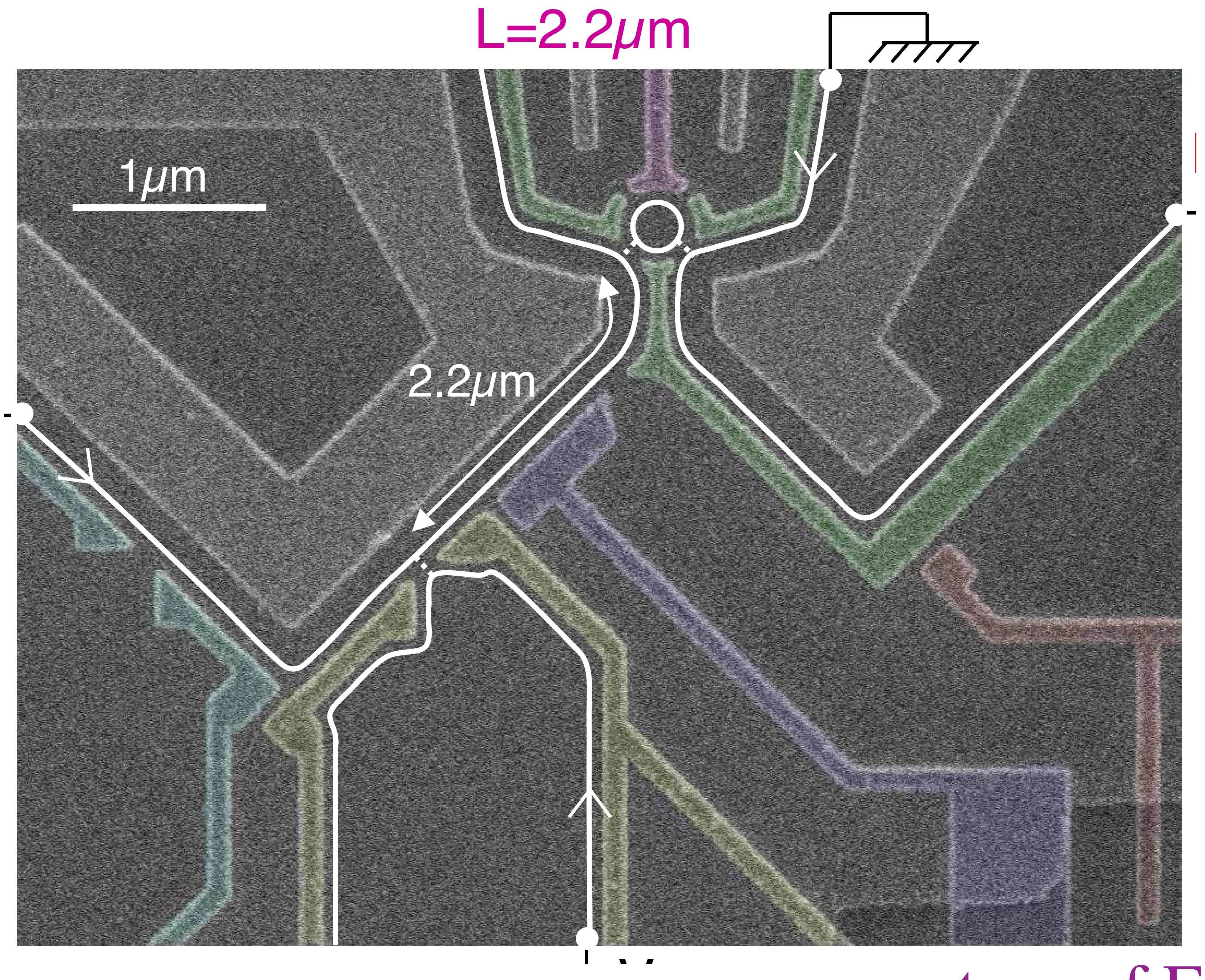

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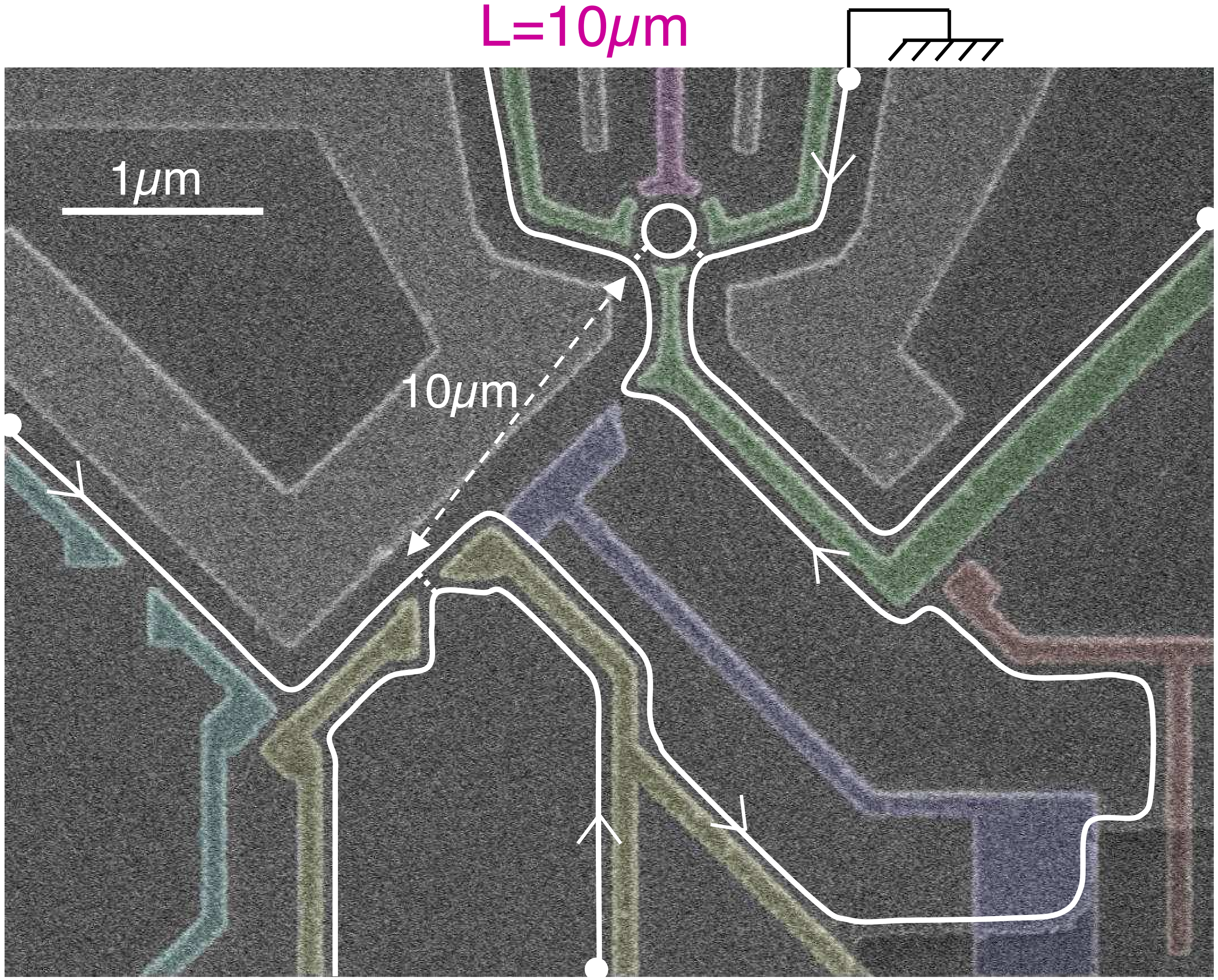

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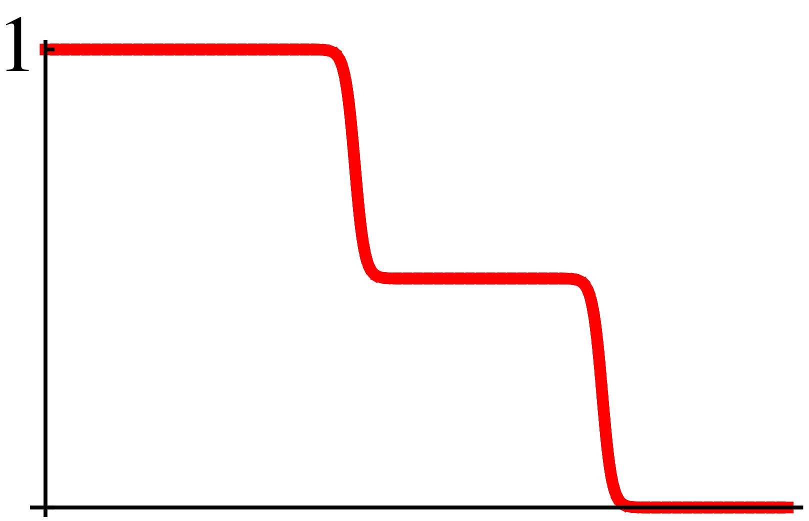
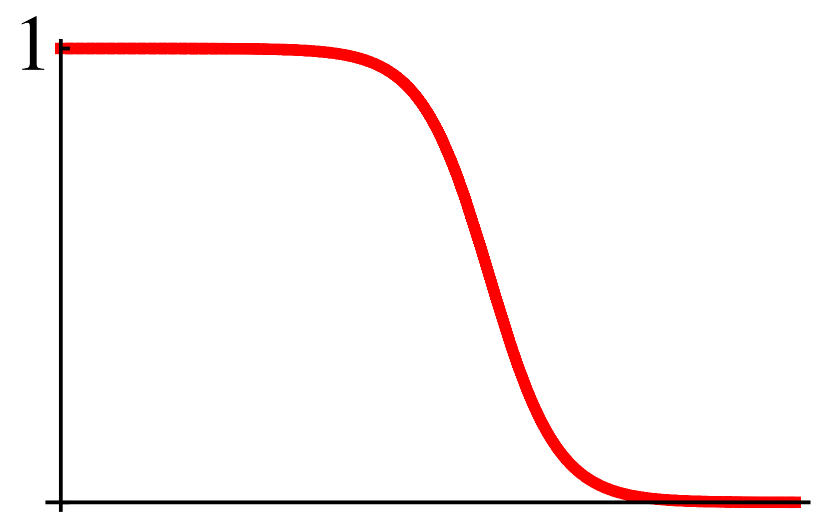
$$L = \; 0.8\mu\mathrm{m}, \; 2.2\mu\mathrm{m}, \; 4\mu\mathrm{m}, \; 10\mu\mathrm{m}, \; 30\mu\mathrm{m}$$

No relaxation!

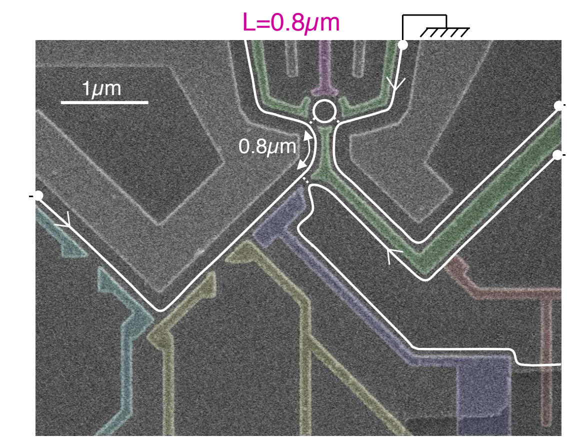
Full relaxation!

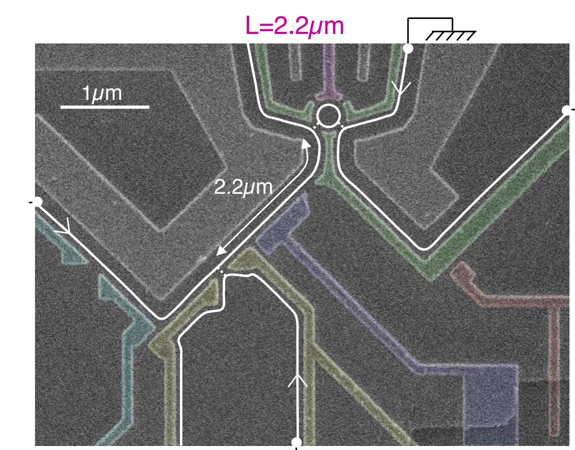


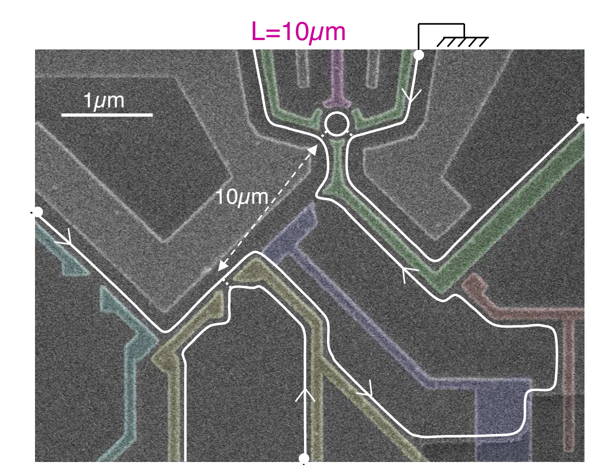


Non-equilibrium
step distribution!

Heated equilibrium
Fermi function!





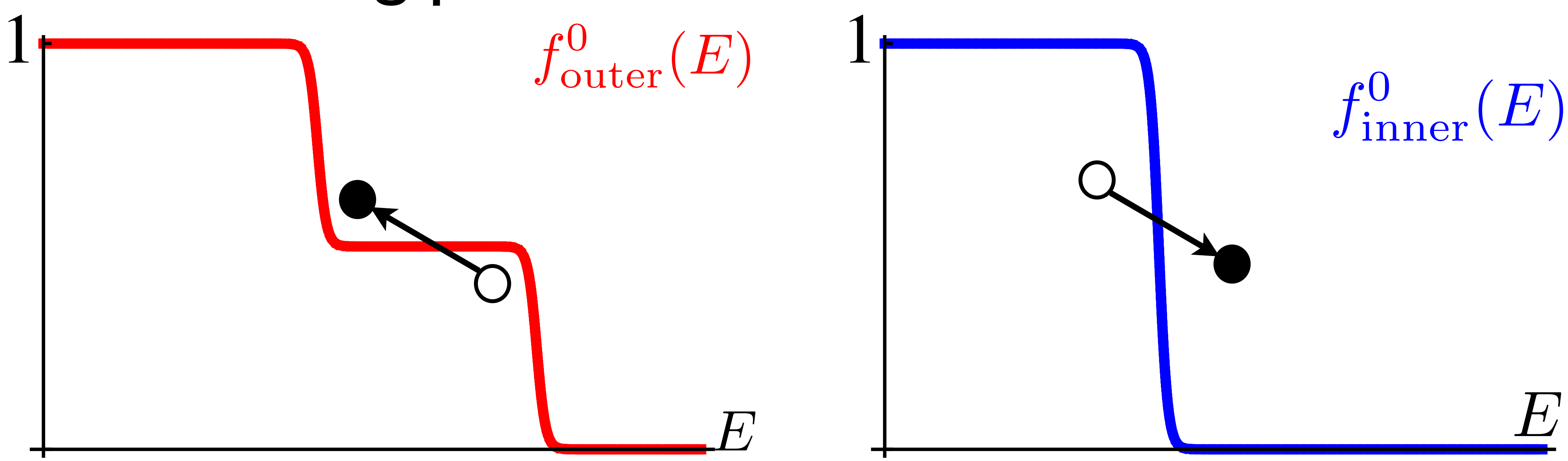


courtesy of F. Pierre *et al*

How does the electronic distribution function relax?

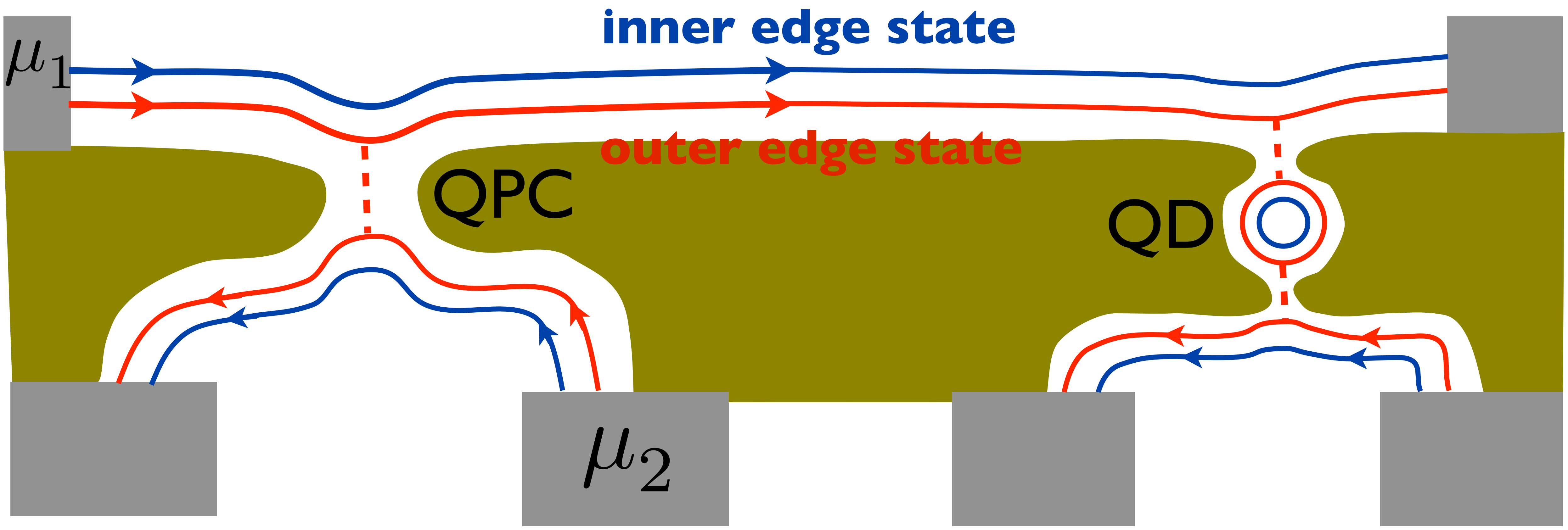
Relaxation due to inter edge state interaction

Basic scattering process:



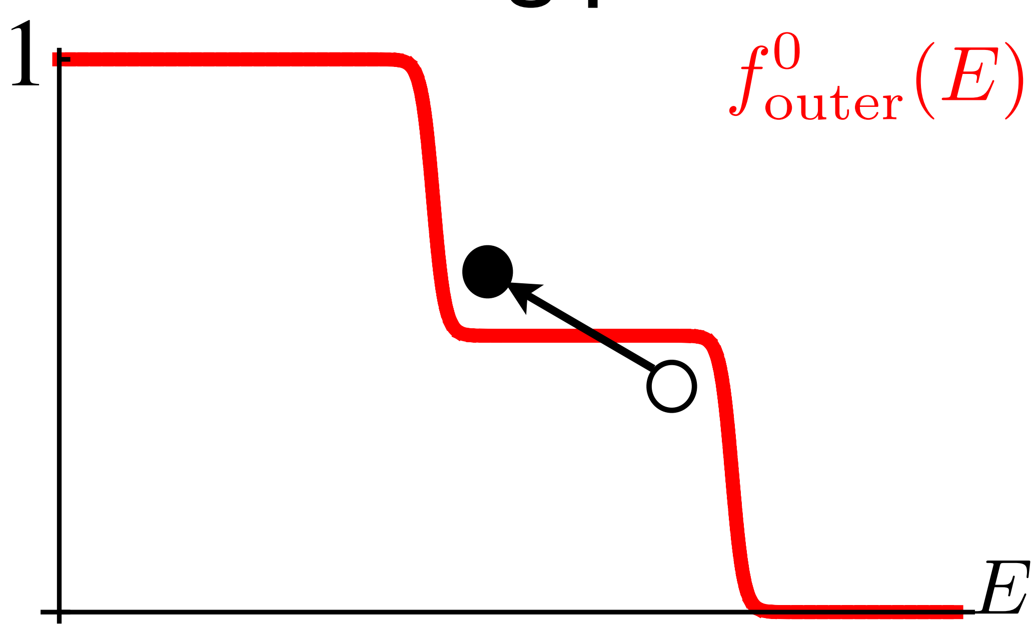
Loses energy

Gains energy

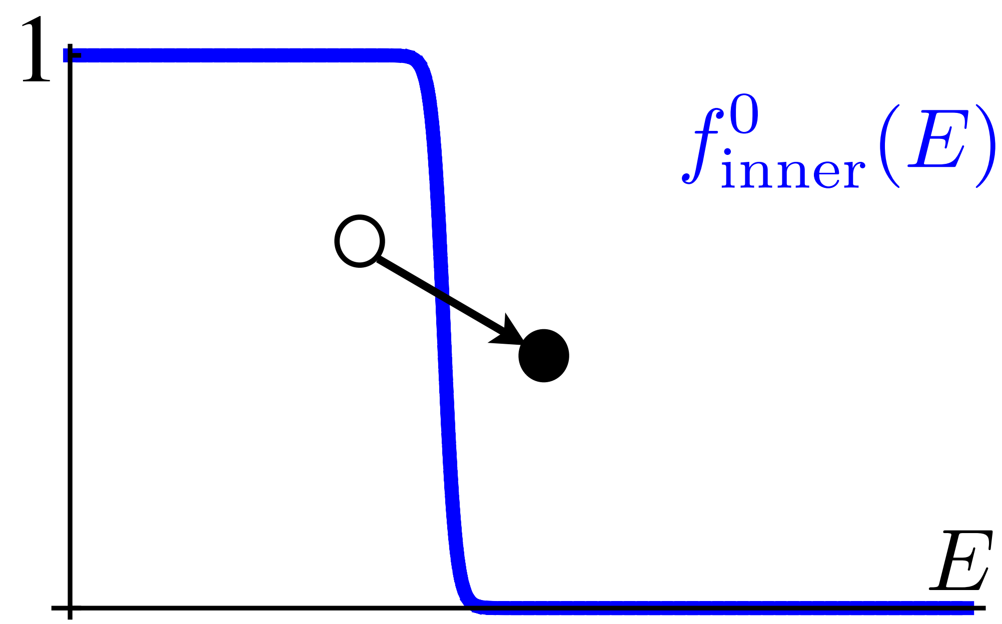


Relaxation due to inter edge state interaction

Basic scattering process:



Loses energy



Gains energy

Kinetic constrains:

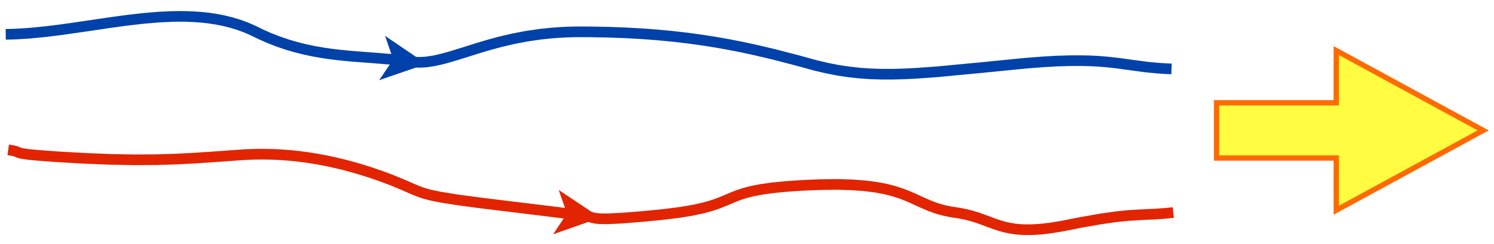
Energy and momentum conservation in 1D → (almost) **No** phase space for electron-electron scattering

However:

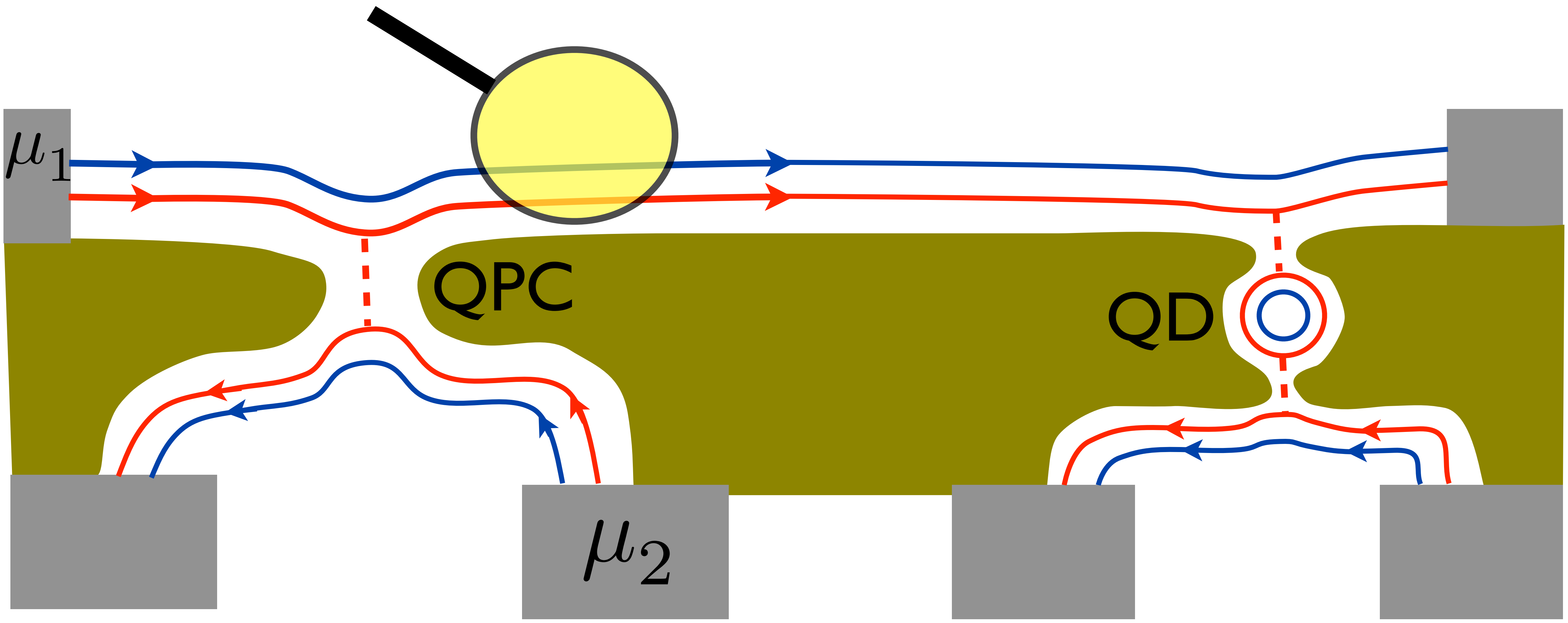
Lack of translational invariance → Momentum **breaking** processes allowed! → Available phase space!!

Lack of translational invariance??

Yes, ***Real*** edge state follow the equipotential lines!



Lack of translational invariance!!!



Relaxation description

Scattering basis states: $\chi_{\alpha E}(x) \qquad \alpha = o, i$

Hamiltonian: $H = H_0 + H_{\rm int}$

Single-particle part: $H_0 = \sum_\alpha \int \mathrm{d}E\ E\ a^\dagger_{\alpha E} a_{\alpha E}$

Inter edge state interaction:

$$H_{\rm int} = \frac{1}{2}\sum_\alpha \int \mathrm{d}E_1 \mathrm{d}E_2 \mathrm{d}E_{1'} \mathrm{d}E_{2'} V^{\alpha\bar\alpha}_{E_{1'},E_{2'},E_1 E_2} a^\dagger_{\alpha E_{1'}} a^\dagger_{\bar\alpha E_{2'}} a_{\bar\alpha E_2} a_{\alpha E_1}$$

Interaction matrix element!
-includes the non-translational invariance

Weak interaction approach:

Perturbation theory in the interaction!

Note: The edges are an open system connected to large Fermi reservoirs, i.e. *not* an isolated 1D infinite system.

Perturbative result I

Distribution function: $f_\alpha^{(2)}(E) = f_\alpha^0(E) + \delta f_\alpha^{(2)}(E)$

$$\delta f_\alpha^{(2)}(E) = (2\pi)^2\hbar \int \mathrm{d}\omega\, \mathrm{d}E' |V^{\alpha\bar{\alpha}}_{EE'+\hbar\omega, E+\hbar\omega E'}|^2$$
$$\times \left[f_\alpha^0(E+\hbar\omega)[1-f_\alpha^0(E)] f_{\bar{\alpha}}^0(E')[1-f_{\bar{\alpha}}^0(E'+\hbar\omega)] - f_\alpha^0(E)[1-f_\alpha^0(E+\hbar\omega)] f_{\bar{\alpha}}^0(E'+\hbar\omega)[1-f_{\bar{\alpha}}^0(E')] \right]$$

Geometrical Averaging
over the edge state irregularities

$$\overline{|V^{\alpha\bar{\alpha}}_{E_{1'}E_{2'},E_1E_2}|^2}_{\Delta k \neq 0} \propto e^{\left[-(\Delta k \ell_p)^2/2\right]}$$

Δk = amount of broken momentum conservation

ℓ_p = momentum breaking correlation length

Side remark: Noise description possible!

$$\delta f_\alpha^{(2)}(E) = 2\pi \int_{-\infty}^{\infty} \mathrm{d}\omega \left[f_\alpha^0(E+\hbar\omega)[1-f_\alpha^0(E)] S^a_{\delta U_\alpha \delta U_\alpha}(E, E+\hbar\omega, \omega) - f_\alpha^0(E)[1-f_\alpha^0(E+\hbar\omega)] S^e_{\delta U_\alpha \delta U_\alpha}(E+\hbar\omega, E, \omega) \right]$$

Absorption and ***emission***
potential fluctuation spectra

Perturbative result II: Analytic results

Distribution function: $$f_\alpha^{(2)}(E) = f_\alpha^0(E) + \delta f_\alpha^{(2)}(E)$$

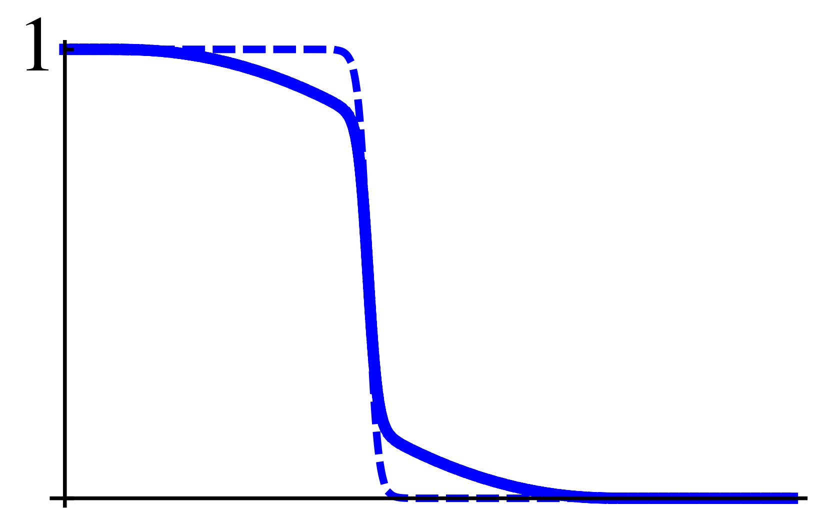
Limit of *no*-momentun conservation: $(\ell_p \longrightarrow 0\ )$

$$\delta f_o^{(2)}(E) = -\gamma^2 \mathcal{T}(1-\mathcal{T})(\mu_2-\mu_1)[f^0_{\mu_2}(E) - f^0_{\mu_1}(E)]\Big[E - \frac{1}{2}(\mu_1+\mu_2)\Big]$$

QPC noise!

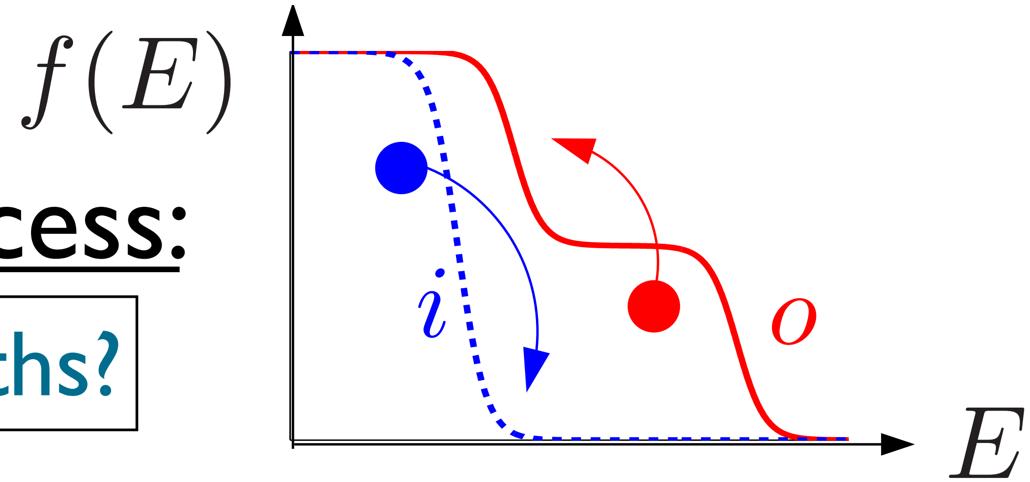
$$\delta f_i^{(2)}(E) = \frac{\gamma^2}{2}\mathcal{T}(1-\mathcal{T})\Big\{ -[f^0_{\mu_i}(E) - f^0_{\mu_i^-}(E)]\big[(\pi k_{\mathrm{B}} T)^2 + (E-\mu_i^-)^2\big] + [f^0_{\mu_i^+}(E) - f^0_{\mu_i}(E)]\big[(\pi k_{\mathrm{B}} T)^2 + (E-\mu_i^+)^2\big]\Big\}$$

Outer edge state



Inner edge state

Fits well the basic scattering process:



However: What to do for longer lengths?

Beyond perturbation Theory I: Basic considerations

Expectation: Fully relaxed distributions are Fermi functions

What are the effective chemical potentials and temperatures?

No particle exchange between the edge states ⇒

$$\mu_{\text{out}}^{L=\infty} = \mu_1 + \mathcal{T}(\mu_2 - \mu_1)$$
$$\mu_{\text{inner}}^{L=\infty} = \mu_{\text{inner}}^{L=0}$$

Energy exchange between the edge states, *but* conservation of total energy ⇒ Conservation of total **Energy current**

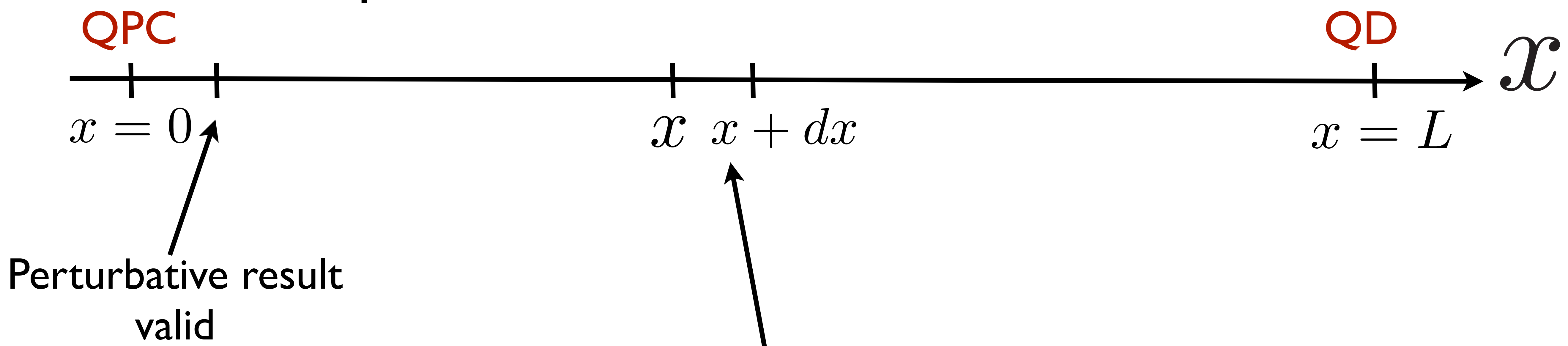
\+ Maximizing the ***entropy***

⇒

$$k_B T_{\text{inner}}^{L=\infty} = k_B T_{\text{outer}}^{L=\infty} = \sqrt{(k_B T)^2 + \frac{3}{2\pi^2}\mathcal{T}(1-\mathcal{T})(\mu_1-\mu_2)^2}$$

Beyond perturbation Theory II

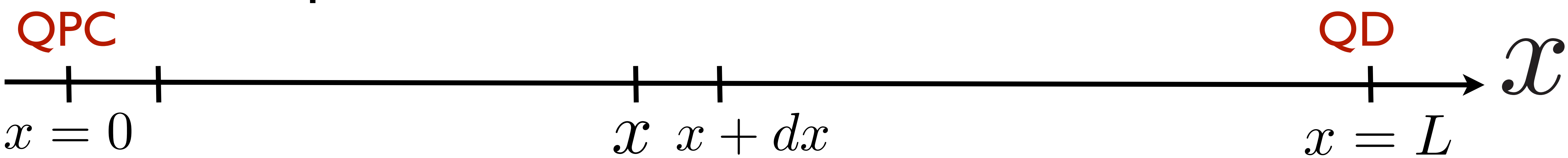
Iteration of the perturbative result:

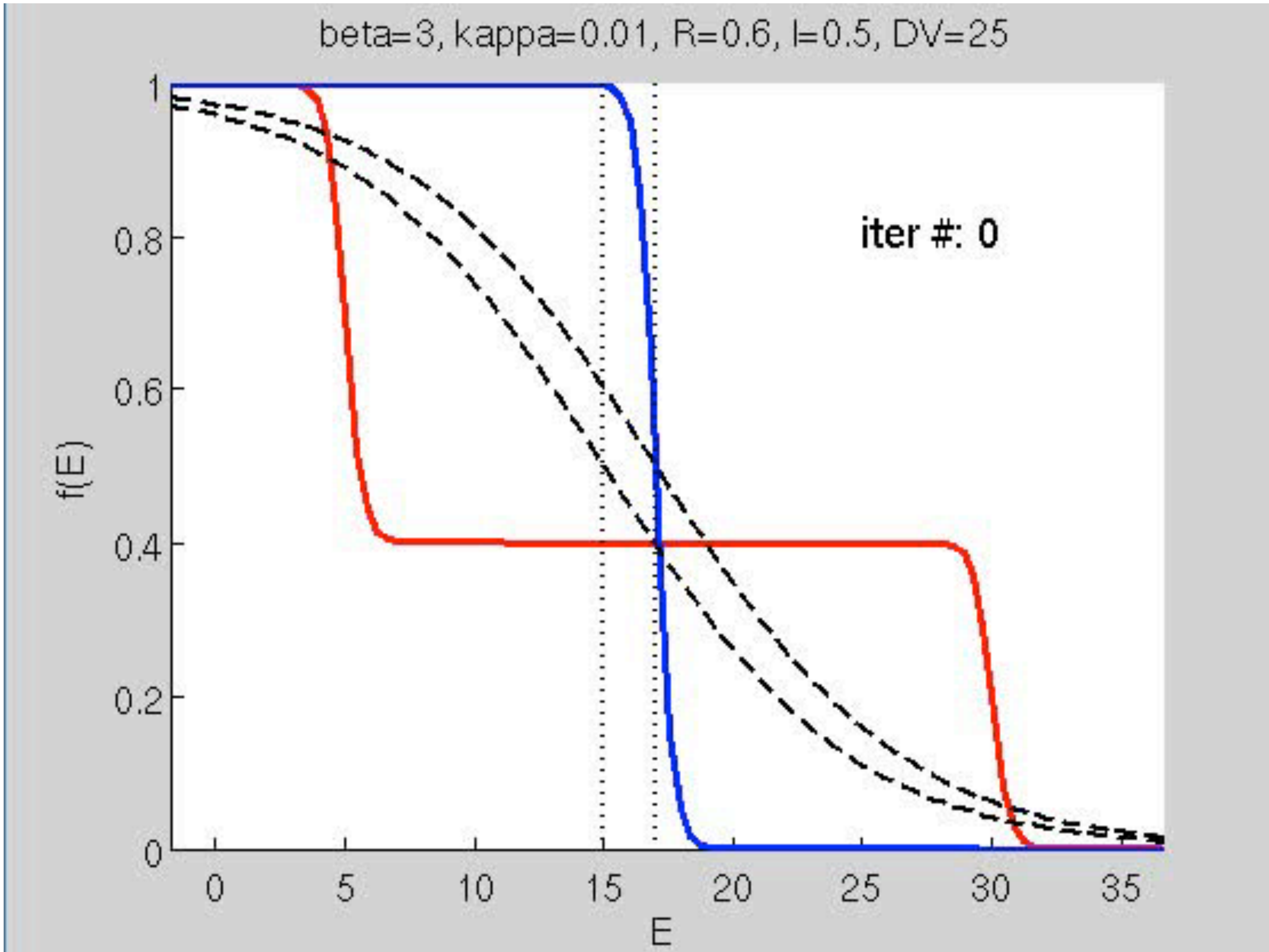


At each step,
insert the previous result as initial conditions.

Beyond perturbation Theory III

Iteration of the perturbative result:

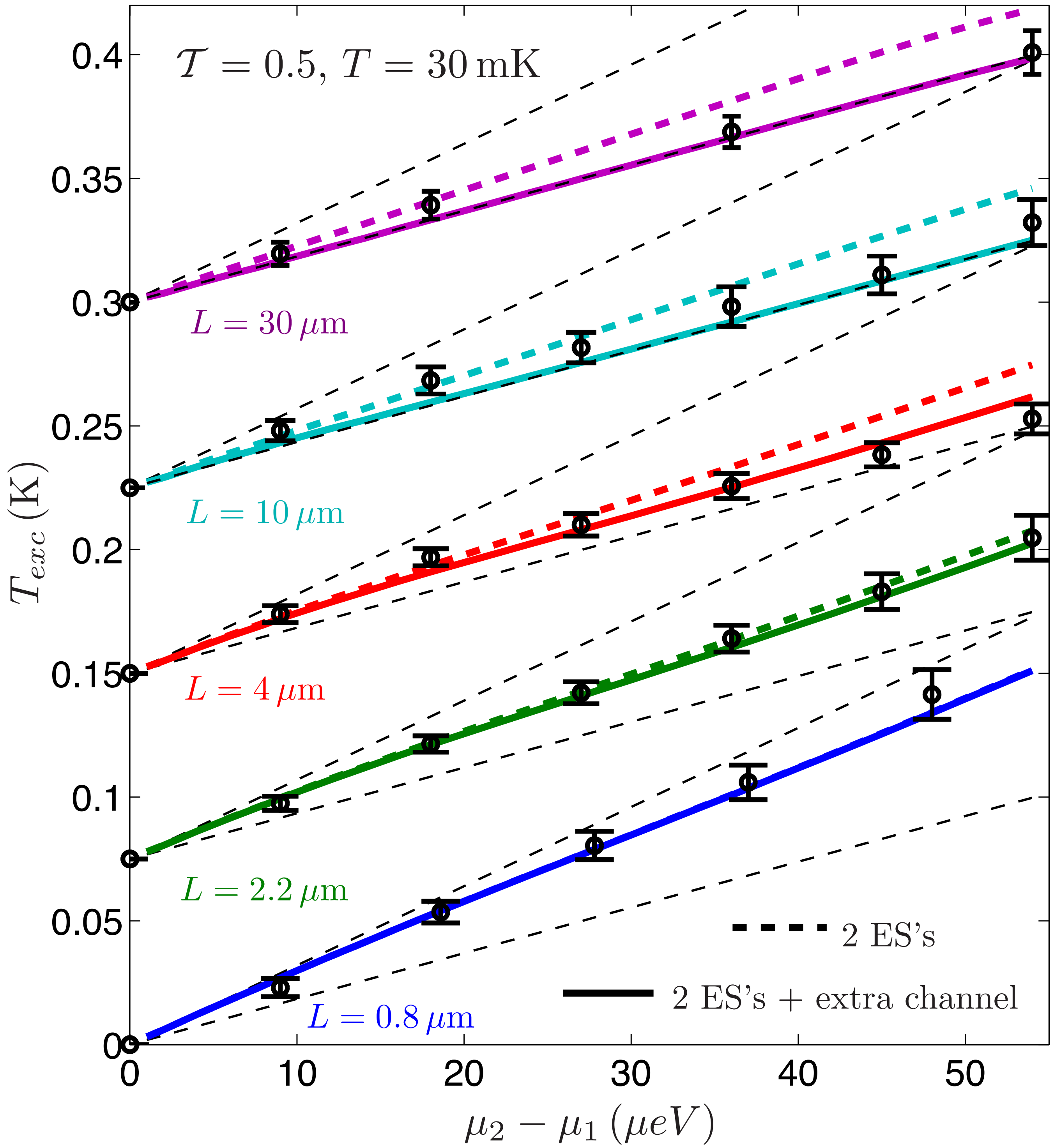




Open questions: Extra relaxation channel?

Excess temperature: $k_{\mathrm{B}}T_{exc,\alpha} \equiv \sqrt{\frac{6}{\pi^2}\int \mathrm{d}E\big[f_\alpha(E) - \theta(\tilde{\mu}_\alpha - E)\big](E-\tilde{\mu}_\alpha) - (k_{\mathrm{B}}T)^2}$

- measures the energy in the non-thermal excitations in $f_\alpha(E)$



Experimental Data courtesy of F. Pierre *et al*

Summary

- Create and measure an non-equilibrium electron distribution of an edge state

- Small review of the experiments by *Pierre et al.*

- Description of the physics and modeling of the relaxation

- Seems to agree with the experiment....
  But: What could the extra relaxation channel be?

Ref.: Lunde. Nigg and Büttiker, Phys. Rev. B Rapid Comm. **81**, 041311 (2010)

see also: Degiovanni *et al*. Phys. Rev. B Rapid Comm. **81** 121302 (2010)

Acknowledgement: **C. Altimiras**, **H. le Sueur** and **F. Pierre** for discussions and sharing experimental data and knowlegde, and **P. Degiovanni** for theoretical discussions.